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Two-loop QED corrections to the di-muon production in e+ e- collisions and related processes

Jonathan Ronca

In collaboration with:
R. Bonciani, A. Broggio, S. Di Vita, A. Ferroglia, S. Laporta, P. Mastrolia, L. Mattiazzi, M. Passera, A. Primo, U. Schubert, W. J. Torres Bobadilla, and F. Tramontano

July 07, 2022

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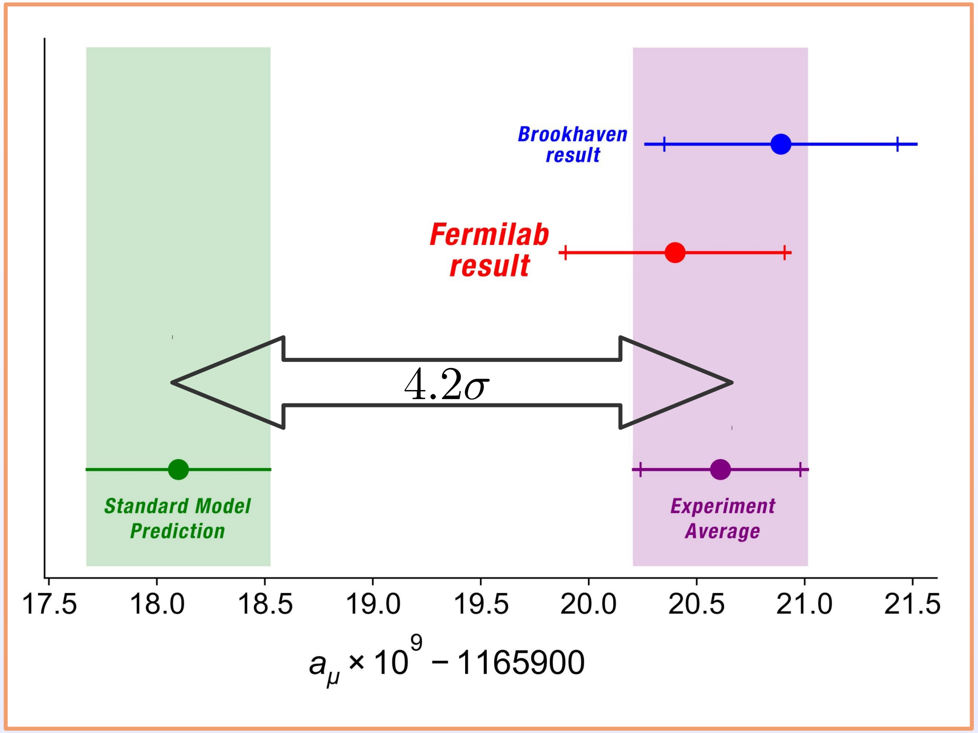
Motivation :: Muon (g - 2)



[Muon g-2 Collaboration (2021)]

$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

Standard Model prediction

$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$

Significant discrepancy between the **experiment** and the **SM prediction**

$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 251\ (59) \times 10^{-11}$

Motivation :: Muon (g - 2)

a_μ^{QED} = 116584718.931 (19)(100)(23) x 10^{-11}
a_μ^{EW} = 153.6 (1.0) x 10^{-11}
a_μ^{HLO} = 6931 (40) x 10^{-11}

Affected by the **largest** theoretical error

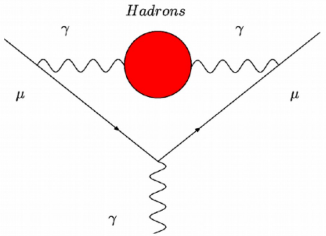
To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at ≲ 10ppm!

Large QED background

⇓

High-precision calculation of **electron-muon elastic scattering in QED**

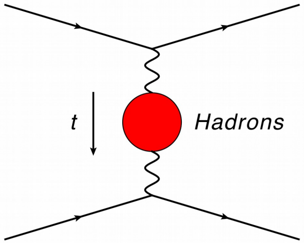
- **Leading hadronic contribution computed via the usual dispersive (timelike) formula:**



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{\rm had}^{(0)}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- **Alternatively, simply exchanging the x and s integrations:**



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: measure a_μ^{HLO} via scattering data!

