Towards identifying a minimal flavor symmetry behind neutrino oscillations

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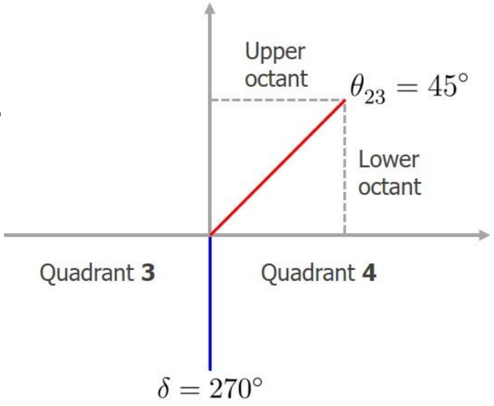
- experimental hints
- mu-tau reflection
- seesaw extension
- concluding remark

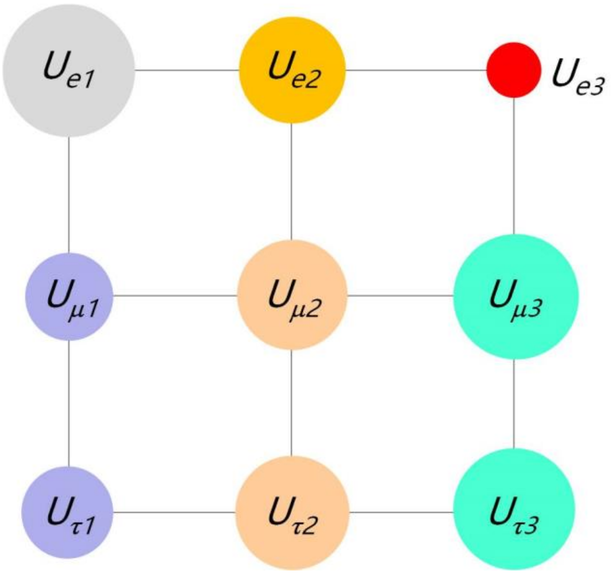


ZZX, JHEP 06 (2022) 034; e-print: 2203.14185

ICHEP 2022, 6—13.7.2022, Bologna, Italy

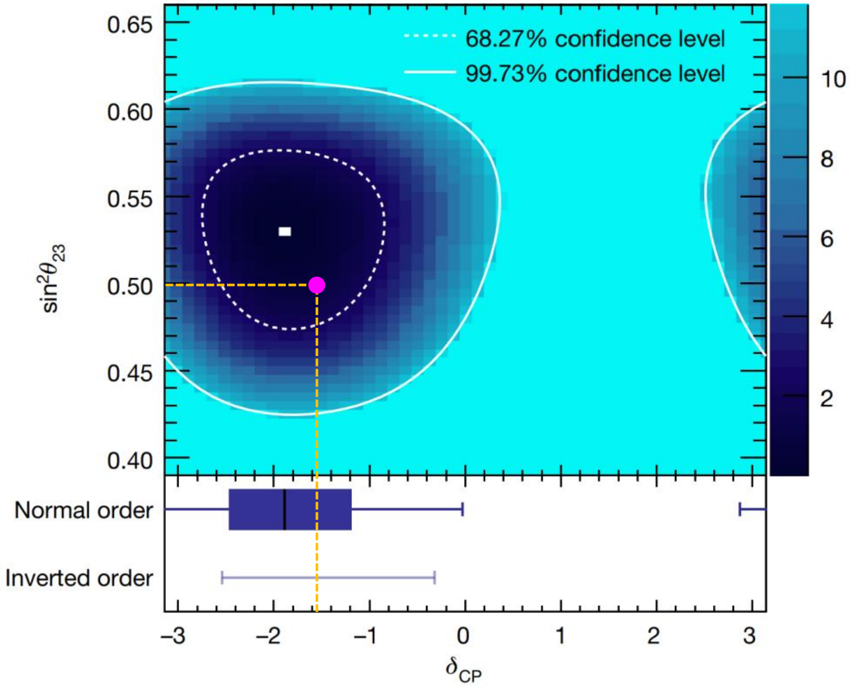
Hints of current experimental data

Nine Moduli of the PMNS matrix elements constrained from the three-flavor global analysis: F. Capozzi et al (2107.00532); M. Gonzalez-Garcia et al (2111.03086)



the area of each circle = the element's modulus

The T2K data on the octant of theta (23) and the quadrant of delta (Nature 2020)



