

Constraints on leptoquarks from charged-lepton-flavour-violating τ processes

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Based on the published work [Husek et al., 2022]



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Overview

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Preceding project

Use of the **SMEFT** up to **D-6 operators** to analyse τ -involved processes [Husek et al., 2021]

- Hadronic τ decays:

$$\begin{aligned}\tau &\rightarrow \ell P : & P = \pi^0, K^0, \bar{K}^0, \eta, \eta' \\ \tau &\rightarrow \ell PP : & PP = \pi^+ \pi^-, K^0 \bar{K}^0, K^+ K^-, \pi^+ K^-, K^+ \pi^- \quad (\ell = e, \mu) \\ \tau &\rightarrow \ell V : & V = \rho^0, \phi, \omega, K^{*0}, \bar{K}^{*0}\end{aligned}$$

- $\ell - \tau$ conversion in nuclei:

$$\ell \ N(A, Z) \longrightarrow \tau \ X \quad (\ell = e, \mu)$$

Current experimental knowledge on τ -involved processes

- Existing limits on hadronic τ decays
 - Belle and BaBar collaborations [Amhis et al., 2017]
 - Experimental prospects
 - Belle II → improve limits for hadronic τ decays by at least one order of magnitude
 - NA64 experiment at CERN → expected sensitivity on $\ell - \tau$ conversion in nuclei
- $$R_{\ell\tau} = \frac{\sigma(\ell + N \rightarrow \tau + X)}{\sigma(\ell + N \rightarrow \ell + X)} \sim 10^{-12} - 10^{-13}, \quad \ell = e, \mu$$

Global numerical analysis — implemented with HEPfit [De Blas et al., 2020] — of these processes based on the experimental limits from **Belle**, **Belle II** and tentatively **NA64**

$$\mathcal{L}_{SM} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_k C_k^{(5)} \mathcal{O}_k^{(5)} + \frac{1}{\Lambda^2} \sum_k C_k^{(6)} \mathcal{O}_k^{(6)} + \mathcal{O}\left(\frac{1}{\Lambda^3}\right) \quad [\text{Grzadkowski et al., 2010}]$$

CLFV operators ($\Lambda = \Lambda_{\text{CLFV}}$) relevant for our analysis [Husek et al., 2021]:

$\Lambda^2 \times \text{Coupling}$	Operator	$\Lambda^2 \times \text{Coupling}$	Operator
$C_{LQ}^{(1)}$	$(\bar{L}_p \gamma_\mu L_r) (\bar{Q}_s \gamma^\mu Q_t)$	$C_{LeQu}^{(1)}$	$(\bar{L}_p^j e_r) \varepsilon_{jk} (\bar{Q}_s^k u_t)$
$C_{LQ}^{(3)}$	$(\bar{L}_p \gamma_\mu \sigma^I L_r) (\bar{Q}_s \gamma^\mu \sigma^I Q_t)$	$C_{LeQu}^{(3)}$	$(\bar{L}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (\bar{Q}_s^k \sigma^{\mu\nu} u_t)$
C_{eu}	$(\bar{e}_p \gamma_\mu e_r) (\bar{u}_s \gamma^\mu u_t)$	$C_{e\varphi}$	$(\varphi^\dagger \varphi) (\bar{L}_p e_r \varphi)$
C_{ed}	$(\bar{e}_p \gamma_\mu e_r) (\bar{d}_s \gamma^\mu d_t)$	$C_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (e_p \gamma^\mu e_r)$
C_{Lu}	$(\bar{L}_p \gamma_\mu L_r) (\bar{u}_s \gamma^\mu u_t)$	$C_{\varphi L}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{L}_p \gamma^\mu L_r)$
C_{Ld}	$(\bar{L}_p \gamma_\mu L_r) (\bar{d}_s \gamma^\mu d_t)$	$C_{\varphi L}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi) (\bar{L}_p \sigma_I \gamma^\mu L_r)$
C_{Qe}	$(\bar{Q}_p \gamma_\mu Q_r) (\bar{e}_s \gamma^\mu e_t)$	C_{eW}	$(\bar{L}_p \sigma^{\mu\nu} e_r) \sigma_I \varphi W_{\mu\nu}^I$
C_{LeQ}	$(\bar{L}_p^j e_r) (\bar{d}_s Q_t^j)$	C_{eB}	$(\bar{L}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$

