XLI International Conference on High Energy Physics

Accidental symmetries in the scalar potential of the Standard Model extended with two Higgs triplets

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IHEP, CAS

2022.7.8

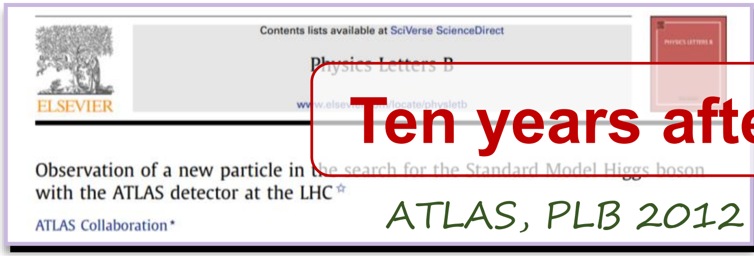
Based on XW, Y. Wang and S. Zhou, JHEP 02 (2022) 059

ICHEP 2022
BOLOGNA

Outline

- Background and Motivation
- Bilinear-field formalism
- Accidental symmetries in 2HTM
- Summary

Background and Motivation


Contents lists available at SciVerse ScienceDirect

Physics Letters B

ELSEVIER

Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC

ATLAS Collaboration

ATLAS, PLB 2012


Contents lists available at SciVerse ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC

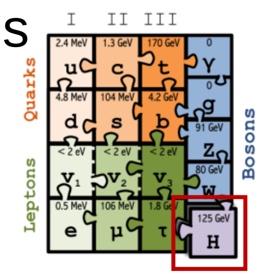
CMS Collaboration

CMS, PLB 2012

Ten years after the discovery…

Huge success

- Prove the validity of Higgs mechanism for spontaneous symmetry breaking
- Paint a clearer portrait of the Higgs boson
- Complete the last piece of the puzzle of the SM



Remaining puzzles

- Nonzero neutrino masses
  - Relation to the inflation in the early Universe
  - Naturalness problem
- Electroweak phase transition
- Matter-antimatter asymmetry

Extended version of the Higgs sector? One Higgs or more?

To be examined over the next fifteen years

CMS, 2207.00043
ATLAS, 2207.00092

Background and Motivation

Two-Higgs-triplet model (2HTM)

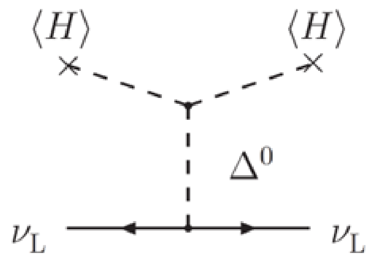
Extend the SM by introducing two triplet scalars with the same hypercharge $Y = -2$

$$\phi_i \equiv (\xi_i^1, \xi_i^2, \xi_i^3)^{\rm T} \ (i = 1, 2)$$

$$\Delta_i = \begin{pmatrix} \xi_3^i & \xi_1^i - {\rm i}\xi_2^i \\ \xi_1^i + {\rm i}\xi_2^i & -\xi_3^i \end{pmatrix} = \sqrt{2}\begin{pmatrix} \Delta^-/\sqrt{2} & \Delta^0 \\ \Delta^{--} & -\Delta^-/\sqrt{2} \end{pmatrix}$$

- Neutrino masses

$$-\mathcal{L}_{\rm Y} = \overline{\ell_{\rm L}} Y_l H E_{\rm R} + \frac{1}{2}\overline{\ell_{\rm L}} Y_{\nu 1} \boldsymbol{\sigma}\cdot\boldsymbol{\phi}_1 {\rm i}\sigma^2 \ell_{\rm L}^{\rm c} + \frac{1}{2}\overline{\ell_{\rm L}} Y_{\nu 2} \boldsymbol{\sigma}\cdot\boldsymbol{\phi}_2 {\rm i}\sigma^2 \ell_{\rm L}^{\rm c} + {\rm h.c.}$$