A mu-tau flavor permutation symmetry

Slight deviations from $\theta_{23} = \pi/4$ and $\delta = -\pi/2$

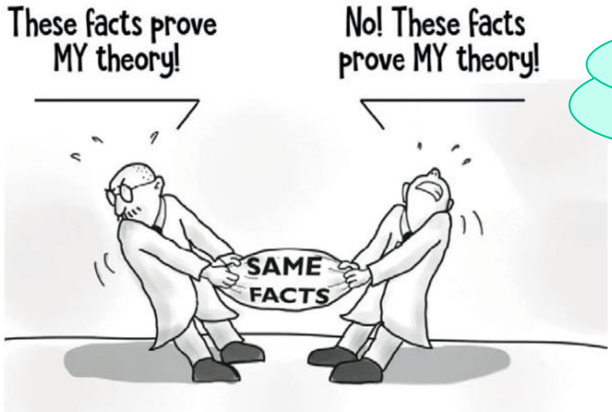
A key conjecture: Behind the observed pattern of lepton flavor mixing should lie a kind of non-Abelian discrete flavor symmetry group, whose CG coefficients might essentially determine some elements of the PMNS matrix

Which symmetry is closer to the truth?

So far a lot of **flavor symmetries** for model building have been considered [two recent reviews, ZZX, 1909.09610 (Phys. Rept. 2020); F. Feruglio, A. Romanino, 1912.06028 (Rev. Mod. Phys. 2021)]

S_3 , S_4 , A_4 , A_5 , D_4 , D_7 , T_7 , T′, Δ(27), Δ(48), …
$U(1)_F$, $SU(2)_F$, modular, translational, ….

A **guiding principle** (**bottom line**) of model building: compatible with current experimental data



You guys should choose the theory that is **most simple** and **straightforward**

a minimal flavor symmetry

symmetry of μ-τ reflection?

P. Harrison, W. Scott 2002

How minimal is minimal: it belongs to a simple symmetry group and its predictions are as close as possible to data.

What is the mu-tau reflection symmetry?

It is a working flavor symmetry requiring the effective Majorana neutrino mass term to be invariant under the transformations of left-handed neutrino fields [ZZX, Z.H. Zhao, 1512.04207 (RPP, 1996)]:

$$\boxed{-\mathcal{L}_{\rm mass} = \frac{1}{2}\overline{\nu_{\rm L}}\, M_\nu \,(\nu_{\rm L})^c + {\rm h.c.}} \longleftarrow \boxed{\nu_{e{\rm L}} \to (\nu_{e{\rm L}})^c\,, \quad \nu_{\mu{\rm L}} \to (\nu_{\tau{\rm L}})^c\,, \quad \nu_{\tau{\rm L}} \to (\nu_{\mu{\rm L}})^c}$$

- **Traditional CP transformation**

$$(t, \mathbf{x}) \longrightarrow (t, -\mathbf{x})$$

$$\left\{\begin{array}{l} \nu_{e{\rm L}} \longrightarrow (\nu_{e{\rm L}})^c \\ \nu_{\mu{\rm L}} \longrightarrow (\nu_{\mu{\rm L}})^c \\ \nu_{\tau{\rm L}} \longrightarrow (\nu_{\tau{\rm L}})^c \end{array}\right.$$

