

# Leptoquark and vector-like quark extended models as the explanation of the muon $g - 2$ anomaly

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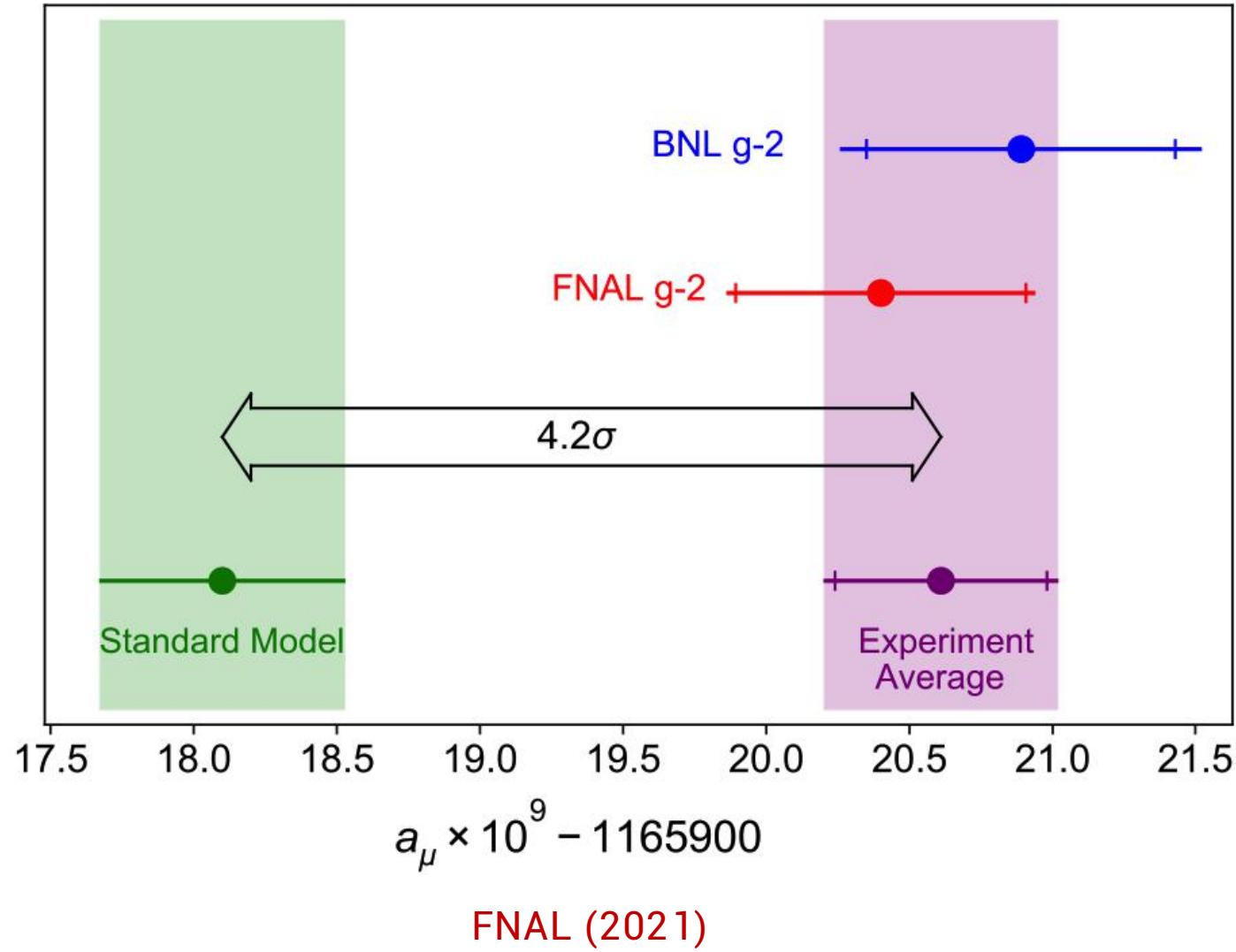
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# 1. $(g - 2)_\mu$ introduction



$$a_\mu(\text{BNL}) = 116592080(63) \times 10^{-11}$$

$$a_\mu(\text{FNAL}) = 116592040(54) \times 10^{-11}$$

$$a_\mu(\text{Exp}) = 116592061(41) \times 10^{-11}$$

$$a_\mu(\text{SM}) = 116591810(43) \times 10^{-11}$$

T. Aoyama et al. (2020)

$$a_\mu(\text{Exp}) - a_\mu(\text{SM}) = (251 \pm 59) \times 10^{-11}$$

4.2  $\sigma$  deviation?

# $(g - 2)_\mu$ prediction in the standard model (SM)

$$a_\mu(\text{SM}) = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{Had}}, \quad a_\mu^{\text{Had}} = a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

$$a_\mu^{\text{QED}} = 116584718.931(104) \times 10^{-11} \quad \text{up to tenth order (5-loop)}$$

$$a_\mu^{\text{EW}} = 153.6(1.0) \times 10^{-11} \quad \text{up to 2-loop}$$

$$a_\mu^{\text{HVP}} = a_\mu^{\text{HVP, LO}} + a_\mu^{\text{HVP, NLO}} + a_\mu^{\text{HVP, NNLO}} = 6845(40) \times 10^{-11}$$

$$a_\mu^{\text{HLbL}} = a_\mu^{\text{HLbL, LO}} + a_\mu^{\text{HLbL, NLO}} = 92(18) \times 10^{-11}$$



$$a_\mu(\text{SM}) = 116591810(43) \times 10^{-11}$$

T. Aoyama et al. (2020)

## developments on this anomaly

- Pin down this anomaly

- Reduce theoretical uncertainty:

- improve the HVP and HLbL calculations

- Reduce experimental uncertainty

- FNAL run 2, J-PARC

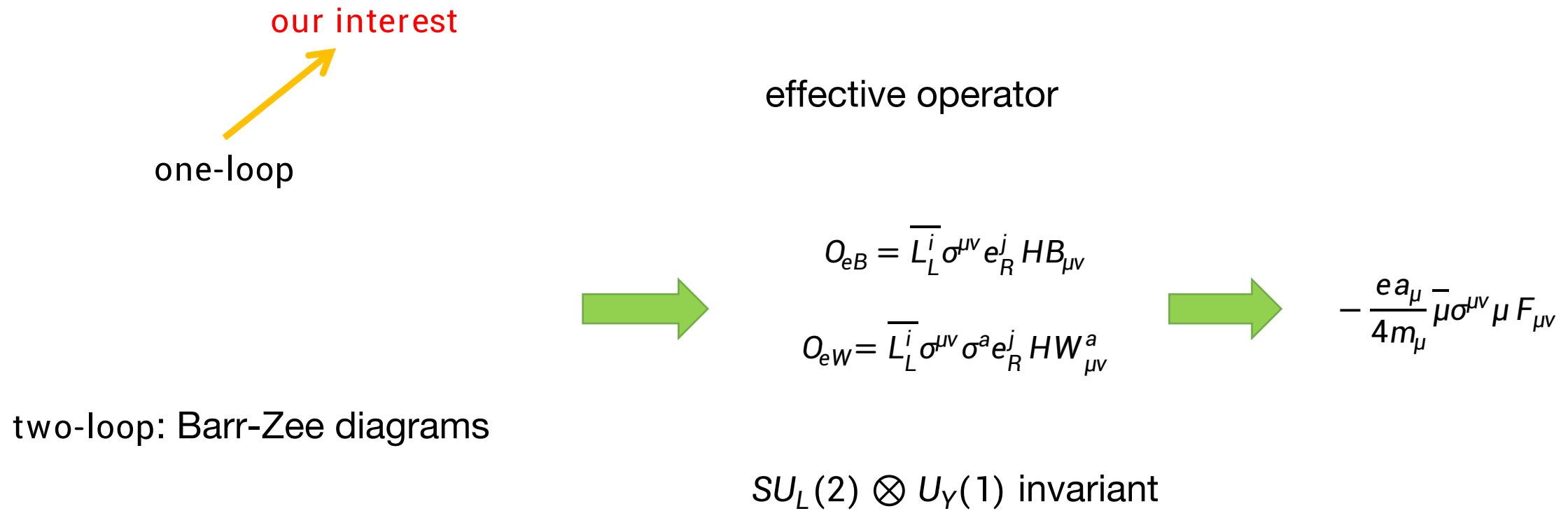
- New physics (NP) explanations

NP implications of this anomaly



our interest

## 2. New physics contributions



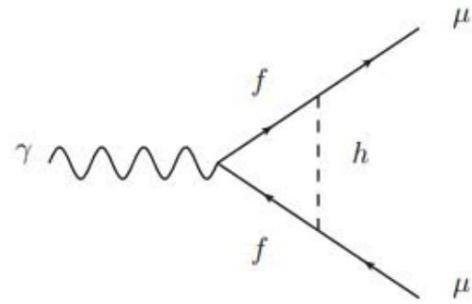
# New physics contributions at one-loop

four basic topologies as the basis of new physics

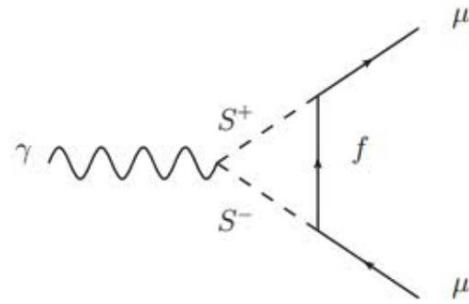
F. Jegerlehner and A. Nyffeler (2009)