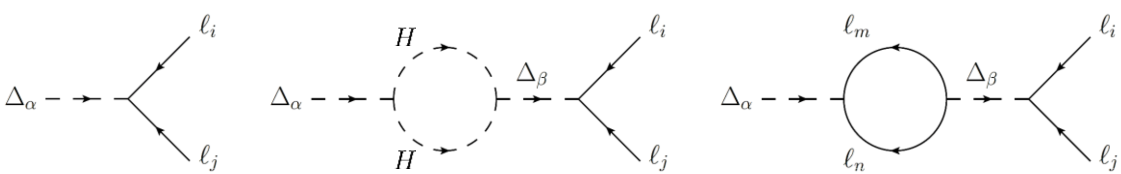


W. Konetschny et al., PLB 1977
M. Magg et al., PLB 1980
J. Schechter et al., PRD 1980
T.P. Cheng et al., PRD 1980
G. Lazarides et al., NPB 1981
R.N. Mohapatra et al., PRD 1981

$\langle \boldsymbol{\phi}_i \rangle = \sqrt{2} v_i$, $\langle H \rangle = v_{\rm H}/\sqrt{2}$ → $\boldsymbol{M_l \equiv Y_l v_{\rm H}/\sqrt{2}}$, $\boldsymbol{M_\nu = \sqrt{2} Y_{\nu 1} v_1 + \sqrt{2} Y_{\nu 2} v_2}$

Tiny neutrino masses can be attributed to the small vev's of $\boldsymbol{\phi}_i$

- Leptogenesis



E. Ma et al., PRL 1998
T. Hambye et al., NPB 2001
D.A. Sierra et al., JCAP 2014

Require at least two Higgs triplets → Guarantee CP violation

- Non-trivial spontaneous CP violation can emerge in the scalar sector

T.D. Lee, PRD 1973; P.M. Ferreira et al., JHEP, 2022

Background and Motivation

Gauge-invariant Lagrangian

$V_{\mathrm{H}} + V_{\phi} + V_{\mathrm{H}\phi}$

$$\mathcal{L}_{\mathrm{2HTM}} = (\mathcal{D}^\mu H)^\dagger(\mathcal{D}_\mu H) + (\mathcal{D}^\mu \phi_1)^\dagger \cdot (\mathcal{D}_\mu \phi_1) + (\mathcal{D}^\mu \phi_2)^\dagger \cdot (\mathcal{D}_\mu \phi_2) - \boxed{V_{\mathrm{2HTM}}}$$

$$V_{\mathrm{H}} = -\mu_{\mathrm{H}}^2 H^\dagger H + \lambda_{\mathrm{H}}(H^\dagger H)^2 \, ,$$

$$\begin{aligned}
V_\phi = & m_{11}^2(\phi_1^* \cdot \phi_1) + m_{22}^2(\phi_2^* \cdot \phi_2) + m_{12}^2(\phi_1^* \cdot \phi_2) + m_{12}^{*2}(\phi_1 \cdot \phi_2^*) + \lambda_1(\phi_1^* \cdot \phi_1)^2 \\
& + \lambda_2(\phi_2^* \cdot \phi_2)^2 + \lambda_3(\phi_1^* \cdot \phi_1)(\phi_2^* \cdot \phi_2) + \lambda_4(\phi_1^* \cdot \phi_2)(\phi_1 \cdot \phi_2^*) + \frac{\lambda_5}{2}(\phi_1^* \cdot \phi_2)^2 \\
& + \frac{\lambda_5^*}{2}(\phi_1 \cdot \phi_2^*)^2 + (\phi_1^* \cdot \phi_1)\left[\lambda_6(\phi_1^* \cdot \phi_2) + \lambda_6^*(\phi_1 \cdot \phi_2^*)\right] \\
& + (\phi_2^* \cdot \phi_2)\left[\lambda_7(\phi_1^* \cdot \phi_2) + \lambda_7^*(\phi_1 \cdot \phi_2^*)\right] + \lambda_8(\phi_1^* \cdot \phi_1^*)(\phi_1 \cdot \phi_1) \\
& + \lambda_9(\phi_2^* \cdot \phi_2^*)(\phi_2 \cdot \phi_2) + \lambda_{10}(\phi_1^* \cdot \phi_2^*)(\phi_1 \cdot \phi_2) \\
& + \lambda_{11}(\phi_1^* \cdot \phi_1^*)(\phi_2 \cdot \phi_2) + \lambda_{11}^*(\phi_1 \cdot \phi_1)(\phi_2^* \cdot \phi_2^*) + \lambda_{12}(\phi_1^* \cdot \phi_1^*)(\phi_1 \cdot \phi_2) \\
& + \lambda_{12}^*(\phi_1 \cdot \phi_1)(\phi_1^* \cdot \phi_2^*) + \lambda_{13}(\phi_2^* \cdot \phi_2^*)(\phi_1 \cdot \phi_2) + \lambda_{13}^*(\phi_2 \cdot \phi_2)(\phi_1^* \cdot \phi_2^*) \, ,
\end{aligned}$$