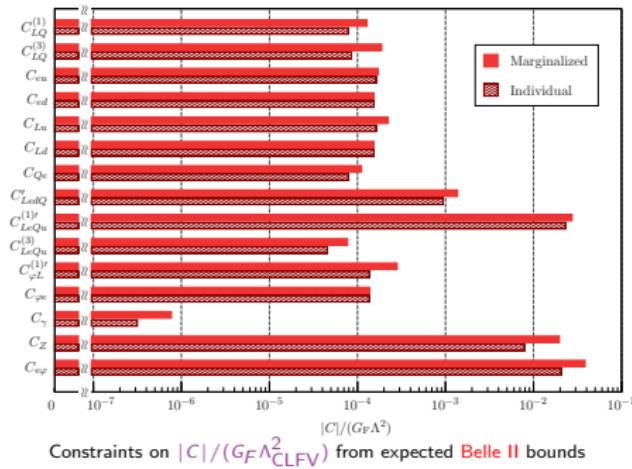
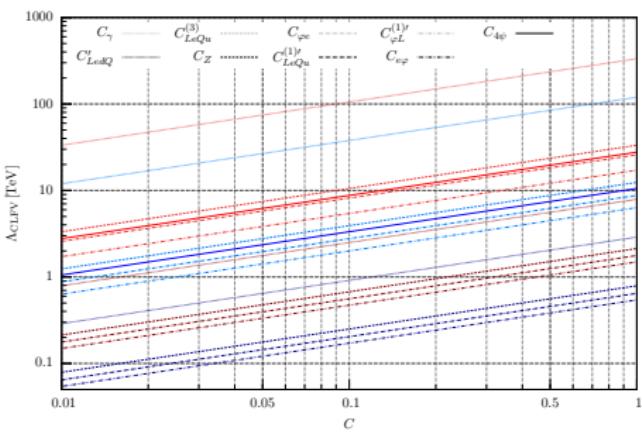
Two scenarios

- ① Quark-flavour diagonal WCs: no FCNC in the quark sector
- ② Equal entries for the full 3×3 quark-flavour WC matrix: FCNC in the quark sector (quark currents like $\bar{c}u$, $\bar{b}s\dots$) is allowed. $\Lambda_{\text{CLFV}} = \Lambda_{\text{FCNC}}$

Most stringent bound found for

$$\mathcal{O}_\gamma \equiv c_W \mathcal{O}_{eB} - s_W \mathcal{O}_{eW} \xrightarrow{C_\gamma \sim 1} \Lambda_{\text{CLFV}}[\text{TeV}] \gtrsim \begin{cases} 120 & \text{Belle} \\ 330 & \text{Belle II} \end{cases}$$

- Belle and Belle II limits on τ decays dominate the analysis
- NA64 expected sensitivity on $\ell\text{-}\tau$ conversion in nuclei is not competitive, but could remove correlations between WCs if improvement of at least two orders of magnitude ($R_{\tau\ell} \sim 10^{-15}$)



Leptoquark Lagrangian

The most general framework: **5 scalar** and **5 vectorial** leptoquarks based on the representations of matter fields under the **SM gauge group** [Doršner et al., 2016] [Crivellin et al., 2021]

$$\mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{LQ-}\psi} + \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{\text{LQ-H}}$$

Keep only terms responsible for **CLFV**: LQ- ψ , γ -mediated, Z -mediated and H -mediated