M. Lindner, M. Platscher, and F. S. Queiroz (2016)

neutral scalar



singly charged scalar



combination



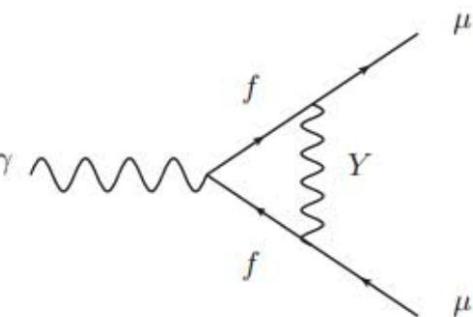
axion / dark photon

doubly charged scalar / vector

scalar / vector leptoquark (LQ)

supersymmetric particles

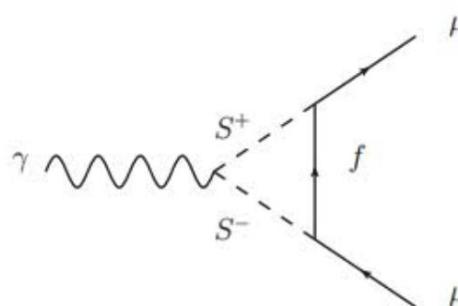
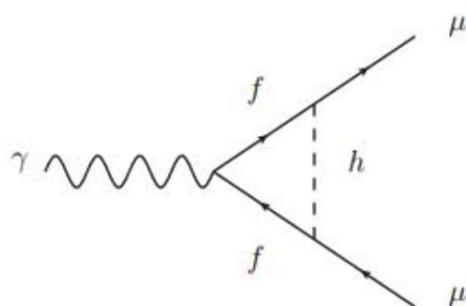
...



neutral vector

singly charged vector

# analysis of the scalar mediator case



two topologies

four independent integrals as the basis

neutral scalar  $h$  (not SM Higgs)

singly charged scalar  $S$

$$\Delta a_\mu = \frac{m_\mu^2}{8\pi^2} \left\{ \frac{(|y_L^h|^2 + |y_R^h|^2) I_{LL}^h + [(y_L^h)(y_R^h)^* + (y_R^h)(y_L^h)^*] I_{LR}^h}{m_h^2} + \frac{(|y_L^S|^2 + |y_R^S|^2) I_{LL}^S + [(y_L^S)(y_R^S)^* + (y_R^S)(y_L^S)^*] I_{LR}^S}{m_S^2} \right\}$$

$$I_{LL}^h = \frac{1}{2} \int_0^1 dx \frac{x^2(1-x)m_h^2}{xm_f^2 + (1-x)m_h^2 - x(1-x)m_\mu^2}$$

$$I_{LL}^S = -\frac{1}{2} \int_0^1 dx \frac{x(1-x)^2 m_S^2}{xm_f^2 + (1-x)m_S^2 - x(1-x)m_\mu^2}$$

$$I_{LR}^h = \frac{m_f}{2m_\mu} \int_0^1 dx \frac{x^2 m_h^2}{xm_f^2 + (1-x)m_h^2 - x(1-x)m_\mu^2}$$

$$I_{LR}^S = -\frac{m_f}{2m_\mu} \int_0^1 dx \frac{x(1-x)m_S^2}{xm_f^2 + (1-x)m_S^2 - x(1-x)m_\mu^2}$$

## properties of the four integrals

- chiral symmetry

$$\lim_{m_\mu \rightarrow 0} \frac{m_\mu^2}{m_s^2} * \{I_{LL}^h, I_{LR}^h, I_{LL}^S, I_{LR}^S\} = 0$$

- decoupling behaviour

$$\lim_{m_s \rightarrow \infty} \frac{m_\mu^2}{m_s^2} * \{I_{LL}^h, I_{LR}^h, I_{LL}^S, I_{LR}^S\} = 0$$

- threshold behaviour

$$m_\mu > m_{h/S} + m_f \quad \longrightarrow \quad \text{Im}\{I_{LL}^h, I_{LR}^h, I_{LL}^S, I_{LR}^S\} \neq 0$$

- expansion results

approximation choice depend on the specific models

one example:  $m_\mu \ll m_{h/S}, m_f$



$$\begin{aligned} I_{LL}^h &\rightarrow f_{LL}^q(x), & I_{LR}^h &\rightarrow \frac{m_f}{m_\mu} \cdot f_{LL}^q(x) & x \equiv \frac{m_f^2}{m_h^2} \\ I_{LL}^S &\rightarrow f_{LL}^S(x), & I_{LR}^S &\rightarrow \frac{m_f}{m_\mu} \cdot f_{LL}^S(x) & x \equiv \frac{m_f^2}{m_s^2} \end{aligned}$$

$$f_{LL}^q(x) = \frac{2 + 3x - 6x^2 + x^3 + 6x \log(x)}{12(1-x)^4}, \quad f_{LR}^q(x) = -\frac{3 - 4x + x^2 + 2 \log(x)}{4(1-x)^3}$$

$$f_{LL}^S(x) = -\frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log(x)}{12(1-x)^4}, \quad f_{LR}^S(x) = -\frac{1 - x^2 + 2x \log(x)}{4(1-x)^3}$$

# new physics models

- light scalar mediator: axion

- heavy scalar mediator

$m_\mu = m_f \ll m_{h/S}$  scenario

$$\Delta a_\mu \approx \frac{1}{8\pi^2} \left\{ (|y_L^h|^2 + |y_R^h|^2) \frac{m_\mu^2}{6m_h^2} + [(y_L^h)(y_R^h)^* + (y_R^h)(y_L^h)^*] \frac{m_\mu^2}{m_h^2} \left( \log \frac{m_h}{m_\mu} - \frac{3}{4} \right) \right. \\ \left. - (|y_L^S|^2 + |y_R^S|^2) \frac{m_\mu^2}{12m_S^2} - [(y_L^S)(y_R^S)^* + (y_R^S)(y_L^S)^*] \frac{m_\mu^2}{4m_S^2} \right\}$$

$m_\mu \ll m_f, m_{h/S}$  scenario

$$\Delta a_\mu \approx \frac{1}{8\pi^2} \left\{ (|y_L^h|^2 + |y_R^h|^2) \frac{m_\mu^2}{m_h^2} f_{LL}^q \left( \frac{m_f^2}{m_h^2} \right) + [(y_L^h)(y_R^h)^* + (y_R^h)(y_L^h)^*] \frac{m_\mu m_f}{m_h^2} f_{LR}^q \left( \frac{m_f^2}{m_h^2} \right) \right. \\ \left. + (|y_L^S|^2 + |y_R^S|^2) \frac{m_\mu^2}{m_S^2} f_{LL}^S \left( \frac{m_f^2}{m_S^2} \right) + [(y_L^S)(y_R^S)^* + (y_R^S)(y_L^S)^*] \frac{m_\mu m_f}{m_S^2} f_{LR}^q \left( \frac{m_f^2}{m_S^2} \right) \right\}$$



$m_\mu \ll m_f \ll m_{h/S}$

$$f_{LL}^q(x) \rightarrow \frac{1}{6}, \quad f_{LR}^q(x) \rightarrow -\frac{3 + 2 \log(x)}{4} \\ f_{LL}^S(x) \rightarrow -\frac{1}{12}, \quad f_{LR}^S(x) \rightarrow -\frac{1}{4}$$

# numerical estimation of heavy mediator case

$$\Delta a_\mu = (251 \pm 59) \times 10^{-11}$$

$m_u = m_f \ll m_{h/S}$  scenario

$$O(\Delta a_\mu) \sim \frac{y^2}{16\pi^2} \frac{m_\mu^2}{m_{h/S}^2} \rightarrow \frac{m_{h/S}}{y} \sim O(100 \text{GeV})$$

- $m_{h/s} \sim \text{GeV}, \quad y \sim 0.01$
- $m_{h/s} \sim 10\text{GeV}, \quad y \sim 0.1$
- $m_{h/s} \sim 100\text{GeV}, \quad y \sim 1$
- $m_{h/s} \sim \text{TeV}$  is not favored, unitarity violation

$m_\mu \ll m_f, m_{h/S}$  scenario

chiral enhancement  $\frac{m_f}{m_\mu}$

$$O(\Delta a_\mu) \sim \frac{y_L y_R}{16\pi^2} \frac{m_\mu m_f}{m_{h/S}^2}$$

How is the chiral enhancement produced?