- **mu-tau-interchanging CP transformation**

$$(t, \mathbf{x}) \longrightarrow (t, -\mathbf{x})$$

$$\left\{\begin{array}{l} \nu_{e{\rm L}} \longrightarrow (\nu_{e{\rm L}})^c \\ \nu_{\mu{\rm L}} \longrightarrow (\nu_{\tau{\rm L}})^c \\ \nu_{\tau{\rm L}} \longrightarrow (\nu_{\mu{\rm L}})^c \end{array}\right.$$

Invariance: $\boxed{M_\nu = M_\nu^*}$ **CP conserving** $\boxed{M_\nu = \mathcal{P} M_\nu^* \mathcal{P}}$ **CP violating**

Constraints on the flavor structure of three Majorana neutrinos:

$$M_\nu = \begin{pmatrix} C & D & D^* \\ D & A & B \\ D^* & B & A^* \end{pmatrix}$$

$$\nu_e \qquad \nu_\mu \leftrightarrow \nu_\tau^c$$

$$\mathcal{P} = \mathcal{P}^T = \mathcal{P}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

mu-tau permutation

$$\theta_{23} = \pi/4\,,\ \delta = \pm\pi/2$$

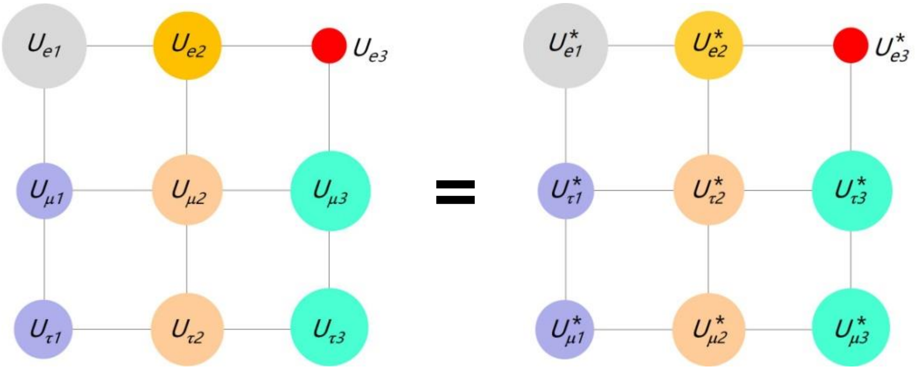
How to identify the mu-tau reflection?

Different from the previous works, here let's start purely from the PMNS matrix constrained by data

A data-driven conjecture: $\boxed{U = \mathcal{P} U^* \zeta}$

$$\mathcal{P} = \mathcal{P}^T = \mathcal{P}^\dagger = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\zeta = \mathrm{Diag}\{\eta_1, \eta_2, \eta_3\} \text{ with } \eta_i = \pm 1$$



In the basis where the flavor eigenstates of 3 charged leptons are identified with their mass eigenstates, we have

the effective Majorana neutrino mass matrix

$$\boxed{M_\nu = U D_\nu U^T = \mathcal{P} U^* \zeta D_\nu \zeta U^\dagger \mathcal{P} = \mathcal{P}\left(U D_\nu U^T\right)^* \mathcal{P} = \mathcal{P} M_\nu^* \mathcal{P}}$$

$$D_\nu \equiv \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

Substitute this into the mass term:

$$\boxed{-\mathcal{L}_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}}\, M_\nu \left(\nu_{\rm L}\right)^c + {\rm h.c.}} \Rightarrow \boxed{-\mathcal{L}'_{\rm mass} = \frac{1}{2} \overline{\nu_{\rm L}} \left(\mathcal{P} M_\nu^* \mathcal{P}\right) \left(\nu_{\rm L}\right)^c + {\rm h.c.} = \frac{1}{2} \overline{\left[\mathcal{P}(\nu_{\rm L})^c\right]}\, M_\nu \left[\mathcal{P}\nu_{\rm L}\right] + {\rm h.c.}}$$