$$\mathcal{L}_{\text{LQ-}\psi} \supset$$

LQ type	SM symmetries	Lagrangian
S_3	$(\bar{\mathbf{3}}, \mathbf{3}, 1/3)$	$+ Y_{3,ij}^{\text{LL}} \bar{Q}_L^{Ci,a} \epsilon^{ab} (\tau_k S_3^k)^{bc} L_L^{j,c} + \text{h.c.}$
R_2	$(\mathbf{3}, \mathbf{2}, 7/6)$	$- Y_{2,ij}^{\text{RL}} \bar{u}_R^i R_2^a \epsilon^{ab} L_L^{j,b} + Y_{2,ij}^{\text{LR}} \bar{e}_R^i R_2^{a\dagger} Q_L^{j,a} + \text{h.c.}$
\tilde{R}_2	$(\mathbf{3}, \mathbf{2}, 1/6)$	$- \tilde{Y}_{2,ij}^{\text{RL}} \bar{d}_R^i \tilde{R}_2^a \epsilon^{ab} L_L^{j,b} + \text{h.c.}$
\tilde{S}_1	$(\bar{\mathbf{3}}, \mathbf{1}, 4/3)$	$+ \tilde{Y}_{1,ij}^{\text{RR}} \bar{d}_R^{C,i} \tilde{S}_1 e_R^j + \text{h.c.}$
S_1	$(\bar{\mathbf{3}}, \mathbf{1}, 1/3)$	$+ Y_{1,ij}^{\text{LL}} \bar{Q}_L^{Ci,a} S_1 \epsilon^{ab} L_L^{j,b} + Y_{1,ij}^{\text{RR}} \bar{u}_R^{Ci} S_1 e_R^j + \text{h.c.}$
U_3	$(\mathbf{3}, \mathbf{3}, 2/3)$	$+ X_{3,ij}^{\text{LL}} \bar{Q}_L^{Ci,a} \gamma^\mu (\tau_k U_{3,\mu}^k)^{ab} L_L^{j,b} + \text{h.c.}$
V_2	$(\bar{\mathbf{3}}, \mathbf{2}, 5/6)$	$+ X_{2,ij}^{\text{RL}} \bar{d}_R^i \gamma^\mu V_{2,\mu}^a \epsilon^{ab} L_L^{j,b} + X_{2,ij}^{\text{LR}} \bar{Q}_L^{Ci,a} \gamma^\mu V_{2,\mu}^b e_R^j + \text{h.c.}$
\tilde{V}_2	$(\bar{\mathbf{3}}, \mathbf{2}, -1/6)$	$+ \tilde{X}_{2,ij}^{\text{RL}} \bar{u}_R^{C,i} \gamma^\mu \tilde{V}_{2,\mu}^b \epsilon^{ab} L_L^{j,a} + \text{h.c.}$
\tilde{U}_1	$(\mathbf{3}, \mathbf{1}, 5/3)$	$+ \tilde{X}_{1,ij}^{\text{RR}} \bar{u}_R^i \gamma^\mu \tilde{U}_{1,\mu} e_R^j + \text{h.c.}$
U_1	$(\mathbf{3}, \mathbf{1}, 2/3)$	$+ X_{1,ij}^{\text{LL}} \bar{Q}_L^{Ci,a} \gamma^\mu U_{1,\mu} L_L^{j,a} + X_{1,ij}^{\text{RR}} \bar{d}_R^i \gamma^\mu U_{1,\mu} e_R^j + \text{h.c.}$

$$\mathcal{L}_S \supset$$

$$\mathcal{L}_S^\gamma = ie \sum_{\text{Scalar LQs}} Q_S \left[\left(\partial_\mu S^\dagger \right) S - S^\dagger \left(\partial_\mu S \right) \right] A^\mu$$

EFT of leptoquarks

1. Matching $\mathcal{L}_{\text{LQ-}\psi}$ to 4ψ SMEFT operators

We integrate out the leptoquarks at tree level to match the 4-fermion operators of the SMEFT

- Take the total derivative of the action resulting from $\mathcal{L}_{\text{UV}} \rightarrow \text{EOM}$ of the LQs
- M_S and M_V large \rightarrow expansion in momenta \rightarrow substituting rules for the LQ fields
- Insert these relations into the Lagrangian

$$\begin{aligned}\mathcal{L}_S^{\text{eff}} &\supset \frac{Y_{d,ij}^{\chi_1\chi_2} Y_{d,mn}^{\chi_3\chi_4}}{M_S^2} (\bar{\psi}_{\chi_1}^i \psi_{\chi_2}^{j*})(\bar{\psi}_{\chi_4}^m \psi_{\chi_3}^n), \\ \mathcal{L}_V^{\text{eff}} &\supset \frac{X_{d,ij}^{\chi_1\chi_2} X_{d,mn}^{\chi_3\chi_4}}{M_V^2} (\bar{\psi}_{\chi_1}^i \gamma_\mu \psi_{\chi_2}^{j*})(\bar{\psi}_{\chi_4}^m \gamma^\mu \psi_{\chi_3}^n).\end{aligned}$$