# vector-like lepton LQ

$$\frac{m_L}{m_\mu}$$

$$m_f \sim \text{GeV}$$

$$m_f \sim 10\text{GeV}$$

$$\frac{m_{h/s}}{\sqrt{y_L y_R}} \sim O(\text{TeV}) - O(10\text{TeV})$$

$m_f \sim 100\text{GeV}$

3

10 of 10

# TeV and heavier mediator is viable

Note: logarithm can have some effects on the estimation

### 3. LQ extended models

minimal LQ models

$SU(3)_C \times SU(2)_L \times U(1)_Y$ representation	label	$F$	$y_L^{\text{LQ}\mu t}$	$y_R^{\text{LQ}\mu t}$
$(\bar{3}, 3, 1/3)$	$S_3$	-2	0	$\mathcal{O}(y)$
$(3, 2, 7/6)$	$R_2$	0	$\mathcal{O}(y)$	$\mathcal{O}(y)$
$(3, 2, 1/6)$	$\tilde{R}_2$	0	0	0
$(\bar{3}, 1, 4/3)$	$\tilde{S}_1$	-2	0	0
$(\bar{3}, 1, 1/3)$	$S_1$	-2	$\mathcal{O}(y)$	$\mathcal{O}(y)$

chiral enhancements

$R_2$        $S_1$

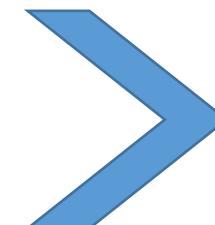
D. Chakraverty, D. Choudhury, and A. Datta (2001)  
K. Cheung (2001)

two LQs

I. Dorsner, S. Fajfer, and O. Sumensari (2019)  
D. Zhang (2021)       $\tilde{R}_2 + S_1$

three LQs

A. Crivellin, B. Fuks, and L. Schnell (2022)       $R_2 + R_2 + R_2$   
S. L. Chen, W. W. Jiang, and Z. K. Liu (2022)       $\tilde{R}_2 + S_1 + S_3$



B anomalies  
neutrino mass

# LQ+vector-like quark (VLQ) models

with up-type quark chiral enhancements

Shi-Ping He, Phys. Rev. D 105 (2022), 035017, arXiv:2112.13490

one LQ + one VLQ

$$\mathcal{L}_{F=0} \supset \bar{\mu}(y_L^{S_A \mu q_A} \omega_- + y_R^{S_A \mu q_A} \omega_+) q_A S_A + \text{h.c.}$$

$$\mathcal{L}_{F=2} \supset \bar{\mu}(y_L^{S_B \mu q_B} \omega_- + y_R^{S_B \mu q_B} \omega_+) q_B^C S_B + \text{h.c.}$$

third generation quark  
heavy quark

LQ	VLQ	related input parameters	mixing angle relation
$R_2$	$T_{L,R}$	$y_L^{R_2 \mu t}, y_R^{R_2 \mu t}, y_R^{R_2 \mu T}, \theta_L, m_T$	$\tan \theta_R = \frac{m_t}{m_T} \tan \theta_L$
	$(X, T)_{L,R}$	$y_L^{R_2 \mu t}, y_R^{R_2 \mu t}, \theta_R, m_T$	$\tan \theta_L = \frac{m_t}{m_T} \tan \theta_R$
	$(T, B)_{L,R}$	$y_L^{R_2 \mu t}, y_R^{R_2 \mu t}, y_L^{R_2 \mu T}, \theta_R^t, \theta_R^b, m_T$	$\tan \theta_L^{t(b)} = \frac{m_t^{t(b)}}{m_{T(B)}} \tan \theta_R^{t(b)}$
	$(X, T, B)_{L,R}$	$y_L^{R_2 \mu t}, y_R^{R_2 \mu t}, y_R^{R_2 \mu T}, \theta_L, m_T$	$\tan \theta_R^{t(b)} = \frac{m_t^{t(b)}}{m_{T(B)}} \tan \theta_L^{t(b)}, \sin 2\theta_L^b = \frac{\sqrt{2}(m_T^2 - m_t^2)}{m_B^2 - m_b^2} \sin 2\theta_L^t$
	$(T, B, Y)_{L,R}$	$y_L^{R_2 \mu t}, y_R^{R_2 \mu t}, \theta_L, m_T$	$\tan \theta_R^{t(b)} = \frac{m_t^{t(b)}}{m_{T(B)}} \tan \theta_L^{t(b)}, \sin 2\theta_L^b = -\frac{m_T^2 - m_t^2}{\sqrt{2}(m_B^2 - m_b^2)} \sin 2\theta_L^t$
$S_1$	$T_{L,R}$	$y_L^{S_1 \mu t}, y_R^{S_1 \mu t}, y_L^{S_1 \mu T}, \theta_L, m_T$	$\tan \theta_R = \frac{m_t}{m_T} \tan \theta_L$
	$(X, T)_{L,R}$	$y_L^{S_1 \mu t}, y_R^{S_1 \mu t}, \theta_R, m_T$	$\tan \theta_L = \frac{m_t}{m_T} \tan \theta_R$
	$(T, B)_{L,R}$	$y_L^{S_1 \mu t}, y_R^{S_1 \mu t}, y_R^{S_1 \mu T}, \theta_R^t, \theta_R^b, m_T$	$\tan \theta_L^{t(b)} = \frac{m_t^{t(b)}}{m_{T(B)}} \tan \theta_R^{t(b)}$
	$(X, T, B)_{L,R}$	$y_L^{S_1 \mu t}, y_R^{S_1 \mu t}, \theta_L, m_T$	$\tan \theta_R^{t(b)} = \frac{m_t^{t(b)}}{m_{T(B)}} \tan \theta_L^{t(b)}, \sin 2\theta_L^b = \frac{\sqrt{2}(m_T^2 - m_t^2)}{m_B^2 - m_b^2} \sin 2\theta_L^t$
	$(T, B, Y)_{L,R}$	$y_L^{S_1 \mu t}, y_R^{S_1 \mu t}, \theta_L, m_T$	$\tan \theta_R^{t(b)} = \frac{m_t^{t(b)}}{m_{T(B)}} \tan \theta_L^{t(b)}, \sin 2\theta_L^b = -\frac{m_T^2 - m_t^2}{\sqrt{2}(m_B^2 - m_b^2)} \sin 2\theta_L^t$
$S_3$	$(X, T, B)_{L,R}$	$y_L^{S_3 \mu T}, y_R^{S_3 \mu t}, \theta_L, m_T$	$\tan \theta_R^{t(b)} = \frac{m_t^{t(b)}}{m_{T(B)}} \tan \theta_L^{t(b)}, \sin 2\theta_L^b = \frac{\sqrt{2}(m_T^2 - m_t^2)}{m_B^2 - m_b^2} \sin 2\theta_L^t$

$$[ \begin{array}{cc} T & \sqrt{2}X \\ \sqrt{2}B & -T \end{array} ]$$

$$-\sqrt{2}T$$



$$[ \begin{array}{cc} B & \sqrt{2}T \\ \sqrt{2}Y & -B \end{array} ]$$

convention difference  
from other Refs.  
But it has no effects on  
the physical quantities.

# $\mu$ LQ Yukawa interactions with top and T quarks

LQ	VLQ	$\bar{\mu}_R t_L$	$\bar{\mu}_L t_R$	$\bar{\mu}_R T_L$	$\bar{\mu}_L T_R$
$R_2$	$T_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R + y_R^{R_2\mu T} c_R$
	$(X, T)_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R$
	$(T, B)_{L,R}$	$y_L^{R_2\mu t} c_L - y_L^{R_2\mu T} s_L$	$y_R^{R_2\mu t} c_R$	$y_L^{R_2\mu t} s_L + y_L^{R_2\mu T} c_L$	$y_R^{R_2\mu t} s_R$
	$(X, T, B)_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R - y_R^{R_2\mu T} s_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R + y_R^{R_2\mu T} c_R$
	$(T, B, Y)_{L,R}$	$y_L^{R_2\mu t} c_L$	$y_R^{R_2\mu t} c_R$	$y_L^{R_2\mu t} s_L$	$y_R^{R_2\mu t} s_R$
LQ	VLQ	$\bar{\mu}_R(t_R)^C$	$\bar{\mu}_L(t_L)^C$	$\bar{\mu}_R(T_R)^C$	$\bar{\mu}_L(T_L)^C$
$S_1$	$T_{L,R}$	$y_L^{S_1\mu t} c_R - y_L^{S_1\mu T} s_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R + y_L^{S_1\mu T} c_R$	$y_R^{S_1\mu t} s_L$
	$(X, T)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L$
	$(T, B)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L - y_R^{S_1\mu T} s_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L + y_R^{S_1\mu T} c_L$
	$(X, T, B)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L$
	$(T, B, Y)_{L,R}$	$y_L^{S_1\mu t} c_R$	$y_R^{S_1\mu t} c_L$	$y_L^{S_1\mu t} s_R$	$y_R^{S_1\mu t} s_L$
$S_3$	$(X, T, B)_{L,R}$	$-y_L^{S_3\mu T} s_R$	$y_R^{S_3\mu t} c_L$	$y_L^{S_3\mu T} c_R$	$y_R^{S_3\mu t} s_L$