Then the invariance $\mathcal{L}'_{\rm mass} = \mathcal{L}_{\rm mass}$ **leads us to the μ-τ reflection transformation** $\boxed{\nu_{\rm L} \rightarrow \mathcal{P}(\nu_{\rm L})^c}$ **. QED**

In the seesaw case: a novel prediction

In the canonical seesaw mechanism with three right-handed neutrinos and lepton number violation

Leptonic weak cc-interaction:

$$\boxed{-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\,\overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}}\,\gamma^\mu \left[U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix}_{\rm L} \right] W^-_\mu + {\rm h.c.}}$$

$$UU^\dagger + RR^\dagger = I$$

neutrino oscillations ⟵ **light**   **heavy** ⟶ **collider physics**, **leptogenesis**, **LFV**

The 3 × 3 PMNS matrix is not exactly unitary, but precision electroweak measurements and neutrino oscillation data have constrained its unitarity to be good at the ≤ 1 % level. So even in the presence of slight unitarity violation one may still make the conjecture (ZZX, 2203.14185):

$$|U_{\mu i}| = |U_{\tau i}| \quad (i = 1, 2, 3) \longrightarrow \boxed{U = \mathcal{P} U^* \zeta}$$

$$D_N \equiv {\rm Diag}\{M_1, M_2, M_3\}$$

The exact seesaw bridge:

$$\boxed{U D_\nu U^T + R D_N R^T = \mathbf{0}}$$

$$D_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$$

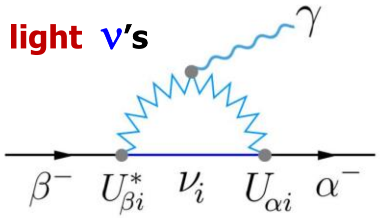
$$\boxed{R = \mathcal{P} R^* \zeta'} \longrightarrow |R_{\mu i}| = |R_{\tau i}|$$

novel prediction

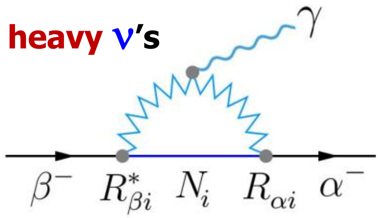
An application of this prediction

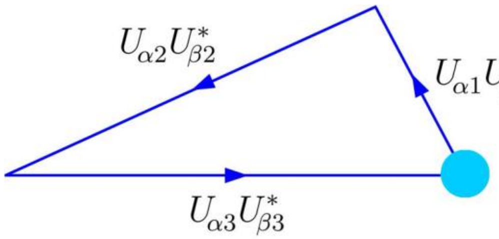
It may help constrain unitarity of the 3×3 PMNS matrix through the charged lepton flavor violation

light ν's



heavy ν's





$$= \sum_{i=1}^{3}\left(R_{\alpha i}R^*_{\beta i}\right)$$

In the full seesaw (ZZX, D. Zhang, 2009.09717) or its EFT with one-loop matching (D. Zhang, S. Zhou, 2107.12133):

$$\xi_{\alpha\beta} \equiv \frac{\Gamma(\beta^- \to \alpha^- + \gamma)}{\Gamma(\beta^- \to \alpha^- + \overline{\nu}_\alpha + \nu_\beta)} \simeq \frac{3\alpha_{\rm em}}{2\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6} + \frac{1}{4}\cdot\frac{m_i^2}{M_W^2}\right) - \frac{1}{3}\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right|^2 \simeq \frac{3\alpha_{\rm em}}{8\pi}\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right|^2$$

which allows us to constrain the unitarity hexagon using current experimental data on three radiative cLFV decays:

$$\left|\sum_{i=1}^{3} U_{\alpha i}U^*_{\beta i}\right| = \left|\sum_{i=1}^{3} R_{\alpha i}R^*_{\beta i}\right| \simeq \sqrt{\frac{8\pi\xi_{\alpha\beta}}{3\alpha_{\rm em}}} \simeq 33.88\sqrt{\xi_{\alpha\beta}}$$