M Passera EPFL 07.06.2021

Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

Motivation :: Muon (g - 2)

$a_\mu^{QED} = 116584718.931\ (19)(100)(23) \times 10^{-11}$
$a_\mu^{EW} = 153.6\ (1.0) \times 10^{-11}$
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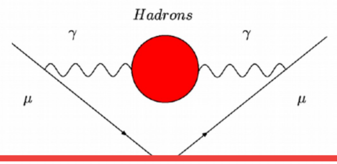
- Leading hadronic contribution computed via the usual dispersive (timelike) formula:



$$a_\mu^{HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{had}^{(0)}(s)$$

$$K(s) = \int_0^1 dx \ldots \frac{x^2\,(1-x)}{\ldots)\,(s/m_\mu^2)}$$

To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the cross sections in the signal and normalisation r… known at ≲ 10ppm!

Theory for muon-electron scattering @ 10 ppm*

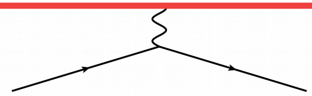
A report of the MUonE theory initiative

P. Banerjee[1], C. M. Carloni Calame[2], M. Chiesa[3], S. Di Vita[4], T. Engel[1,5], M. Fael[6], S. Laporta[7,8], P. Mastrolia[7,8], G. Montagna[9,2], O. Nicrosini[2], G. Ossola[10], M. Passera[8], F. Piccinini[2], A. Primo[5], J. Ronca[11], A. Signer[a,1,5], W. J. Torres Bobadilla[11], L. Trentadue[12,13], Y. Ulrich[a,1,5], G. Venanzoni[14]

…ons:

$\ldots\alpha_{had}[t(x)]$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$



Large QED back…und

More details on tomorrow talks by

Ettore Budassi and **Riccardo Pilato**

@Strong interactions and Hadron Physics session

muon elastic scattering in QED

$\Delta\alpha_{had}(t)$ is the hadronic contri… spacelike region: measure a_μ…

FL 07.06.2021

[Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, Passera, Piccinini, Tenchini, Trentadue, Venanzoni (2017)]

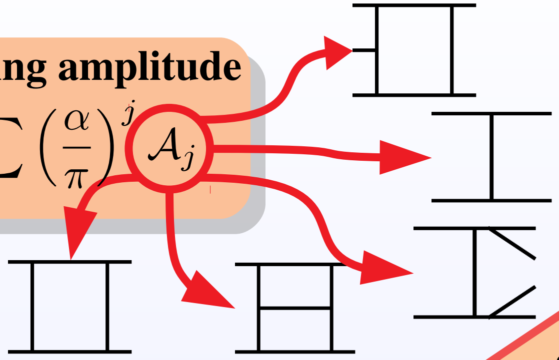
Cross Section and Scattering Amplitudes in pQFT

Target observable: **cross section**

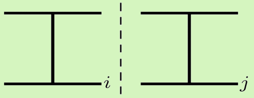
$$\sigma(2\to 2) = \alpha^2 \left[\sum_{j=0}^{n} \left(\frac{\alpha}{\pi}\right)^j \sigma_{\mathrm{N^jLO}} + O(\alpha^{n+1}) \right]$$

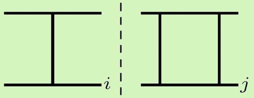
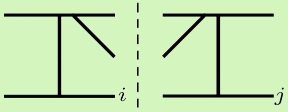
related to the **scattering amplitude**

$$\mathcal{A} = 4\pi\alpha S_\epsilon \mu^{-2\epsilon} \sum_j \left(\frac{\alpha}{\pi}\right)^j \mathcal{A}_j$$

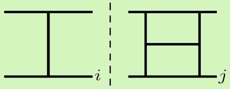


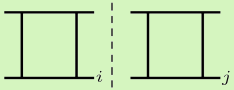
Feynman diagrams

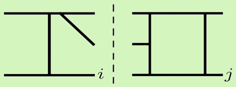
@LO $\sigma_{\mathrm{LO}} = \int \sum_{ij}$  $d\mathrm{PS}_2$

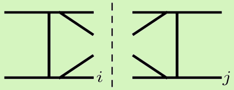
@NLO $\sigma^{\mathrm{V}}_{\mathrm{NLO}} = \int 2\,\mathrm{Re} \left(\sum_{ij} \right.$  $\left. \right) d\mathrm{PS}_2$ $\quad \sigma^{\mathrm{R}}_{\mathrm{NLO}} = \int \sum_{ij}$  $d\mathrm{PS}_3$

Virtual **Real**

@NNLO $\sigma^{\mathrm{VV,2L}}_{\mathrm{NNLO}} = \int 2\,\mathrm{Re} \left(\sum_{ij} \right.$  $\left. \right) d\mathrm{PS}_2$