# $\mu$ LQ Yukawa interactions with bottom and B quarks

LQ	VLQ	$\overline{\mu_R} b_L$	$\overline{\mu_L} b_R$	$\overline{\mu_R} B_L$	$\overline{\mu_L} B_R$
$R_2$	$T_{L,R}$	$y_L^{R_2\mu t}$	0	$\times$	$\times$
	$(X, T)_{L,R}$	$y_L^{R_2\mu t}$	0	$\times$	$\times$
	$(T, B)_{L,R}$	$y_L^{R_2\mu t} c_L^b - y_L^{R_2\mu T} s_L^b$	0	$y_L^{R_2\mu T} c_L^b + y_L^{R_2\mu t} s_L^b$	0
	$(X, T, B)_{L,R}$	$y_L^{R_2\mu t} c_L^b$	$-\sqrt{2}y_R^{R_2\mu T} s_R^b$	$y_L^{R_2\mu t} s_L^b$	$\sqrt{2}y_R^{R_2\mu T} c_R^b$
	$(T, B, Y)_{L,R}$	$y_L^{R_2\mu t} c_L^b$	0	$y_L^{R_2\mu t} s_L^b$	0
LQ	VLQ	$\overline{\mu_R}(b_R)^C$	$\overline{\mu_L}(b_L)^C$	$\overline{\mu_R}(B_R)^C$	$\overline{\mu_L}(B_L)^C$
$S_1$	$T_{L,R}$	0	0	$\times$	$\times$
	$(X, T)_{L,R}$	0	0	$\times$	$\times$
	$(T, B)_{L,R}$	0	0	0	0
	$(X, T, B)_{L,R}$	0	0	0	0
	$(T, B, Y)_{L,R}$	0	0	0	0
$S_3$	$(X, T, B)_{L,R}$	$-y_L^{S_3\mu T} s_R^b$	$\sqrt{2}y_R^{S_3\mu t} c_L^b$	$y_L^{S_3\mu T} c_R^b$	$\sqrt{2}y_R^{S_3\mu t} s_L^b$

Erratum xxx (2022)

# contributions to the $(g - 2)_\mu$

approximate results

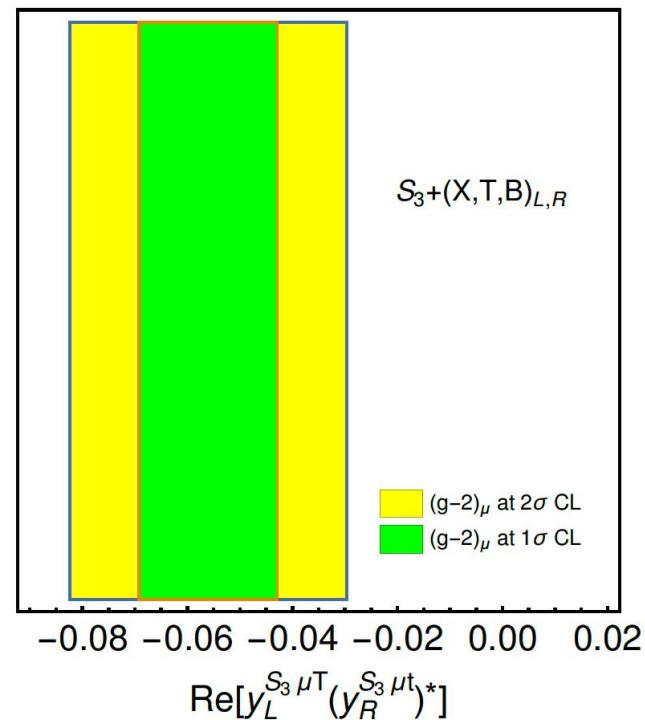
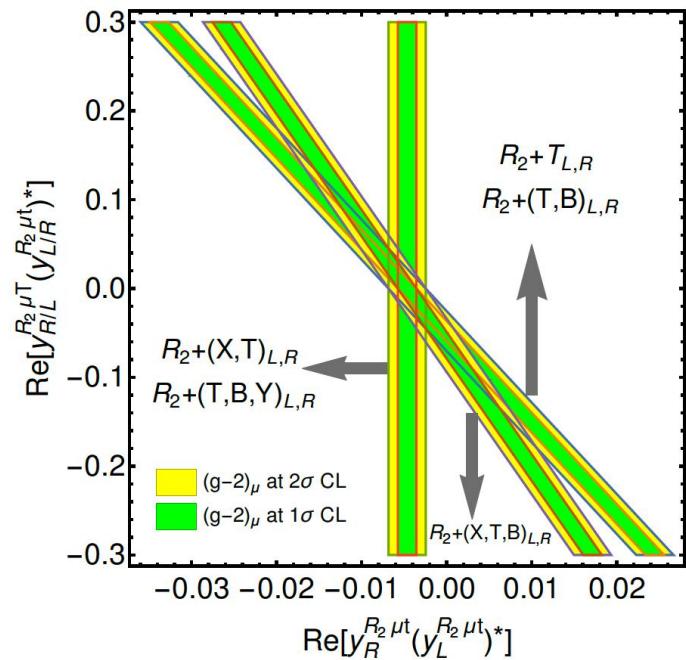
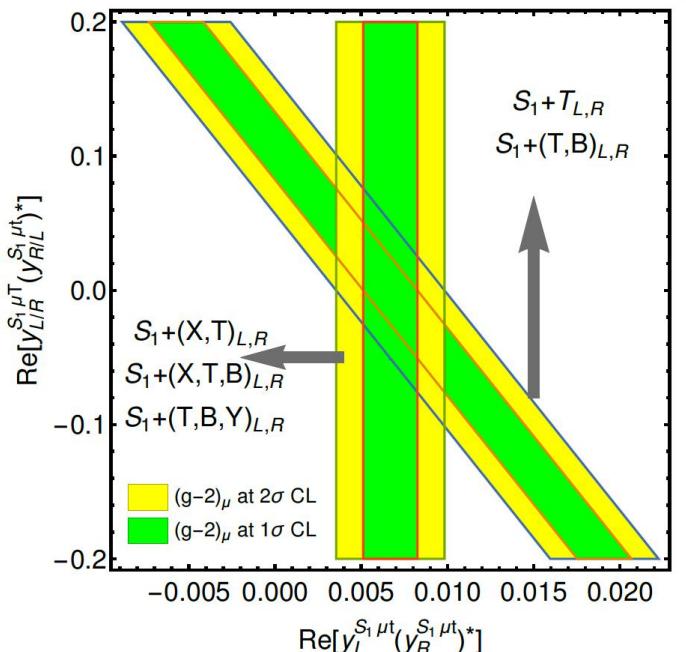
chirally enhanced parts