$$\begin{aligned}
V_{\mathrm{H}\phi} = & \lambda_{14}(H^\dagger H)(\phi_1^* \cdot \phi_1) + \lambda_{15}(H^\dagger H)(\phi_2^* \cdot \phi_2) + \lambda_{16}(H^\dagger H)(\phi_1^* \cdot \phi_2) \\
& + \lambda_{16}^*(H^\dagger H)(\phi_1 \cdot \phi_2^*) + \lambda_{17}(H^\dagger \mathrm{i}\boldsymbol{\sigma} H) \cdot (\phi_1^* \times \phi_1) + \lambda_{18}(H^\dagger \mathrm{i}\boldsymbol{\sigma} H) \cdot (\phi_2^* \times \phi_2) \\
& + \lambda_{19}(H^\dagger \mathrm{i}\boldsymbol{\sigma} H) \cdot (\phi_1^* \times \phi_2) + \lambda_{19}^*(H^\dagger \mathrm{i}\boldsymbol{\sigma} H) \cdot (\phi_2^* \times \phi_1) \\
& + (\mu_1 H^{\mathrm{T}} \mathrm{i}\sigma_2 \boldsymbol{\sigma} \cdot \phi_1 H + \mu_2 H^{\mathrm{T}} \mathrm{i}\sigma_2 \boldsymbol{\sigma} \cdot \phi_2 H + \mathrm{h.c.})
\end{aligned}$$

Quite complex

Background and Motivation

Accidental symmetry

*"… It often happens that condition of renormalizability is so stringent that the effective Lagrangian **automatically** obeys one or more symmetries, which are not symmetries of the underlying theory, and may therefore be violated by the suppressed non-renormalizable terms in the effective Lagrangian…"*

—— S. Weinberg, "The Quantum theory of fields. Vol. 1: Foundations"

A little bit different from the **accidental symmetry** involved in this work:

Specific relations among coupling constants ⟺ Accidental symmetries: Symmetries that **automatically** exist in the scalar potential apart from the gauge symmetry

- Higgs family symmetry: $\phi_i \rightarrow \Lambda_{ij}\phi_j$
- Generalized CP symmetry: $\phi_i \rightarrow \Lambda_{ij}\phi_j^*$

Reduce the number of free parameters

Enhance the predictive power of the theory

Example $V \supset (\phi_1^* \cdot \phi_1 + \phi_2^* \cdot \phi_2)^2$

Keeps invariant under

$$\phi_1 \rightarrow +\phi_1 \cos\theta + \phi_2 \sin\theta$$

$$\phi_2 \rightarrow -\phi_1 \sin\theta + \phi_2 \cos\theta$$

Bilinear-field formalism

Convert ϕ_1 and ϕ_2 into a vector R^μ in the bilinear space

Preliminary: 2HDM

$$\mathrm{SU}(2)\otimes\mathrm{SU}(2)\to\mathrm{SO}(1,3)$$

$$\Phi=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} \xrightarrow{R^\mu=\phi^\dagger\sigma^\mu\phi} R^\mu=\begin{pmatrix}\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2\\ \phi_1^\dagger\phi_2+\phi_2^\dagger\phi_1\\ -\mathrm{i}\left(\phi_1^\dagger\phi_2-\phi_2^\dagger\phi_1\right)\\ \phi_1^\dagger\phi_1-\phi_2^\dagger\phi_2\end{pmatrix}$$

M. Maniatis et al, EPJC 2006
C.C. Nishi, PRD 2006
I.P. Ivanov, PLB 2006
I. P. Ivanov, PRD 2007, 2008

Minkowski space

Reveal the geometrical properties of the 2HDM

The 2HTM case

$\boldsymbol{\phi}$ and $\boldsymbol{\phi}^*$ transform in the same way under the $\mathrm{SU}(2)_\mathrm{L}$ group (adjoint rep.)

$$\Phi=\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_1^*\\ \phi_2^*\end{pmatrix}$$

- Necessary for us to rewrite the terms like $(\phi_i^\dagger\phi_j^*)(\phi_m^\mathrm{T}\phi_n)$
- Investigate family and CP symmetries simultaneously