2. Flavour considerations

- Enhancement of flavour violation in the third family
- Equal entries quark-flavour WC matrix \rightarrow quark-flavour blind Yukawas

$$Y_d^{\chi_1\chi_2} = \begin{pmatrix} y_d^{\chi_1\chi_2} & y_d^{\chi_1\chi_2} & y_{d\tau}^{\chi_1\chi_2} \\ y_d^{\chi_1\chi_2} & y_d^{\chi_1\chi_2} & y_{d\tau}^{\chi_1\chi_2} \\ y_d^{\chi_1\chi_2} & y_d^{\chi_1\chi_2} & y_{d\tau}^{\chi_1\chi_2} \end{pmatrix}, \quad y_d^{\chi_1\chi_2} \neq y_{d\tau}^{\chi_1\chi_2}$$

Results: scalar leptoquarks

$$\Lambda_{\text{CLFV}} = M_S$$

$$yy = f(C_i)$$

$$y_3^{\text{LL}} y_{3\tau}^{\text{LL}} = C_{LQ}^{(1)} + C_{LQ}^{(3)},$$

$$y_2^{\text{RL}} y_{2\tau}^{\text{RL}} = -2C_{Lu},$$

$$y_1^{\text{LL}} y_{1\tau}^{\text{LL}} = C_{LQ}^{(1)} - 3C_{LQ}^{(3)},$$

$$\tilde{y}_2^{\text{RL}} \tilde{y}_{2\tau}^{\text{RL}} = -2C_{Ld},$$

$$y_{2\tau}^{\text{RL}} y_2^{\text{LR}} = -C_{LeQu}^{(1)(\ell-\tau)} - 4C_{LeQu}^{(3)(\ell-\tau)},$$

$$y_{1\tau}^{\text{LL}} y_1^{\text{RR}} = C_{LeQu}^{(1)(\ell-\tau)} - 4C_{LeQu}^{(3)(\ell-\tau)},$$

$$y_2^{\text{LR}} y_{2\tau}^{\text{LR}} = -2C_{Qe},$$

$$y_1^{\text{RR}} y_{1\tau}^{\text{RR}} = 2C_{eu},$$

$$\tilde{y}_1^{\text{RR}} \tilde{y}_{1\tau}^{\text{RR}} = 2C_{ed},$$

$$y_2^{\text{RL}} y_{2\tau}^{\text{LR}} = -C_{LeQu}^{(1)(\tau-h)} - 4C_{LeQu}^{(3)(\tau-h)},$$

$$y_1^{\text{LL}} y_{1\tau}^{\text{RR}} = C_{LeQu}^{(1)(\tau-h)} - 4C_{LeQu}^{(3)(\tau-h)}$$

- Pairs of Yukawas $y_{2\tau}^{\text{RL}} y_2^{\text{LR}}$ and $y_{1\tau}^{\text{LL}} y_1^{\text{RR}}$ unconstrained by τ decays $\rightarrow \ell - \tau$ conversion limits considered
- 11 pairs of Yukawas and 11 effective bounds on the WCs

τ decays	Upper bounds on $\frac{ yy' }{M_S^2} [10^{-3} \text{ TeV}^{-2}]$		Lower bounds on $M_S [\text{TeV}]$	
Yukawa pair	Belle	Belle II	Belle	Belle II
$ y_3^{LL} y_{3\tau}^{LL} $	12	1.9	9.1	23
$ y_2^{RL} y_{2\tau}^{RL} $	47	5.0	4.6	14
$ y_2^{LR} y_{2\tau}^{LR} $	17	2.6	7.8	20
$ y_2^{RL} y_{2\tau}^{LR} $	28	3.7	6.0	16
$ \tilde{y}_2^{RL} \tilde{y}_{2\tau}^{RL} , \tilde{y}_1^{RR} \tilde{y}_{1\tau}^{RR} $	20	3.0	7.1	18
$ y_1^{LL} y_{1\tau}^{LL} $	64	7.7	3.9	11
$ y_1^{RR} y_{1\tau}^{RR} $	34	4.1	5.4	16
$ y_1^{LL} y_{1\tau}^{RR} $	28	3.7	6.0	16
$\ell-\tau$ conversion	Upper bounds on $\frac{ yy' }{M_S^2} [10^0 \text{ TeV}^{-2}]$		Lower bounds on $M_S [\text{TeV}]$	
Yukawa pair	$e-\tau$	$\mu-\tau$	$e-\tau$	$\mu-\tau$
$ y_{2\tau}^{RL} y_2^{LR} $	350	2.3	0.054	0.66
$ y_{1\tau}^{LL} y_1^{RR} $	250	1.8	0.063	0.75