$\sigma^{\mathrm{VV,1Lsq}}_{\mathrm{NNLO}} = \int \sum_{ij}$  $d\mathrm{PS}_2$

$\sigma^{\mathrm{RV}}_{\mathrm{NNLO}} = \int 2\,\mathrm{Re} \left(\sum_{ij} \right.$  $\left. \right) d\mathrm{PS}_3$

$\sigma^{\mathrm{RR}}_{\mathrm{NNLO}} = \int \sum_{ij}$  $d\mathrm{PS}_4$

Double-Virtual **Real-Virtual** **Double-Real**

Cross Section and Scattering Amplitudes in pQFT

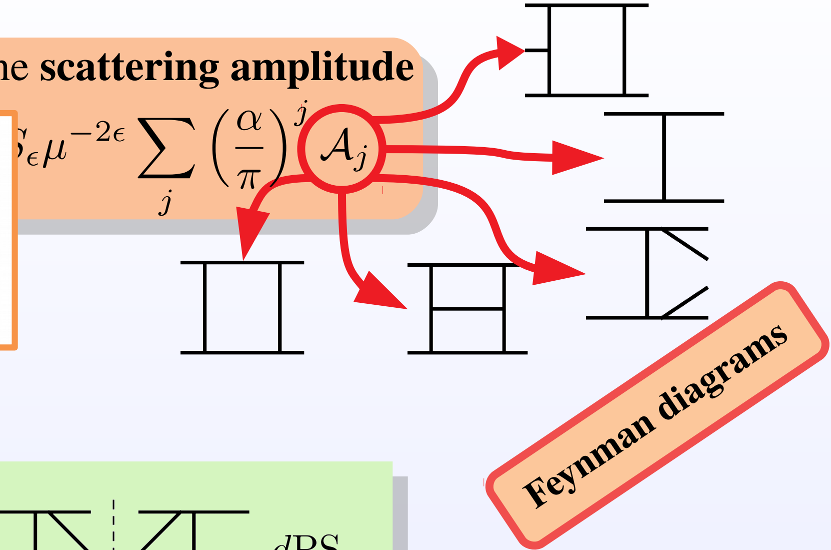
Target observable: **cross section**

PHYSICAL REVIEW LETTERS **128,** 022002 (2022)

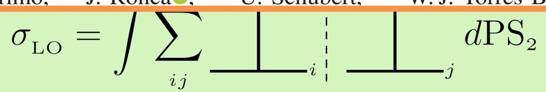
Two-Loop Four-Fermion Scattering Amplitude in QED

R. Bonciani,[1,*] A. Broggio,[2,†] S. Di Vita,[3,4] A. Ferroglia,[5,6,‡] M. K. Mandal,[7,8,§] P. Mastrolia,[8,7,∥] L. Mattiazzi,[7,8,¶] A. Primo,[9,**] J. Ronca,[10,††] U. Schubert,[11,‡‡] W. J. Torres Bobadilla,[12,§§] and F. Tramontano[10,∥∥]