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\mu i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\mu i}\right| < \underline{2.20\times 10^{-5}}$$

$$\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\tau i}\right| < \underline{1.46\times 10^{-2}}$$

$$\left|\sum_{i=1}^{3} U_{\mu i}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{\mu i}R^*_{\tau i}\right| < 1.70\times 10^{-2}$$

Now imposing the mu-tau reflection symmetry, we have

$$\boxed{\left|\sum_{i=1}^{3} U_{ei}U^*_{\tau i}\right| = \left|\sum_{i=1}^{3} R_{ei}R^*_{\tau i}\right| < 2.20\times 10^{-5}}$$

Is there the same flavor symmetry behind?

Yes, let's consider the neutrino mass term in the canonical seesaw mechanism: $M_{\rm D} \equiv Y_\nu \langle H \rangle$

$$\boxed{-\mathcal{L}_\nu = \overline{\ell_{\rm L}} Y_\nu \widetilde{H} N_{\rm R} + \frac{1}{2}\overline{(N_{\rm R})^c} M_{\rm R} N_{\rm R} + {\rm h.c.}} \xrightarrow{\text{SSB}} \boxed{-\mathcal{L}'_\nu = \frac{1}{2}\overline{[\nu_{\rm L} \;\; (N_{\rm R})^c]} \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{bmatrix} (\nu_{\rm L})^c \\ N_{\rm R} \end{bmatrix} + {\rm h.c.}}$$

Diagonalizing the 6×6 neutrino mass matrix:

$$\begin{pmatrix} U & R \\ S & Q \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^T & M_{\rm R} \end{pmatrix} \begin{pmatrix} U & R \\ S & Q \end{pmatrix}^* = \begin{pmatrix} D_\nu & \mathbf{0} \\ \mathbf{0} & D_N \end{pmatrix} \xrightarrow{\text{exact seesaw}} \boxed{U D_\nu U^T + R D_N R^T = \mathbf{0}}$$

Unitarity:

$$\begin{cases} UU^\dagger + RR^\dagger = SS^\dagger + QQ^\dagger = I \\ U^\dagger U + S^\dagger S = R^\dagger R + Q^\dagger Q = I \\ US^\dagger + RQ^\dagger = U^\dagger R + S^\dagger Q = \mathbf{0} \end{cases}$$

seesaw transfer of transformations

$$\boxed{\begin{array}{c} \underbrace{U = \mathcal{P} U^* \zeta \longrightarrow R = \mathcal{P} R^* \zeta'} \\ \Downarrow \\ \overbrace{S = \mathcal{T} S^* \zeta \qquad Q = \mathcal{T} Q^* \zeta'} \end{array}}$$

T **can be arbitrary unitary transformation**

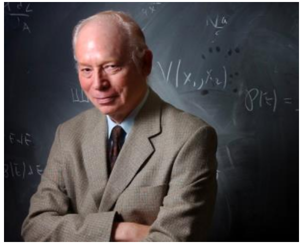
$$\boxed{M_{\rm D} = \mathcal{P} M_{\rm D}^* \mathcal{T} \; , \quad M_{\rm R} = \mathcal{T}^T M_{\rm R}^* \mathcal{T}}$$

Substitute these into the above neutrino mass term and require it to be invariant, we obtain the answer

$$\boxed{\nu_{\rm L} \to \mathcal{P} (\nu_{\rm L})^c \; , \quad N_{\rm R} \to \mathcal{T}^* (N_{\rm R})^c}$$

Are we really in the landscape?

The seesaw picture is well consistent with the spirit of Weinberg's EFT with a unique d=5 operator.