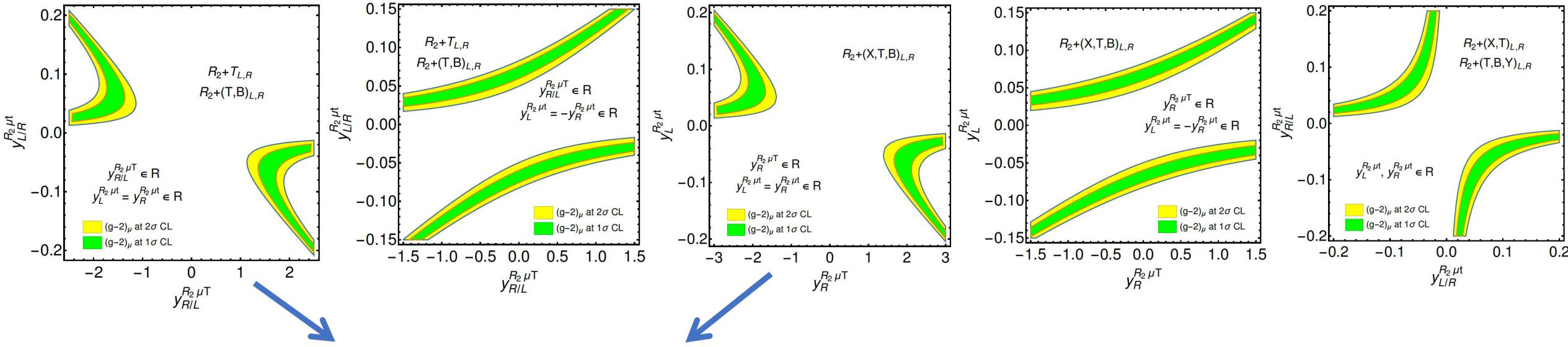
$m_b \ll m_t \ll m_T \approx m_B$

LQ	VLQ	the approximate expressions of $\Delta \bar{a}_\mu$	coupling product order of $T$ compared to $t$
$R_2$	$T_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2}(m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu T} (y_L^{R_2\mu t})^*] s_L + (\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	$s_L$
	$(X, T)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2}(m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] s_L s_R + (\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	$m_t s_R^2 / m_T$
	$(T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2}(m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu T})^*] s_R + (\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	$s_R$
	$(X, T, B)_{L,R}$	$\frac{m_T}{m_t} [f_{LR}^{R_2}(m_T^2/m_{R_2}^2) + 2\tilde{f}_{LR}^{R_2}(m_T^2/m_{R_2}^2)] \cdot \text{Re}[y_R^{R_2\mu T} (y_L^{R_2\mu t})^*] s_L + (\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	$s_L$
	$(T, B, Y)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{R_2}(m_T^2/m_{R_2}^2) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*] s_L s_R + (\frac{1}{4} + \log \frac{m_t^2}{m_{R_2}^2}) \text{Re}[y_R^{R_2\mu t} (y_L^{R_2\mu t})^*]$	$m_t s_L^2 / m_T$
$S_1$	$T_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1}(m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu T} (y_R^{S_1\mu t})^*] s_L - (\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$s_L$
	$(X, T)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1}(m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R - (\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$m_t s_R^2 / m_T$
	$(T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1}(m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu T})^*] s_R - (\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$s_R$
	$(X, T, B)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1}(m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R - (\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$m_t s_L^2 / m_T$
	$(T, B, Y)_{L,R}$	$\frac{m_T}{m_t} f_{LR}^{S_1}(m_T^2/m_{S_1}^2) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*] s_L s_R - (\frac{7}{4} + \log \frac{m_t^2}{m_{S_1}^2}) \text{Re}[y_L^{S_1\mu t} (y_R^{S_1\mu t})^*]$	$m_t s_L^2 / m_T$
$S_3$	$(X, T, B)_{L,R}$	$\frac{m_T}{m_t} [f_{LR}^{S_3}(m_T^2/m_{S_3}^2) + 2\tilde{f}_{LR}^{S_3}(m_T^2/m_{S_3}^2)] \cdot \text{Re}[y_L^{S_3\mu T} (y_R^{S_3\mu t})^*] s_L + (\frac{7}{4} + \log \frac{m_t^2}{m_{S_3}^2}) \text{Re}[y_L^{S_3\mu t} (y_R^{S_3\mu t})^*] s_R$	$m_T / m_t$

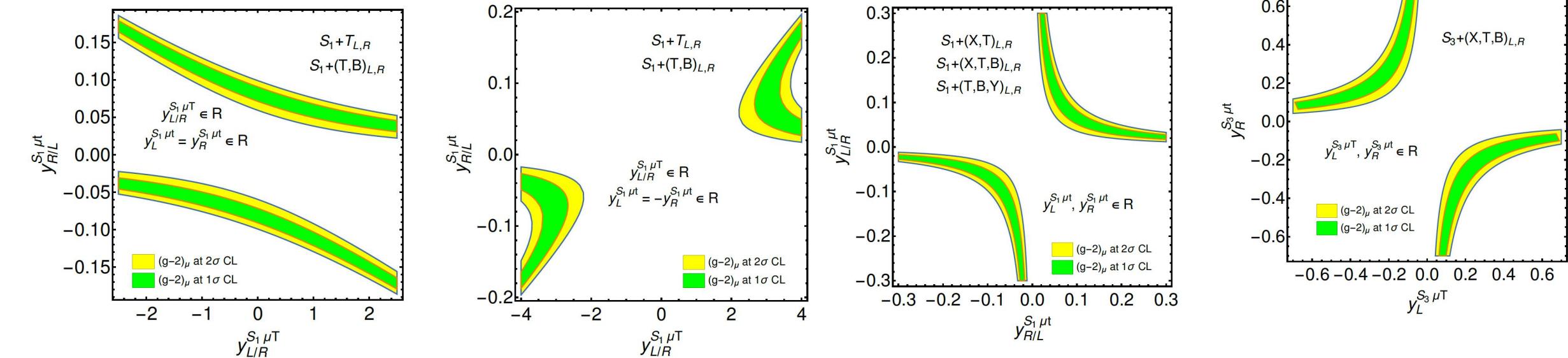
# numerical results

LQ	VLQ	the leading order expressions of $\Delta a_\mu \times 10^7$
$R_2$	$T_{L,R}$	$-0.5238\text{Re}[y_R^{R_2\mu T}(y_L^{R_2\mu t})^*] - 5.393\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(X,T)_{L,R}$	$-5.397\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(T,B)_{L,R}$	$-0.5238\text{Re}[y_L^{R_2\mu T}(y_R^{R_2\mu t})^*] - 5.393\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(X,T,B)_{L,R}$	$-0.3923\text{Re}[y_R^{R_2\mu T}(y_L^{R_2\mu t})^*] - 5.397\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
	$(T,B,Y)_{L,R}$	$-5.397\text{Re}[y_R^{R_2\mu t}(y_L^{R_2\mu t})^*]$
$S_1$	$T_{L,R}$	$0.2331\text{Re}[y_L^{S_1\mu T}(y_R^{S_1\mu t})^*] + 3.754\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
	$(X,T)_{L,R}$	$3.756\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
	$(T,B)_{L,R}$	$0.2331\text{Re}[y_R^{S_1\mu T}(y_L^{S_1\mu t})^*] + 3.754\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
	$(X,T,B)_{L,R}$	$3.756\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
	$(T,B,Y)_{L,R}$	$3.756\text{Re}[y_L^{S_1\mu t}(y_R^{S_1\mu t})^*]$
$S_3$	$(X,T,B)_{L,R}$	$-0.4478\text{Re}[y_L^{S_3\mu T}(y_R^{S_3\mu t})^*]$