"Majorana formalism" $\Phi=\mathrm{C}\Phi^*\,(\mathrm{C}=\sigma^1\otimes\sigma^0\otimes\mathbf{I}_{3\times3})$

Require $R^\mu=\Phi^\dagger\Sigma^\mu\Phi$ $(\Sigma^\mu\equiv\Sigma^\mu_{\alpha\beta}\sigma^\alpha\otimes\sigma^\beta)$ to be invariant under C

R.A. Battye et al., JHEP 2011, A. Pilaftsis, PLB 2012

$$R^\mu=\Phi^\dagger\Sigma^\mu\Phi$$

$$\begin{aligned}
\Sigma^0&=+\frac{1}{2}\sigma^0\otimes\sigma^0\,, & \Sigma^1&=-\frac{1}{2}\sigma^2\otimes\sigma^3\,,\\
\Sigma^2&=-\frac{1}{2}\sigma^1\otimes\sigma^0\,, & \Sigma^3&=+\frac{1}{2}\sigma^2\otimes\sigma^1\,,\\
\Sigma^4&=-\frac{1}{2}\sigma^1\otimes\sigma^3\,, & \Sigma^5&=+\frac{1}{2}\sigma^2\otimes\sigma^0\,,\\
\Sigma^6&=+\frac{1}{2}\sigma^1\otimes\sigma^1\,, & \Sigma^7&=+\frac{1}{2}\sigma^0\otimes\sigma^1\,,\\
\Sigma^8&=-\frac{1}{2}\sigma^3\otimes\sigma^2\,, & \Sigma^9&=+\frac{1}{2}\sigma^0\otimes\sigma^3\,.
\end{aligned}$$

Bilinear-field formalism

Convert ϕ_1 and ϕ_2 into a vector R^μ in the bilinear space

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$$\mathrm{SU}(2) \otimes \mathrm{SU}(2) \rightarrow \mathrm{SO}(1,3)$$

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \xrightarrow{R^\mu = \phi^\dagger \sigma^\mu \phi} R^\mu = \begin{pmatrix} \phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 \\ \phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \\ -\mathrm{i}\left(\phi_1^\dagger \phi_2 - \phi_2^\dagger \phi_1\right) \\ \phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2 \end{pmatrix}$$

M. Maniatis et al, EPJC 2006
C.C. Nishi, PRD 2006
I.P. Ivanov, PLB 2006
I. P. Ivanov, PRD 2007, 2008

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Require $R^\mu = \Phi^\dagger \Sigma^\mu \Phi \; (\Sigma^\mu \equiv \Sigma^\mu_{\alpha\beta} \sigma^\alpha \otimes \sigma^\beta)$ to be invariant under C

$$R^\mu = \begin{pmatrix} \phi_1^* \cdot \phi_1 + \phi_2^* \cdot \phi_2 \\ +\frac{\mathrm{i}}{2}\left(\phi_1^* \cdot \phi_1^* - \phi_1 \cdot \phi_1 - \phi_2^* \cdot \phi_2^* + \phi_2 \cdot \phi_2\right) \\ -\frac{1}{2}\left(\phi_1^* \cdot \phi_1^* + \phi_1 \cdot \phi_1 + \phi_2^* \cdot \phi_2^* + \phi_2 \cdot \phi_2\right) \\ -\mathrm{i}\left(\phi_1^* \cdot \phi_2^* - \phi_1 \cdot \phi_2\right) \\ -\frac{1}{2}\left(\phi_1^* \cdot \phi_1^* - \phi_2^* \cdot \phi_2^* + \phi_1 \cdot \phi_1 - \phi_2 \cdot \phi_2\right) \\ -\frac{\mathrm{i}}{2}\left(\phi_1^* \cdot \phi_1^* + \phi_2^* \cdot \phi_2^* - \phi_1 \cdot \phi_1 - \phi_2 \cdot \phi_2\right) \\ \phi_1^* \cdot \phi_2^* + \phi_1 \cdot \phi_2 \\ \phi_1^* \cdot \phi_2 + \phi_1 \cdot \phi_2^* \\ \mathrm{i}\left(\phi_1^* \cdot \phi_2 - \phi_1 \cdot \phi_2^*\right) \\ \phi_1^* \cdot \phi_1 - \phi_2^* \cdot \phi_2 \end{pmatrix}$$