Results: C_γ

To match the C_γ consider gauge couplings of the leptoquarks

- Vector leptoquarks
 - uncertainty on their origin → not considered [Gonderinger and Ramsey-Musolf, 2010]
- Scalar leptoquarks
 - coupled through the covariant derivative

Leading-order contribution to C_γ from LQs → one-loop computation of $\ell_1 \rightarrow \ell_2 \gamma$ process

- Integration by regions [Beneke and Smirnov, 1998] [Smirnov, 2002] [Jantzen, 2011]
- Main contribution from two leptoquarks: $R_2^{5/3}$ and S_1

$$\frac{C_\gamma}{\Lambda_{\text{CLFV}}^2} = \frac{e N_C m_t V_{tb}}{32\sqrt{2}\pi^2 v M_S^2} (Q_{LQ} - 3Q_t) y_1 y_2 \longrightarrow \begin{cases} R_2^{5/3} : & Q_{LQ} = 5/3; y_1 y_2 = y_{2\tau}^{RL} y_2^{LR} \\ S_1 : & Q_{LQ} = 1/3; y_1 y_2 = y_{1\tau}^{LL} y_1^{RR} \end{cases}$$

$C_\gamma / \Lambda_{\text{CLFV}}^2$	Upper bounds on $\frac{ yy' }{M_S^2} [10^{-3} \text{ TeV}^{-2}]$		Lower bounds on $M_S [\text{TeV}]$	
Yukawa pair	Belle	Belle II	Belle	Belle II
$ y_{2\tau}^{RL} y_2^{LR} $	150	19	3.1	8.6
$ y_{1\tau}^{LL} y_1^{RR} $	21	2.7	8.2	23

Conclusions

Model-independent numerical analysis of SMEFT D-6 operators related to CLFV processes the τ lepton

We studied 28+4 observables

- 14 different LFV τ decay channels into hadrons for each $\ell \rightarrow$ we used current Belle and expected Belle II data (strongest bounds)
- $e - \tau$ and $\mu - \tau$ conversion in Fe(56,26) and Pb(208,82) \rightarrow feasible at NA64
 - not competitive yet, could remove correlations between WCs if improvement of at least two orders of magnitude ($R_{\tau\ell} \sim 10^{-15}$)

We translated the bounds on the SMEFT parameters into constraints on a general leptoquark framework

Main constraints from four-fermion operators (tree-level LQ contributions)

- Strongest bound on scalar LQs: $|y_3^{\text{LL}} y_{3\tau}^{\text{LL}}| \sim 1 \longrightarrow M_S \gtrsim 23\text{TeV}$ (Belle II)
- Slightly stronger bounds on vector LQs
- Some pairs of Yukawas only bounded from $\ell - \tau$ conversion

Main bound on C_γ (one-loop LQ contribution) from τ decays helps improving the low-bounded pair of Yukawas