not favoured by perturbative unitarity

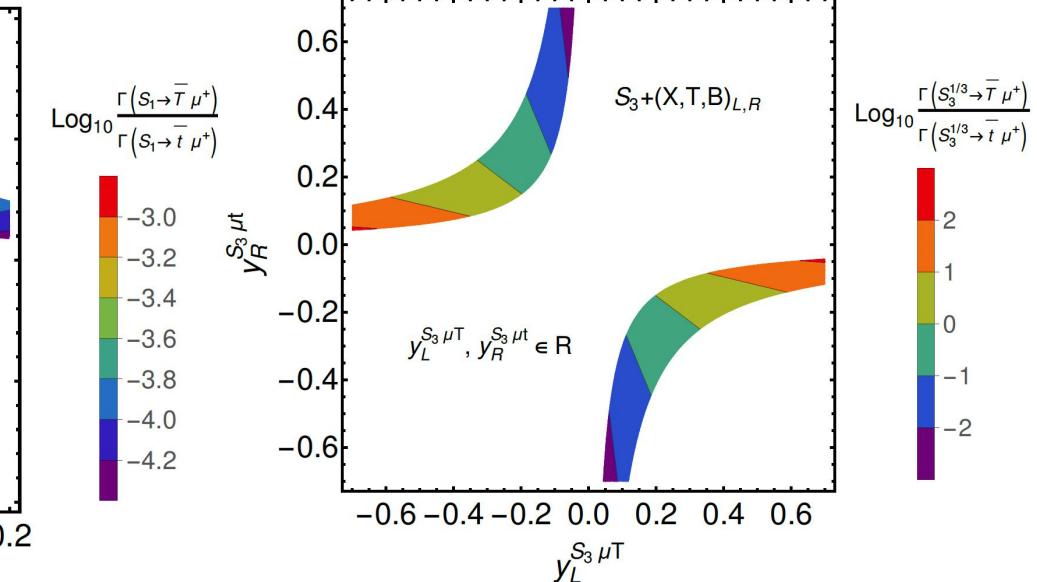
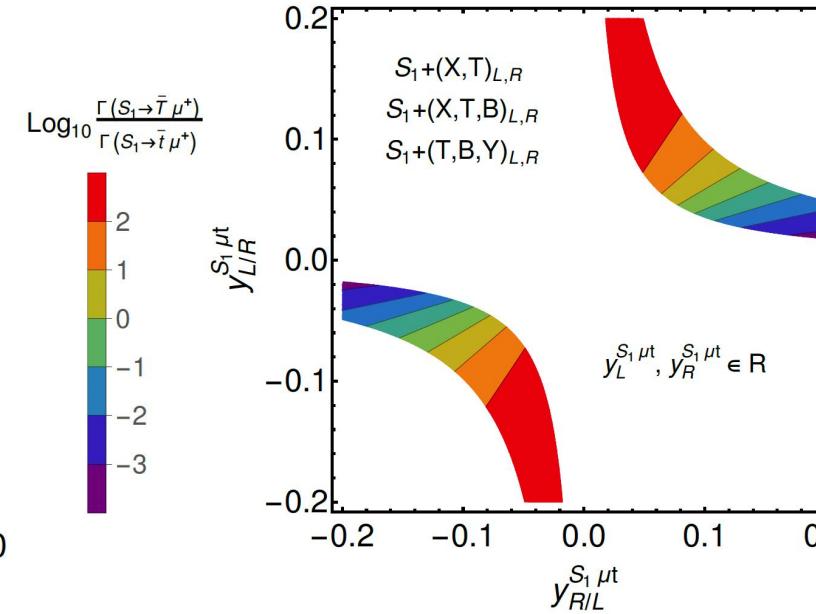
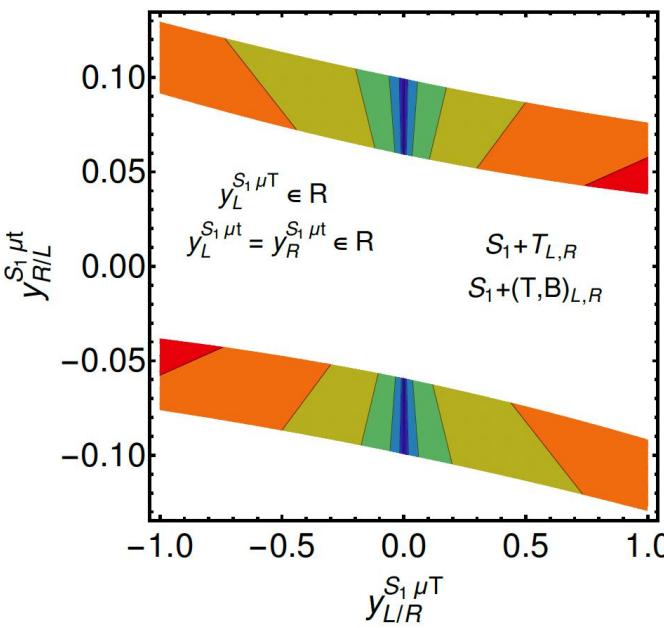
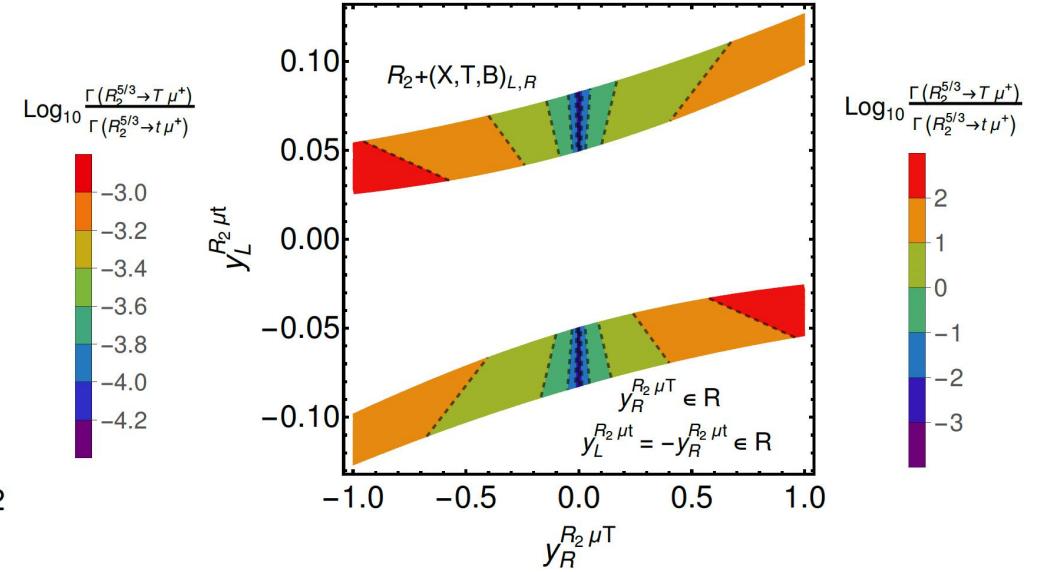
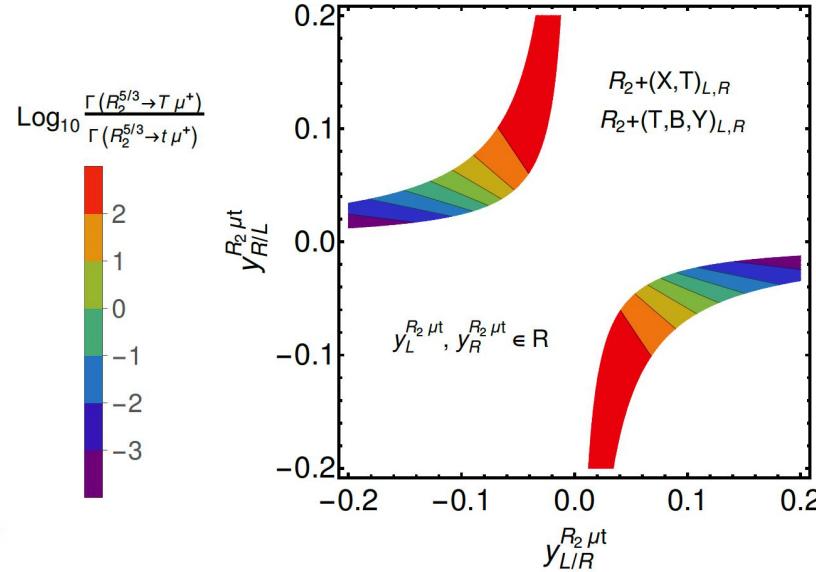
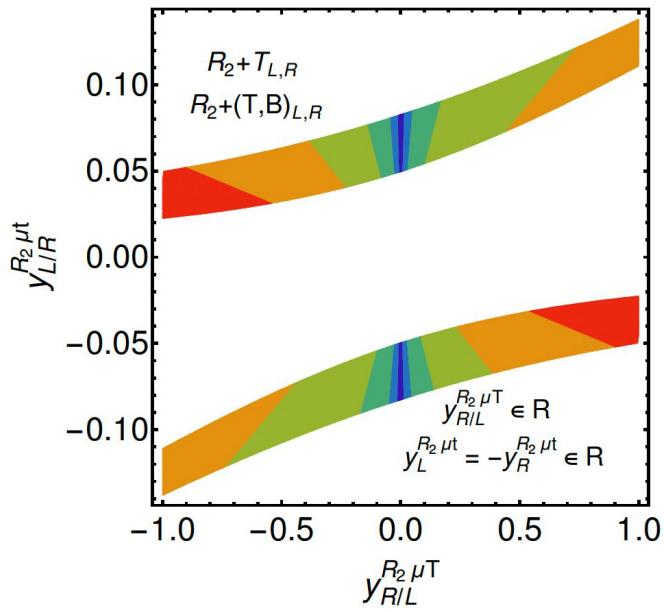


# Collider phenomenology

one LQ + one VLQ

new LQ decay channel  
 $T\mu$

LQ	VLQ	the approximate expressions of $\frac{\Gamma(LQ \rightarrow T\mu)}{\Gamma(LQ \rightarrow t\mu)}$	suppress or not
$R_2$	$T_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2  y_R^{R_2\mu T} ^2 / ( y_L^{R_2\mu t} ^2 +  y_R^{R_2\mu t} ^2)$	No
	$(X, T)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2  y_R^{R_2\mu t} ^2 s_R^2 / ( y_L^{R_2\mu t} ^2 +  y_R^{R_2\mu t} ^2)$	$s_R^2$
	$(T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2  y_L^{R_2\mu T} ^2 / ( y_L^{R_2\mu t} ^2 +  y_R^{R_2\mu t} ^2)$	No
	$(X, T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2  y_R^{R_2\mu T} ^2 / ( y_L^{R_2\mu t} ^2 +  y_R^{R_2\mu t} ^2)$	No
	$(T, B, Y)_{L,R}$	$(1 - \frac{m_T^2}{m_{R_2}^2})^2  y_L^{R_2\mu t} ^2 s_L^2 / ( y_L^{R_2\mu t} ^2 +  y_R^{R_2\mu t} ^2)$	$s_L^2$
$S_1$	$T_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2  y_L^{S_1\mu T} ^2 / ( y_L^{S_1\mu t} ^2 +  y_R^{S_1\mu t} ^2)$	No
	$(X, T)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2  y_L^{S_1\mu t} ^2 s_R^2 / ( y_L^{S_1\mu t} ^2 +  y_R^{S_1\mu t} ^2)$	$s_R^2$
	$(T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2  y_R^{S_1\mu T} ^2 / ( y_L^{S_1\mu t} ^2 +  y_R^{S_1\mu t} ^2)$	No
	$(X, T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2  y_R^{S_1\mu t} ^2 s_L^2 / ( y_L^{S_1\mu t} ^2 +  y_R^{S_1\mu t} ^2)$	$s_L^2$
	$(T, B, Y)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_1}^2})^2  y_R^{S_1\mu t} ^2 s_L^2 / ( y_L^{S_1\mu t} ^2 +  y_R^{S_1\mu t} ^2)$	$s_L^2$
$S_3$	$(X, T, B)_{L,R}$	$(1 - \frac{m_T^2}{m_{S_3}^2})^2  y_L^{S_3\mu T} ^2 /  y_R^{S_3\mu t} ^2$	No



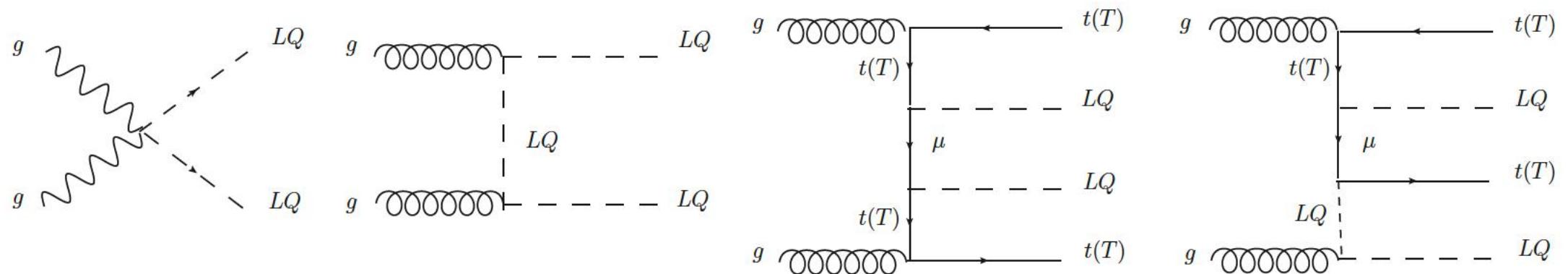
For detailed numerical results of one LQ + two VLQs, refer to our paper