R.A. Battye et al., JHEP 2011, A. Pilaftsis, PLB 2012

Bilinear-field formalism

The 2HTM case

Pure-triplet scalar potential

$$V_\phi = \frac{1}{2} M_\mu R^\mu + \frac{1}{4} L_{\mu\nu} R^\mu R^\nu$$

$$M_\mu = \left(m_{11}^2 + m_{22}^2,\ 0,\ \cdots,\ 0,\ 2\mathrm{Re}\, m_{12}^2,\ 2\mathrm{Im}\, m_{12}^2,\ m_{11}^2 - m_{22}^2\right)$$

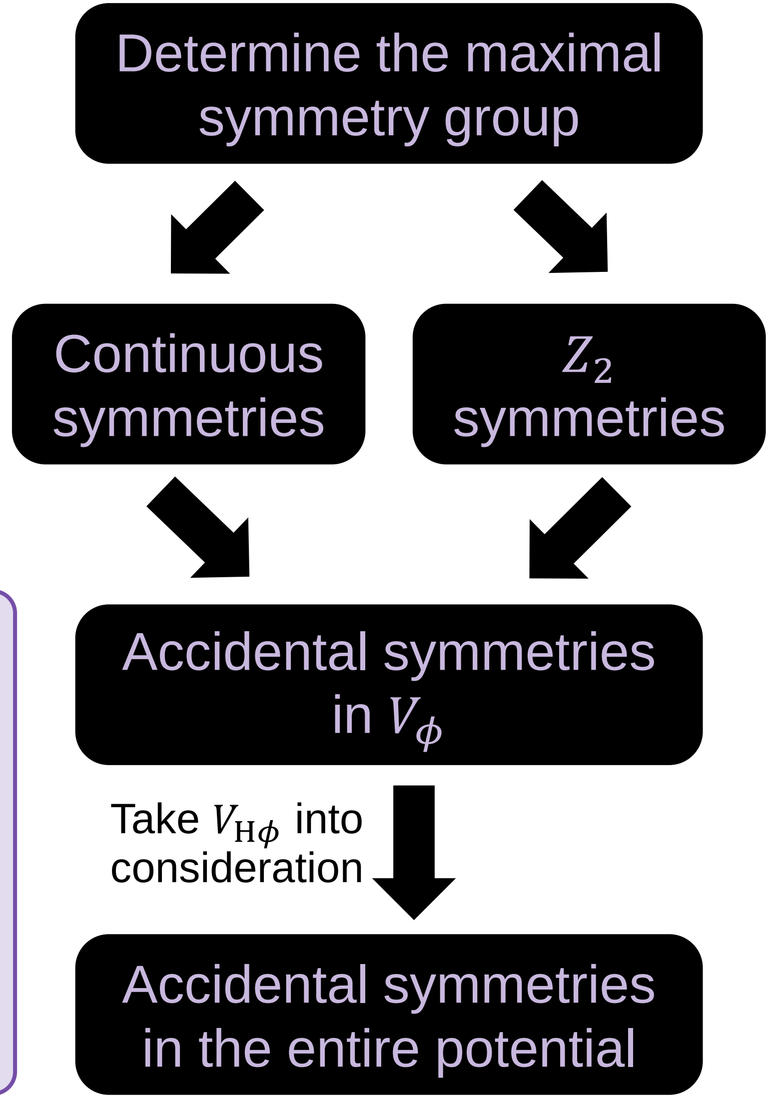
$$L_{\mu\nu} = \begin{pmatrix} \lambda_1 + \lambda_2 + \lambda_3 & \mathbf{0}_{1\times 3} & \mathbf{0}_{1\times 3} & P \\ \mathbf{0}_{3\times 1} & K_1 & K_2 & \mathbf{0}_{3\times 3} \\ \mathbf{0}_{3\times 1} & K_2^{\mathrm{T}} & K_1 & \mathbf{0}_{3\times 3} \\ P^{\mathrm{T}} & \mathbf{0}_{3\times 3} & \mathbf{0}_{3\times 3} & Q \end{pmatrix}$$

$$P \equiv \begin{pmatrix} \mathrm{Re}(\lambda_6 + \lambda_7) & \mathrm{Im}(\lambda_6 + \lambda_7) & \lambda_1 - \lambda_2 \end{pmatrix}$$