GRACIAS
SPASSIBO ENHACHALNUVA
DANKSCHEEN

ARIGATO
MERASTAMY
SALJITHO
GÖZAIMASHITA
EFCHARISTO
AGUYJE
FAKAUE

SHUKURIA
TAVTAPOCH MEDAHAGSE
SHUKURIA

JUSPAXAR
BAKSA

TASHAKKUR ATU
CHISLTU
YAQHANYELAY
NADEELJA MATTEKA YUBACAKATAH
DAHNEYIBAD

YAQHANYELAY
NADEELJA MATTEKA YUBACAKATAH
ATTO AHNIA UNAHCEHESI

MAAKE
MAAKE

GRAZIE
SPASSIBO DENKAU-JAR
MERASERI ENDAU CIR ODAO

MEHRBANI
EKHMET
PALDIES

PALDIES

TINGKI

THANK YOU
HATOB GUN
HAKKEU HAKKEU

BOLZIN
MERONCHAR

MERCI

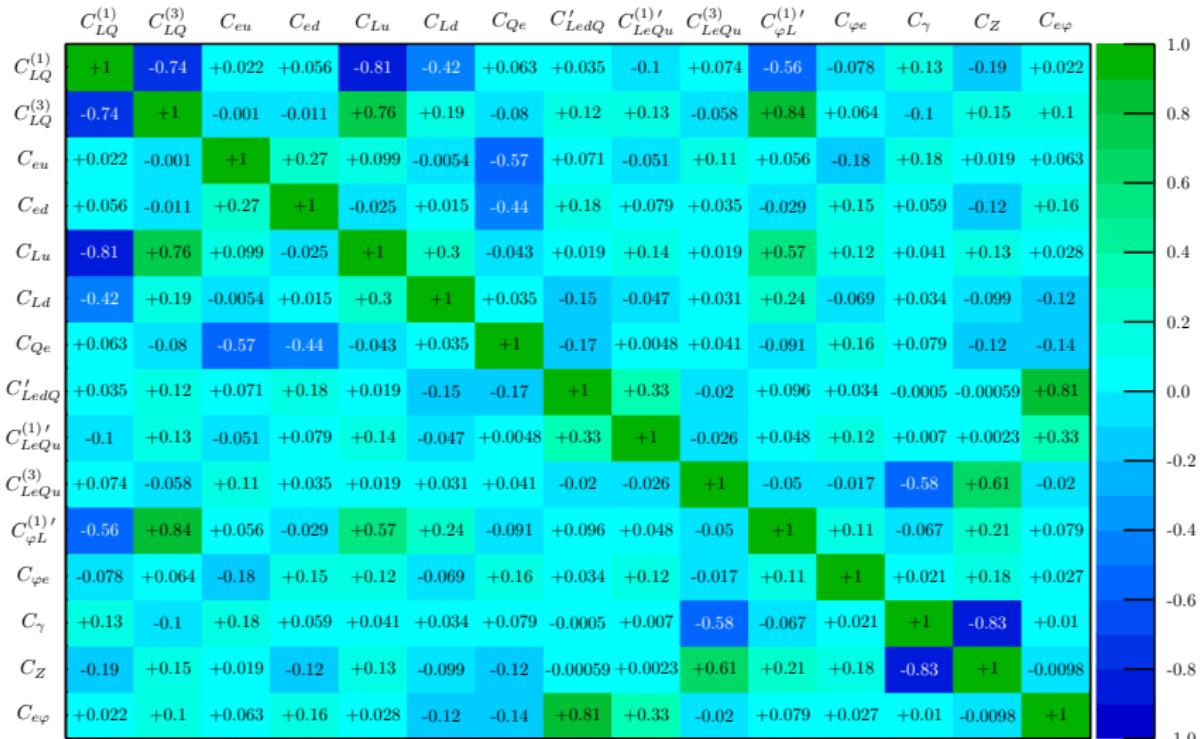
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Correlation matrix: Belle



Results: set-up

Bounds from CLFV τ processes translated to the leptoquark framework → bounds on $yy'/M_{S,V}^2$

Two independent scenarios → all scalar or vector leptoquarks at the same time

- Same energy scale for all leptoquarks within the same scenario → equal masses

Previous 2nd scenario assumption $\Lambda_{\text{CLFV}} = \Lambda_{\text{FCNC}}$ is better motivated

Bounds mainly from four-fermion operators of the SMEFT

- Gauge boson couplings to LQs leads to constraints on the same pairs of Yukawas
- Gauge bosons contribute through loop processes → lower sensitivity (except for the C_γ ; see below)

Results: vector leptoquarks

$$\Lambda_{\text{CLFV}} = m_V$$

- We start with 11 pairs of Yukawas but 9 effective bounds on the WCs

$$\frac{1}{2} C_{LdQ}^{(\ell-\tau)} = x_{1\tau}^{\text{LL}} x_1^{\text{RR}} - x_{2\tau}^{\text{RL}} x_2^{\text{LR}} \equiv x_{1,2}^{(\ell-\tau)}, \quad \frac{1}{2} C_{LdQ}^{(\tau-h)} = x_1^{\text{LL}} x_{1\tau}^{\text{RR}} - x_2^{\text{RL}} x_{2\tau}^{\text{LR}} \equiv x_{1,2}^{(\tau-h)}$$

↓

$$x_3^{\text{LL}} x_{3\tau}^{\text{LL}} = \frac{1}{2}(C_{LQ}^{(3)} - C_{LQ}^{(1)}), \quad x_2^{\text{RL}} x_{2\tau}^{\text{RL}} = C_{Ld}, \quad x_2^{\text{LR}} x_{2\tau}^{\text{LR}} = C_{Qe},$$

$$\tilde{x}_2^{\text{RL}} \tilde{x}_{2\tau}^{\text{RL}} = C_{Lu}, \quad x_1^{\text{LL}} x_{1\tau}^{\text{LL}} = -\frac{1}{2}(C_{LQ}^{(1)} + 3C_{LQ}^{(3)}), \quad x_1^{\text{RR}} x_{1\tau}^{\text{RR}} = -C_{ed},$$

$$\tilde{x}_1^{\text{RR}} \tilde{x}_{1\tau}^{\text{RR}} = -C_{eu}, \quad x_{1,2}^{(\tau-h)} = \frac{1}{2} C_{LdQ}^{(\tau-h)}, \quad x_{1,2}^{(\ell-\tau)} = \frac{1}{2} C_{LdQ}^{(\ell-\tau)}.$$