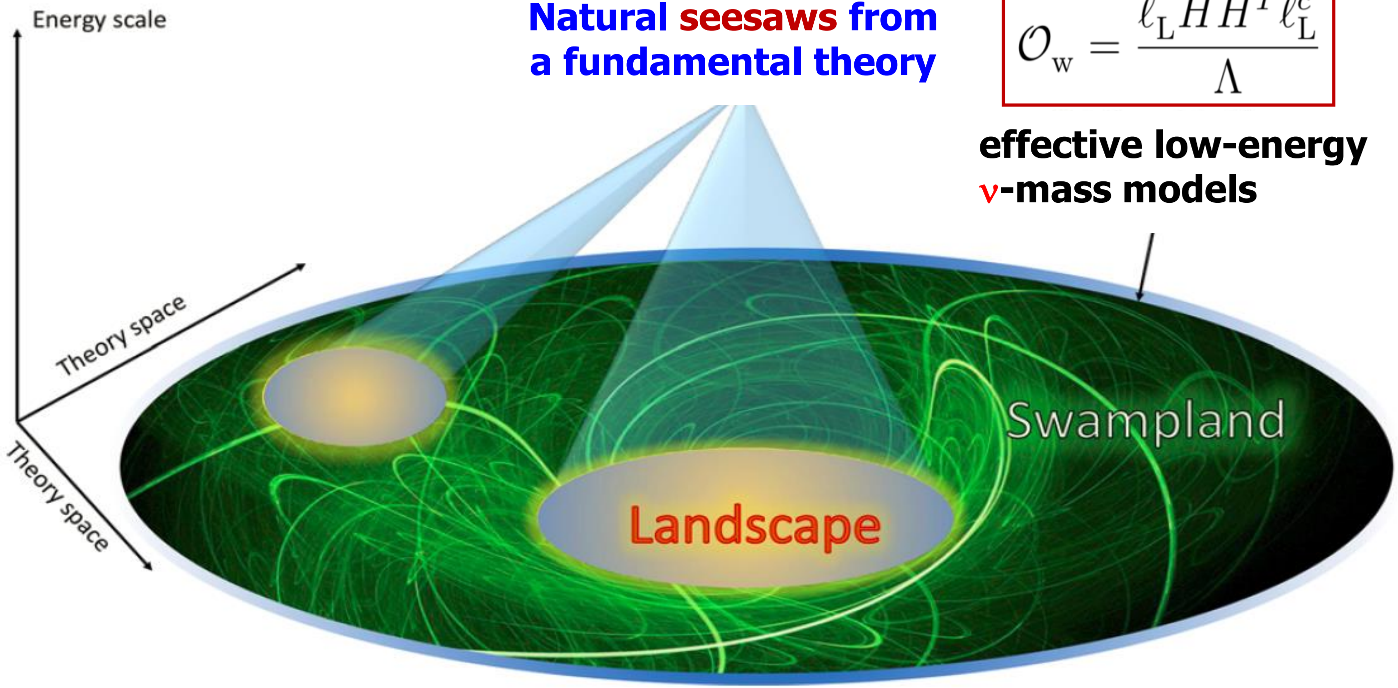


Natural seesaws from a fundamental theory

$$\mathcal{O}_{\rm W} = \frac{\overline{\ell_{\rm L}}\widetilde{H}\widetilde{H}^T \ell^c_{\rm L}}{\Lambda}$$

effective low-energy ν-mass models

Cumrun Vafa's swampland conjecture



A combination of the mu-tau reflection symmetry and the canonical seesaw at a super high energy scale may help understand the origin of tiny neutrino masses and large flavor mixing. This minimal flavor symmetry can be softly broken via renormalization-group-equation (RGE) running effects.

Concluding remarks

- Starting from current neutrino oscillation data, we have identified a minimal flavor symmetry of Majorana neutrinos: the mu-tau reflection symmetry.

- In this case one may resolve the octant of θ_{23} and the quadrant of δ via soft symmetry breaking, e.g., with the help of the RGE-induced quantum corrections.

- Combining this simple flavor symmetry with the *canonical seesaw* can help constrain the flavor textures and lead to some interesting consequences.

- Further precision measurements will test such a symmetry and the viable neutrino mass models into which it can be naturally embedded.