$$K_1 \equiv \begin{pmatrix} \lambda_8 + \lambda_9 - 2\,\mathrm{Re}\,\lambda_{11} & -2\,\mathrm{Im}\,\lambda_{11} & -\mathrm{Re}(\lambda_{12} - \lambda_{13}) \\ -2\,\mathrm{Im}\,\lambda_{11} & \lambda_8 + \lambda_9 + 2\,\mathrm{Re}\,\lambda_{11} & -\mathrm{Im}(\lambda_{12} + \lambda_{13}) \\ -\mathrm{Re}(\lambda_{12} - \lambda_{13}) & -\mathrm{Im}(\lambda_{12} + \lambda_{13}) & \lambda_{10} \end{pmatrix}$$

$$K_2 \equiv \begin{pmatrix} 0 & -(\lambda_8 - \lambda_9) & +\mathrm{Im}(\lambda_{12} - \lambda_{13}) \\ \lambda_8 - \lambda_9 & 0 & -\mathrm{Re}(\lambda_{12} + \lambda_{13}) \\ -\mathrm{Im}(\lambda_{12} - \lambda_{13}) & \mathrm{Re}(\lambda_{12} + \lambda_{13}) & 0 \end{pmatrix}$$

$$Q \equiv \begin{pmatrix} \lambda_4 + \mathrm{Re}\,\lambda_5 & \mathrm{Im}\,\lambda_5 & \mathrm{Re}(\lambda_6 - \lambda_7) \\ \mathrm{Im}\,\lambda_5 & \lambda_4 - \mathrm{Re}\,\lambda_5 & \mathrm{Im}(\lambda_6 - \lambda_7) \\ \mathrm{Re}(\lambda_6 - \lambda_7) & \mathrm{Im}(\lambda_6 - \lambda_7) & \lambda_1 + \lambda_2 - \lambda_3 \end{pmatrix}$$



Accidental symmetries in 2HTM

The maximal symmetry group

Symmetry transformation in $\mathbf{\Phi}$-space $U \in \mathrm{U}(4)$

Majorana condition $\mathbf{C^{-1} J^a C = -(J^a)^*}$

$$
\begin{aligned}
J^1 &= \frac{1}{2}\sigma^3 \otimes \sigma^3 , \quad J^2 = \frac{1}{2}\sigma^3 \otimes \sigma^1 , \quad J^3 = \frac{1}{2}\sigma^0 \otimes \sigma^2 , \\
J^4 &= \frac{1}{2}\sigma^3 \otimes \sigma^0 , \quad J^5 = \frac{1}{2}\sigma^1 \otimes \sigma^2 , \quad J^6 = \frac{1}{2}\sigma^2 \otimes \sigma^2 .
\end{aligned}
$$

Lie algebra

$$
[J^i, J^j] = \mathrm{i}\epsilon^{ijk} J^k , \quad [J^{i+3}, J^{j+3}] = \mathrm{i}\epsilon^{ijk} J^{k+3} , \quad [J^i, J^{j+3}] = 0 , \quad (i,j,k = 1,2,3)
$$

Isomorphic to $SU(2) \otimes SU(2)$ (Φ space)

$\mathbf{\Phi}$-space $\xrightarrow{\delta R^i = \mathrm{i}\theta_a \Phi^\dagger [\Sigma^i, J^a]\Phi}$ ***R^i*-space**

2-1 correspondence

$\mathrm{SO(4)} \simeq [\mathrm{SU(2)} \times \mathrm{SU(2)}]/\mathrm{Z}_2$

Representation matrices T^a in the R^i-space

$$
(T^a)_{ij} = \mathrm{Tr}\left([\Sigma^i, J^a]\Sigma^j\right) , \quad (\text{for } i,j = 1,2,\cdots,9)
$$

$$
T^1 = \begin{pmatrix}
0 & +\mathrm{i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\mathrm{i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & +\mathrm{i} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\mathrm{i} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & +\mathrm{i} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\mathrm{i} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

Two SO(3) rotations

Accidental symmetries in 2HTM

Continuous symmetries

Accidental symmetries ⟷ **Relations among coupling constants**

Take the SO(4) symmetry for instance:

R^i: rank-two tensor r^{ij} **M_i: rank-two tensor M_{ij}** **L_{ij}: rank-four tensor $L_{im,jn}$**

$$\begin{pmatrix} R^1 & R^2 & R^3 \\ R^4 & R^5 & R^6 \\ R^7 & R^8 & R^9 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 & 2\,\mathrm{Re}\, m_{12}^2 \\ 0 & 0 & 2\,\mathrm{Im}\, m_{12}^2 \\ 0 & 0 & m_{11}^2 - m_{22}^2 \end{pmatrix} \qquad \begin{pmatrix} (K_1)_{ij} & (K_2)_{ij} & \mathbf{0}_{3\times3} \\ (K_2^{\mathrm{T}})_{ij} & (K_1)_{ij} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & Q_{ij} \end{pmatrix}$$

SO(3): $J^{1,2,3}$

SO(3): $J^{4,5,6}$

One should require $M_{ij} = (K_2)_{ij} = 0, \quad P = \mathbf{0}, \quad K_1 = Q \propto \mathbf{I}_{3\times3}$

$$m_{11}^2 = m_{22}^2\,, \quad m_{12}^2 = 0\,, \quad \lambda_1 = \lambda_2\,, \quad \lambda_3 = 2\lambda_1 - 2\lambda_8\,,$$
$$\lambda_4 = \lambda_{10} = 2\lambda_8 = 2\lambda_9\,, \quad \lambda_5 = \lambda_6 = \lambda_7 = \lambda_{11} = \lambda_{12} = \lambda_{13} = 0\,.$$

$$\begin{aligned} V_{\phi,\,\mathrm{SO}(4)} = {} & m_{11}^2(\phi_1^*\cdot\phi_1 + \phi_2^*\cdot\phi_2) + \lambda_1(\phi_1^*\cdot\phi_1 + \phi_2^*\cdot\phi_2)^2 \\ & + 2\lambda_8\left[(\phi_1^*\cdot\phi_2)(\phi_1\cdot\phi_2^*) - (\phi_1^*\cdot\phi_1)(\phi_2^*\cdot\phi_2)\right] \\ & + \lambda_8\left[(\phi_1^*\cdot\phi_1^*)(\phi_1\cdot\phi_1) + 2(\phi_1^*\cdot\phi_2^*)(\phi_1\cdot\phi_2) + (\phi_2^*\cdot\phi_2^*)(\phi_2\cdot\phi_2)\right] \end{aligned}$$

Only three independent parameters m_{11}^2, λ_1 and λ_8 are left

Analyze all the subgroups

SO(4)
↓
O(3) × O(2)
↓
O(2) × O(2)
⋮

Accidental symmetries in 2HTM

Z_2 symmetries

Quartic potential $V_\phi^{(4)} = \frac{1}{4} L_{\mu\nu} \boxed{R^\mu R^\nu}$ $R^\mu \to -R^\mu$ → Z_2 symmetry

Acting a Z_2 transformation on Φ, r^{ij} exhibits three patterns

$$(\mathbf{a}) \begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix} \quad (\mathbf{b}) \begin{pmatrix} - & - & - \\ + & + & + \\ - & - & - \end{pmatrix} \quad (\mathbf{c}) \begin{pmatrix} + & - & - \\ + & - & - \\ + & - & - \end{pmatrix}$$

(a) $\phi_1 \to -\phi_1,\ \phi_2 \to \phi_2$
(b) $\phi_1 \to -\phi_2,\ \phi_2 \to \phi_1$
(c) $\phi_1 \to -\phi_2^*,\ \phi_2 \to \phi_1^*$

Entire symmetry = Continuous symmetry + Z_2 symmetry

Symmetries of the full potential

Three different kinds of doublet-triplet-mixing terms

Similar as $\phi_i^* \cdot \phi_j$

- $(H^\dagger H)(\phi_i^* \cdot \phi_j)$ Maximal symmetry group $SO(4)$
- $(H^\dagger \mathrm{i}\boldsymbol{\sigma} H) \cdot (\phi_i^* \times \phi_j)$ Maximal symmetry group $O(3)^i \otimes O(2)^j$
- $H^{\mathrm{T}} \mathrm{i}\sigma_2 \boldsymbol{\sigma} \cdot \phi_i H$ Violate all the symmetries except $SO(2)$ and Z_2