- $x_{1,2}^{(\ell-\tau)}$ unconstrained by τ decays $\rightarrow \ell - \tau$ conversion limits considered
- 9 pairs of Yukawas and 9 effective bounds on the WCs

τ decays	Upper bounds on $\frac{ xx' }{M_V^2} [10^{-3} \text{ TeV}^{-2}]$		Lower bounds on $M_V [\text{TeV}]$	
Yukawa pair	Belle	Belle II	Belle	Belle II
$ x_3^{\text{LL}} x_{3\tau}^{\text{LL}} $	15	1.7	8.2	25
$ x_2^{\text{RL}} x_{2\tau}^{\text{RL}} , x_1^{\text{RR}} x_{1\tau}^{\text{RR}} $	10	1.5	10	26
$ x_2^{\text{LR}} x_{2\tau}^{\text{LR}} $	8.3	1.3	11	28
$ \tilde{x}_2^{\text{RL}} \tilde{x}_{2\tau}^{\text{RL}} $	24	2.5	6.5	20
$ x_1^{\text{LL}} x_{1\tau}^{\text{LL}} $	22	3.1	6.7	18
$ \tilde{x}_1^{\text{RR}} \tilde{x}_{1\tau}^{\text{RR}} $	17	2.1	7.7	22
$ x_{1,2}^{\tau h} $	3.1	0.42	18	49

$\ell-\tau$ conversion	Upper bounds on $\frac{ xx' }{M_V^2} [10^0 \text{ TeV}^{-2}]$		Lower bounds on $M_V [\text{TeV}]$	
Yukawa pair	$e-\tau$	$\mu-\tau$	$e-\tau$	$\mu-\tau$
$ x_{1,2}^{\ell-\tau} $	330	1.5	0.055	0.83

Gauge couplings of vector leptoquarks

Vector leptoquarks interaction with the photon depend on their nature → **gauge bosons or not** of a higher energy theory

- there can exist an anomalous magnetic moment coupling

$$\mathcal{L}_{V,\gamma} = -ieQ_V \left([V_{\mu\nu}^\dagger V^\nu - V_{\mu\nu} V^{\nu\dagger}] A^\mu - (1 - \kappa) V_\mu^\dagger V_\nu F^{\mu\nu} \right)$$

- gauge boson → $\kappa = 0$
 - three-gauge-boson vertex

If gauge boson, propagator:

$$\frac{-ig^{\mu\nu}}{k^2 - m_V^2 + i\epsilon},$$

otherwise

$$\frac{-i}{k^2 - m_V^2 + i\epsilon} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_V^2} \right)$$

Second term introduces **extra divergences** to the loop computations of $\ell_1 \rightarrow \ell_2 \gamma$

- they do not cancel as for scalar leptoquarks

If gauge boson, possible contributions to the loop from other degrees of freedom in the UV completion