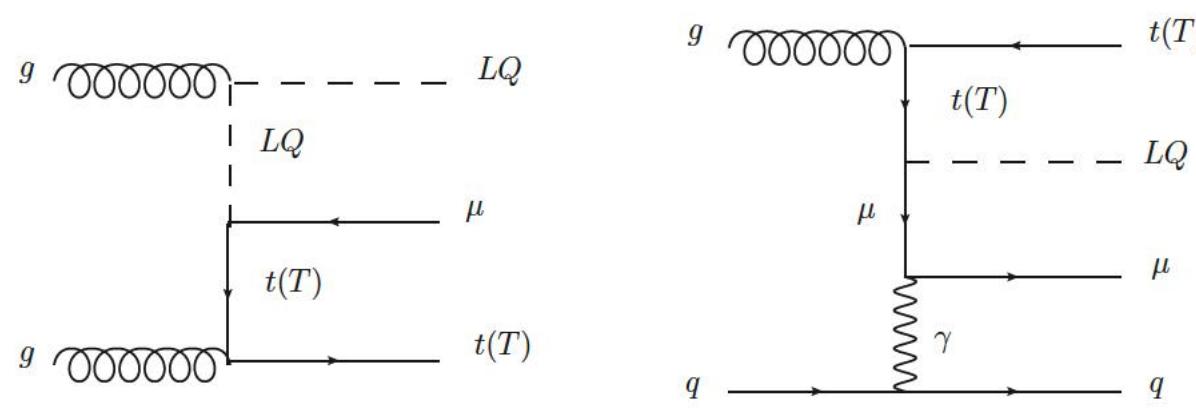
Shi-Ping He (2022)

# LQ production channels

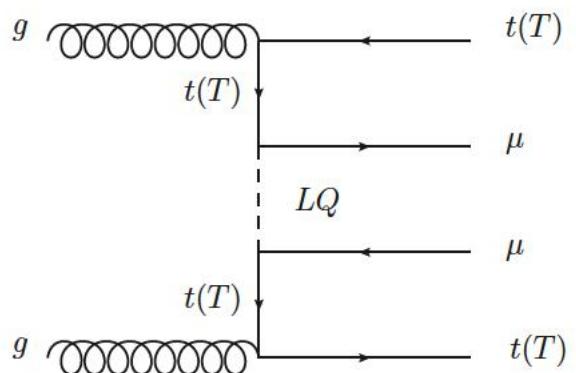
LQ pair



single LQ



off-shell LQ



collider signatures:  
multi- $t$  and multi- $\mu$

$$R_2^{5/3} \rightarrow t \mu^+$$

$$R_2^{5/3} \rightarrow T \mu^+$$

$bW^+, tZ, t\bar{h}$

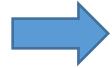
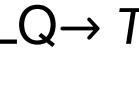
$$S_1/S_3^{1/3} \rightarrow \bar{t} \mu^+$$

$$S_1/S_3^{1/3} \rightarrow \bar{T} \mu^+$$

$bW^+, tZ, t\bar{h}$

$$\begin{aligned} R_2^{2/3} &\rightarrow b/B \mu^+ \\ S_3^{4/3} &\rightarrow \bar{b}/\bar{B} \mu^+ \\ S_3^{-2/3} &\rightarrow \bar{X} \mu^+ \end{aligned}$$

# Summary

- introduction to the  $(g - 2)_\mu$  light scalar mediator  
heavy scalar mediator  chiral enhancement
- one LQ + one VLQ models  $R_2 + T_{L,R}/(X,T)_{L,R}/(T,B)_{L,R}/(X,T,B)_{L,R}/(T,B,Y)_{L,R}$   
 $S_1 + T_{L,R}/(X,T)_{L,R}/(T,B)_{L,R}/(X,T,B)_{L,R}/(T,B,Y)_{L,R}$   
 $S_3 + (X,T,B)_{L,R}$
- phenomenology under the constraints from  $(g - 2)_\mu$    
 $LQ \rightarrow T\mu$   
multi-t and multi- $\mu$  signatures

## Further studies

- $S_3 + (X,T,B)_{L,R}$  model for  $(g - 2)_\mu$  and W mass simultaneously

LQ contributions  
VLQ contributions  W mass

Shi-Ping He, arXiv:2205.02088

Thank you!

谢谢!

Grazie!

# Backups

$$\bar{\mu}(y_L^{S_A \mu q_A} \omega_- + y_R^{S_A \mu q_A} \omega_+) q_A S_A + \text{h.c.}$$

$$\begin{aligned} \Delta a_\mu = & -\frac{N_C m_\mu^2}{8\pi^2 m_{S_A}^2} \{ (|y_L^{S_A \mu q_A}|^2 + |y_R^{S_A \mu q_A}|^2) [Q_{q_A} f_{LL}^q(\frac{m_{q_A}^2}{m_{S_A}^2}) + Q_{S_A} f_{LL}^S(\frac{m_{q_A}^2}{m_{S_A}^2})] \\ & + \frac{2m_{q_A}}{m_\mu} \text{Re}[y_L^{S_A \mu q_A} (y_R^{S_A \mu q_A})^*] \cdot [Q_{q_A} f_{LR}^q(\frac{m_{q_A}^2}{m_{S_A}^2}) + Q_{S_A} f_{LR}^S(\frac{m_{q_A}^2}{m_{S_A}^2})] \} \end{aligned}$$

$$\bar{\mu}(y_L^{S_B \mu q_B} \omega_- + y_R^{S_B \mu q_B} \omega_+) q_B^C S_B + \text{h.c.}$$

$$\begin{aligned} \Delta a_\mu = & -\frac{N_C m_\mu^2}{8\pi^2 m_{S_B}^2} \{ (|y_L^{S_B \mu q_B}|^2 + |y_R^{S_B \mu q_B}|^2) [-Q_{q_B} f_{LL}^q(\frac{m_{q_B}^2}{m_{S_B}^2}) + Q_{S_B} f_{LL}^S(\frac{m_{q_B}^2}{m_{S_B}^2})] \\ & + \frac{2m_{q_B}}{m_\mu} \text{Re}[y_L^{S_B \mu q_B} (y_R^{S_B \mu q_B})^*] \cdot [-Q_{q_B} f_{LR}^q(\frac{m_{q_B}^2}{m_{S_B}^2}) + Q_{S_B} f_{LR}^S(\frac{m_{q_B}^2}{m_{S_B}^2})] \} \end{aligned}$$

$$\bar{\mu} t(R_2^{5/3})^*, \bar{\mu} T(R_2^{5/3})^*$$

$$Q_{q_A} = \frac{2}{3}, Q_{S_A} = -\frac{5}{3}$$

$$\bar{\mu} b(R_2^{2/3})^*, \bar{\mu} B(R_2^{2/3})^*$$

$$Q_{q_A} = -\frac{1}{3}, Q_{S_A} = -\frac{2}{3}$$

$$\bar{\mu} t^c(S_1^{1/3})^*, \bar{\mu} T^c(S_1^{1/3})^*$$

$$Q_{q_B} = \frac{2}{3}, Q_{S_A} = -\frac{1}{3}$$

$$\bar{\mu} t^c(S_3^{1/3})^*, \bar{\mu} T^c(S_3^{1/3})^*$$

$$\bar{\mu} X^c(S_3^{-2/3})^*$$

$$Q_{q_B} = \frac{5}{3}, Q_{S_A} = \frac{2}{3}$$

$\bar{\mu} t (R_2^{5/3})^*$  interaction  
related functions

$$f_{LL}^{R_2}(x) \equiv -2f_{LL}^q(x) + 5f_{LL}^S(x) = -\frac{3 - 8x + x^2 + 4x^3 + 2x(2 - 5x)\log(x)}{4(1-x)^4}$$

$$f_{LR}^{R_2}(x) \equiv -2f_{LR}^q(x) + 5f_{LR}^S(x) = \frac{1 - 8x + 7x^2 + (4 - 10x)\log(x)}{4(1-x)^3}$$

$\bar{\mu} t^c (S_1^{1/3})^*$  interaction  
related functions

$$f_{LL}^{S_1}(x) \equiv 2f_{LL}^q(x) + f_{LL}^S(x) = \frac{1 + 4x - 5x^2 + 2x(2 + x)\log(x)}{4(1-x)^4}$$

$$f_{LR}^{S_1}(x) \equiv 2f_{LR}^q(x) + f_{LR}^S(x) = -\frac{7 - 8x + x^2 + (4 + 2x)\log(x)}{4(1-x)^3}$$