$\{J^1, J^2, J^3, J^4\}$

Accidental symmetries in 2HTM

Classification of all the accidental symmetries

Symmetry	Generators	m_{22}^2	m_{12}^2	λ_2	λ_3	λ_4	$\mathrm{Re}\,\lambda_5$	$\lambda_6 = \lambda_7$	λ_{10}	$\mathrm{Re}\,\lambda_{11}$
$\mathrm{SO}(4)$	$J^{1,2,3,4,5,6}$	m_{11}^2	0	λ_1	$2\lambda_1 - 2\lambda_8$	$2\lambda_8$	0	0	$2\lambda_8$	0
$\mathrm{O}(3)^i \otimes \mathrm{O}(2)^j$	$J^{1,2,3,4}$	m_{11}^2	0	λ_1	$2\lambda_1 - \lambda_4$	–	0	0	$2\lambda_8$	0
$\mathrm{O}(2)^i \otimes \mathrm{O}(3)^j$	$J^{1,4,5,6}$	m_{11}^2	0	λ_1	–	$2\lambda_8$	0	0	$2\lambda_1 - \lambda_3$	0
	$J^{2,4,5,6}$	m_{11}^2	0	λ_1	–	$2\lambda_8$	$-2\lambda_1 + \lambda_3 + 2\lambda_8$	0	$2\lambda_1 - \lambda_3$	$+\lambda_1 - \lambda_8 - \lambda_3/2$
	$J^{3,4,5,6}$	m_{11}^2	0	λ_1	–	$2\lambda_8$	$+2\lambda_1 - \lambda_3 - 2\lambda_8$	0	$2\lambda_1 - \lambda_3$	$-\lambda_1 + \lambda_8 + \lambda_3/2$
$\mathrm{O}(3)^j \otimes Z_2$	$J^{4,5,6}$	m_{11}^2	0	λ_1	–	$2\lambda_8$	$-2\,\mathrm{Re}\,\lambda_{11}$	0	$2\lambda_1 - \lambda_3$	–
$\mathrm{O}(2)^i \otimes \mathrm{O}(2)^j$	$J^{1,4}$	–	0	–	–	–	0	0	–	0
	$J^{2,4}$	m_{11}^2	$\mathrm{Im}\, m_{12}^2$	λ_1	–	–	$+2\lambda_1 - \lambda_3 - \lambda_4$	$\mathrm{Im}\,\lambda_6$	$2\lambda_8 - 2\,\mathrm{Re}\,\lambda_{11}$	–
	$J^{3,4}$	m_{11}^2	$\mathrm{Re}\, m_{12}^2$	λ_1	–	–	$-2\lambda_1 + \lambda_3 + \lambda_4$	$\mathrm{Re}\,\lambda_6$	$2\lambda_8 + 2\,\mathrm{Re}\,\lambda_{11}$	–
$\mathrm{O}(2)^i \otimes \mathrm{O}(2)^j \otimes Z_2$	$J^{1,4}$	m_{11}^2	0	λ_1	–	–	0	0	–	0
	$J^{2,4}$	m_{11}^2	0	λ_1	–	–	$+2\lambda_1 - \lambda_3 - \lambda_4$	0	$2\lambda_8 - 2\,\mathrm{Re}\,\lambda_{11}$	–
	$J^{3,4}$	m_{11}^2	0	λ_1	–	–	$-2\lambda_1 + \lambda_3 + \lambda_4$	0	$2\lambda_8 + 2\,\mathrm{Re}\,\lambda_{11}$	–
$\mathrm{SO}(2)^j \otimes (Z_2)^2$	J^4	m_{11}^2	0	λ_1	–	–	–	0	–	–
$\mathrm{O}(2)^j \otimes Z_2$	J^4	m_{11}^2	–	–	–	–	–	–	–	–
		–	–	λ_1	–	–	–	–	–	–
		–	0	–	–	–	–	0	–	–

Together with

$$\mathrm{Im}\,\lambda_5 = 0,\ \lambda_8 = \lambda_9,\ \mathrm{Im}\,\lambda_{11} = 0,\ \lambda_{12} = \lambda_{13} = 0$$

In total eight distinct types of accidental symmetries

Summary

- **Motivations to consider the 2HTM**
  - -- Account for nonzero neutrino masses
  - -- Generate successful leptogenesis
- **Classification of accidental symmetries in the 2HTM**
  - -- The maximal symmetry group is $SO(4)$
  - -- There are in total eight kinds of accidental symmetries
- **Accidental symmetries are useful**
  - -- Construct predictive models with less parameters
  - -- Investigate vacuum stability conditions and find out the vacuum solutions in the 2HTM
  - -- Study topological structures of the 2HTM

Thank you!