LQ	$C_{LQ}^{(1), klmn}$	$C_{LQ}^{(3), klmn}$	$C_{LeQu}^{(1), klmn}$	$C_{LeQu}^{(3), klmn}$	C_{Qe}^{klmn}	C_{Lu}^{klmn}	C_{Ld}^{klmn}	C_{eu}^{klmn}	C_{ed}^{klmn}	C_{LedQ}^{klmn}
S_3	$+\frac{3}{4} Y_{3,nl}^{\text{LL}} Y_{3,mk}^{\text{LL}}$	$+\frac{1}{4} Y_{3,nl}^{\text{LL}} Y_{3,mk}^{\text{LL}}$	\times	\times	\times	\times	\times	\times	\times	\times
R_2	\times	\times	$-\frac{1}{2} Y_{2,nk}^{\text{RL}} Y_{2,lm}^{\text{LR}}$	$-\frac{1}{8} Y_{2,nk}^{\text{RL}} Y_{2,lm}^{\text{LR}}$	$-\frac{1}{2} Y_{2,ml}^{\text{LR}} Y_{2,nk}^{\text{LR}}$	$-\frac{1}{2} Y_{2,ml}^{\text{RL}} Y_{2,nk}^{\text{RL}}$	\times	\times	\times	\times
\tilde{R}_2	\times	\times	\times	\times	\times	\times	$-\frac{1}{2} \tilde{Y}_{2,ml}^{\text{RL}} \tilde{Y}_{2,nk}^{\text{RL}}$	\times	\times	\times
\tilde{S}_1	\times	\times	$+\frac{1}{2} \tilde{Y}_{1,nl}^{\text{RR}} \tilde{Y}_{1,mk}^{\text{RR}}$	\times						
S_1	$+\frac{1}{4} Y_{1,nl}^{\text{LL}} Y_{1,mk}^{\text{LL}}$	$-\frac{1}{4} Y_{1,nl}^{\text{LL}} Y_{1,mk}^{\text{LL}}$	$+\frac{1}{2} Y_{1,mk}^{\text{LL}} Y_{1,nl}^{\text{RR}}$	$-\frac{1}{8} Y_{1,mk}^{\text{LL}} Y_{1,nl}^{\text{RR}}$	\times	\times	$+\frac{1}{2} Y_{1,nl}^{\text{RR}} Y_{1,mk}^{\text{RR}}$	\times	\times	\times
U_3	$-\frac{3}{2} X_{3,m/}^{\text{LL}} X_{3,nk}^{\text{LL}}$	$+\frac{1}{2} X_{3,m/}^{\text{LL}} X_{3,nk}^{\text{LL}}$	\times	\times	\times	\times	\times	\times	\times	\times
V_2	\times	\times	\times	\times	$+X_{2,ln}^{\text{LR}} X_{2,km}^{\text{LR}}$	\times	$+X_{2,m/}^{\text{RL}} X_{2,mk}^{\text{RL}}$	\times	\times	$-2X_{2,mk}^{\text{RL}} X_{2,nl}^{\text{LR}}$
\tilde{V}_2	\times	\times	\times	\times	$+X_{2,nl}^{\text{RL}} X_{2,mk}^{\text{RL}}$	\times	\times	\times	\times	\times
\tilde{U}_1	\times	\times	\times	\times	\times	\times	$-X_{1,ml}^{\text{RR}} X_{1,nk}^{\text{RR}}$	\times	\times	\times
U_1	$-\frac{1}{2} X_{1,m/}^{\text{LL}} X_{1,nk}^{\text{LL}}$	$-\frac{1}{2} X_{1,m/}^{\text{LL}} X_{1,nk}^{\text{LL}}$	\times	\times	\times	\times	\times	$-X_{1,m/}^{\text{RR}} X_{1,nk}^{\text{RR}}$	$+2X_{1,m/}^{\text{LL}} X_{1,nk}^{\text{RR}}$	\times

Table: Results of the matching of pairs of Yukawa couplings stemming from each leptoquark type to the Wilson coefficients of the four-fermion operators of SMEFT. Notice that, owing to the Fierz rearrangement, we also obtain contributions containing tensorial operators, i.e. $C_{LeQu}^{(3)}$.

$C_\gamma/\Lambda_{\text{CLFV}}^2$	Upper bounds on $\frac{ yy' }{M_S^2} [10^{-3} \text{ TeV}^{-2}]$		Lower bounds on $M_S [\text{TeV}]$	
Yukawa pair	Belle	Belle II	Belle	Belle II
$ y_{2\tau}^{\text{RL}} y_2^{\text{LR}} $	150	19	3.1	8.6
$ y_{1\tau}^{\text{LL}} y_1^{\text{RR}} $	21	2.7	8.2	23

$\tau \rightarrow \ell\gamma$	Upper bounds on $\frac{ yy' }{M_S^2} [10^{-3} \text{ TeV}^{-2}]$		Lower bounds on $M_S [\text{TeV}]$	
Yukawa pair	$\tau \rightarrow e\gamma$	$\tau \rightarrow \mu\gamma$	$\tau \rightarrow e\gamma$	$\tau \rightarrow \mu\gamma$
$ y_{2\tau}^{\text{RL}} y_2^{\text{LR}} $	0.66	0.79	83	75
$ y_{1\tau}^{\text{LL}} y_1^{\text{RR}} $	1.4	1.7	71	64