$\bar{\mu} b (R_2^{2/3})^*$  interaction  
related functions

$$\bar{f}_{LL}^{R_2}(x) \equiv f_{LL}^q(x) + 2f_{LL}^S(x) = \frac{x[5 - 4x - x^2 + (2 + 4x)\log(x)]}{4(1-x)^4}$$

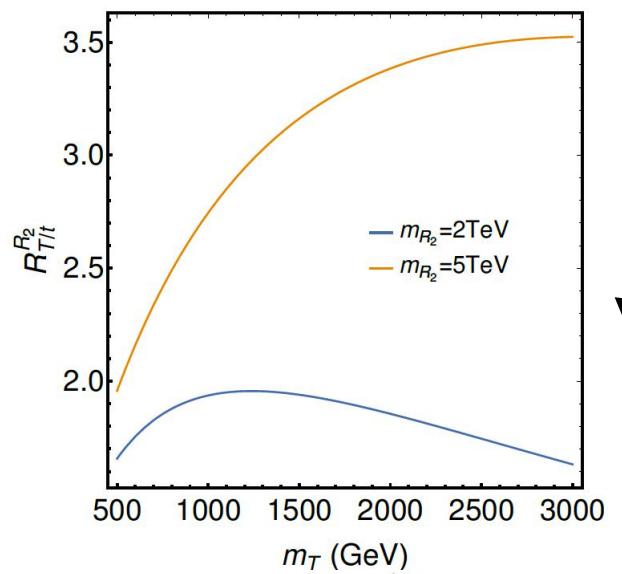
$$\bar{f}_{LR}^{R_2}(x) \equiv f_{LR}^q(x) + 2f_{LR}^S(x) = -\frac{5 - 4x - x^2 + (2 + 4x)\log(x)}{4(1-x)^3}$$

$\bar{\mu} b^c (S_3^{4/3})^*$  interaction  
related functions

$$\bar{f}_{LL}^{S_3}(x) \equiv -f_{LL}^q(x) + 4f_{LL}^S(x) = -\frac{2 - 7x + 2x^2 + 3x^3 + 2x(1 - 4x)\log(x)}{4(1-x)^4}$$

$$\bar{f}_{LR}^{S_3}(x) \equiv -f_{LR}^q(x) + 4f_{LR}^S(x) = -\frac{1 + 4x - 5x^2 - (2 - 8x)\log(x)}{4(1-x)^3}$$

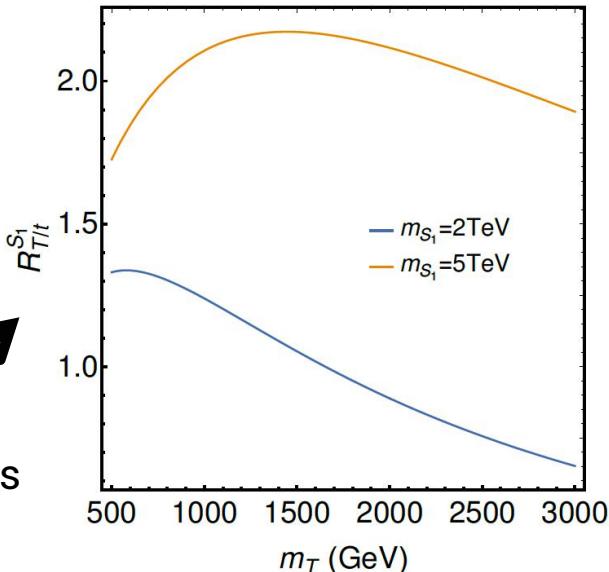
$$R_{T/t}^{R_2}(m_T, m_{R_2}) \equiv \frac{m_T f_{LR}^{R_2}(m_T^2/m_{R_2}^2)}{m_t f_{LR}^{R_2}(m_t^2/m_{R_2}^2)}$$



the ratio of T loop integral  
to the top one

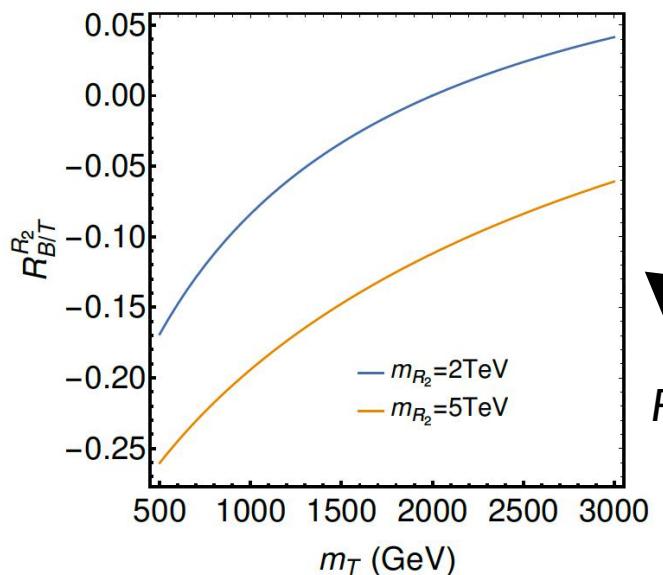
$R_2 + \text{VLQ models}$

$$R_{T/t}^{S_1}(m_T, m_{S_1}) \equiv \frac{m_T f_{LR}^{S_1}(m_T^2/m_{S_1}^2)}{m_t f_{LR}^{S_1}(m_t^2/m_{S_1}^2)}$$



$S_1 + \text{VLQ models}$

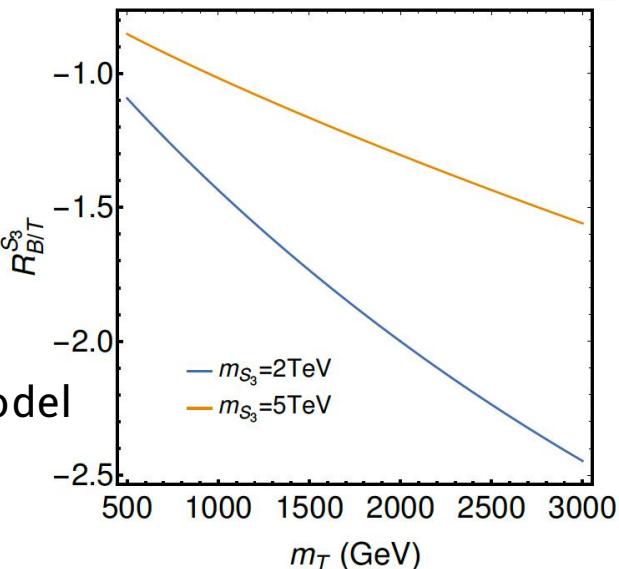
$$R_{B/T}^{R_2}(m_T, m_{R_2}) \equiv \frac{\tilde{f}_{LR}^{R_2}(m_T^2/m_{R_2}^2)}{f_{LR}^{R_2}(m_T^2/m_{R_2}^2)}$$



the ratio of B loop integral  
to the T one

$R_2 + (X, T, B)_{L,R} \text{ model}$

$$R_{B/T}^{S_3}(m_T, m_{S_3}) \equiv \frac{\tilde{f}_{LR}^{S_3}(m_T^2/m_{S_3}^2)}{f_{LR}^{S_3}(m_T^2/m_{S_3}^2)}$$



$S_3 + (X, T, B)_{L,R} \text{ model}$

one LQ + two VLQs

$$R_2/S_1 + T_{L,R} + (T,B)_{L,R}$$

turn off the LQ  $\mu t$  interactions

$\mu$  only interacts with heavy quarks

turn off the  $t - T$  mixings

$$\Delta a_\mu^{R_2+T_{L,R}+(T,B)_{L,R}} \approx \frac{m_\mu^2}{4\pi^2 m_{R_2}^2} \text{Re}[y_R^{R_2\mu T_1} (y_L^{R_2\mu T_2})^*] \left[ -\frac{m_T}{m_\mu} f_{LR}^{R_2} \left( \frac{m_T^2}{m_{R_2}^2} \right) s_L^T c_R^T + \frac{m_{T'}}{m_\mu} f_{LR}^{R_2} \left( \frac{m_{T'}^2}{m_{R_2}^2} \right) s_R^T c_L^T \right]$$

$$\Delta a_\mu^{S_1+T_{L,R}+(T,B)_{L,R}} \approx \frac{m_\mu^2}{4\pi^2 m_{S_1}^2} \text{Re}[y_L^{S_1\mu T_1} (y_R^{S_1\mu T_2})^*] \left[ -\frac{m_T}{m_\mu} f_{LR}^{S_1} \left( \frac{m_T^2}{m_{S_1}^2} \right) s_L^T c_R^T + \frac{m_{T'}}{m_\mu} f_{LR}^{S_1} \left( \frac{m_{T'}^2}{m_{S_1}^2} \right) s_R^T c_L^T \right]$$

cancellation if  $m_T = m_{T'}$  and  $\theta_L^T = \theta_R^T$

constructive interference if  $m_T = m_{T'}$  and  $\theta_L^T = -\theta_R^T$

For detailed numerical results, refer to our paper  Shi-Ping He (2022)