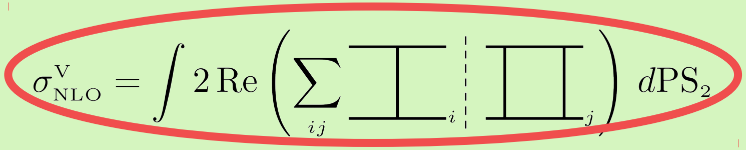
related to the **scattering amplitude**



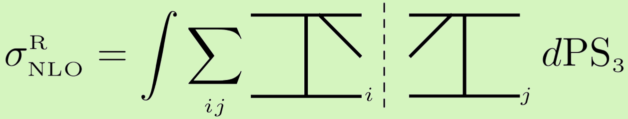
@LO



@NLO

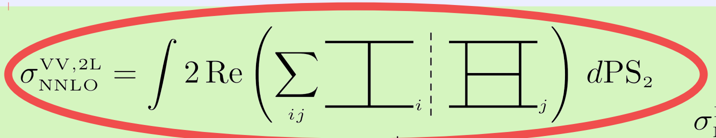


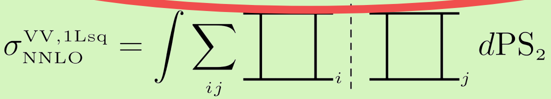
Virtual



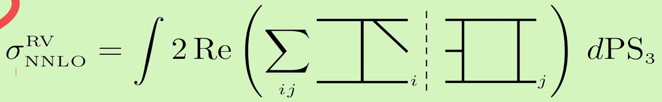
Real

@NNLO

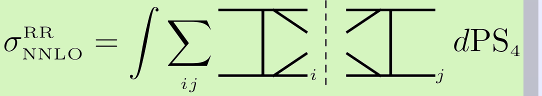




Double-Virtual



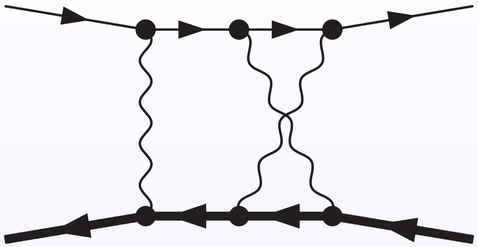
Real-Virtual



Double-Real

Anatomy of $\mu e \to \mu e$ two-loop NNLO QED contributions

$$\mathcal{M}_b^{(n)} = \frac{1}{4}\sum_{\text{spin}} 2\text{Re}[\mathcal{A}^{(0)*}\mathcal{A}^{(n)}]$$
$$= (S_\epsilon)^n \int \prod_{i=1}^{n} \frac{d^d k_i}{(2\pi)^d} \sum_G \frac{N_G}{\prod_{\sigma\in G} D_\sigma}$$



$\in G$

Two-loop Feynman integrals

- 4-point kinematics
- 4 mass-scales variables
  - 2 Mandelstam s, t
  - 2 masses m_e, m_μ

Observation: $\frac{m_e}{m_\mu} \simeq 10^{-5}$

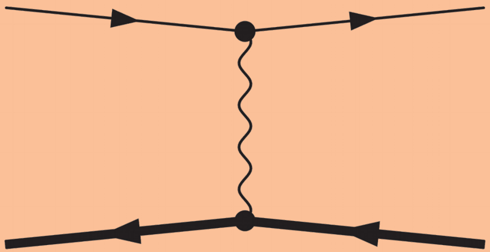
$m_e = 0$
$m_\mu = M$

- One less mass-scale
- Opens the possibility of an *analytical calculation*
- Simplification of the Dirac trace algebra

Feynman integrals are (in general) UV and IR divergent

Using **Dimensional Regularization**: space-time is treated as a free parameter $d = 4 - 2\epsilon$

Crossing: $e^- \mu^+ \to e^- \mu^+$ vs. $e^- e^+ \to \mu^- \mu^+$



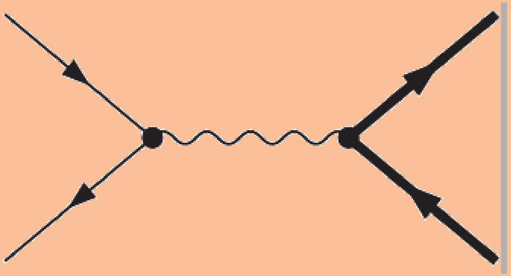
$$e^-(p_1)\,\mu^+(p_2) \to e^-(p_3)\mu^+(p_4)$$

$$p_1^2 = p_3^2 = 0$$
$$p_2^2 = p_4^2 = M^2$$
$$s = (p_1 + p_2)^2$$
$$t = (p_1 - p_3)^2$$
$$u = (p_2 - p_3)^2$$
$$s + t + u = M^2$$

$$\mathcal{M}^{(0)} = \frac{4(s - M^2)^2 + 4st + (d-2)t^2}{t^2}$$

Crossing

$s \to t$
$t \to s$
$u \to u$



$$e^-(p_1)\,e^+(p_2) \to \mu^-(p_3)\mu^+(p_4)$$

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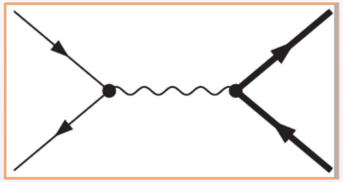
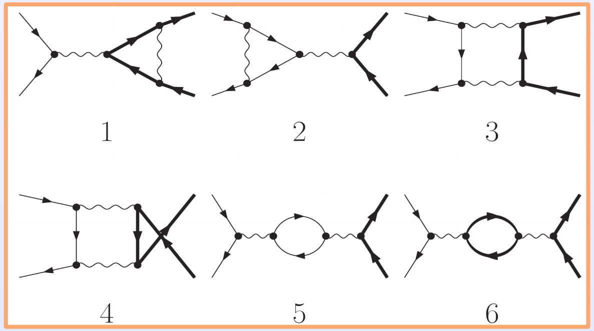
From now on, we consider the cross-related **di-muon production**

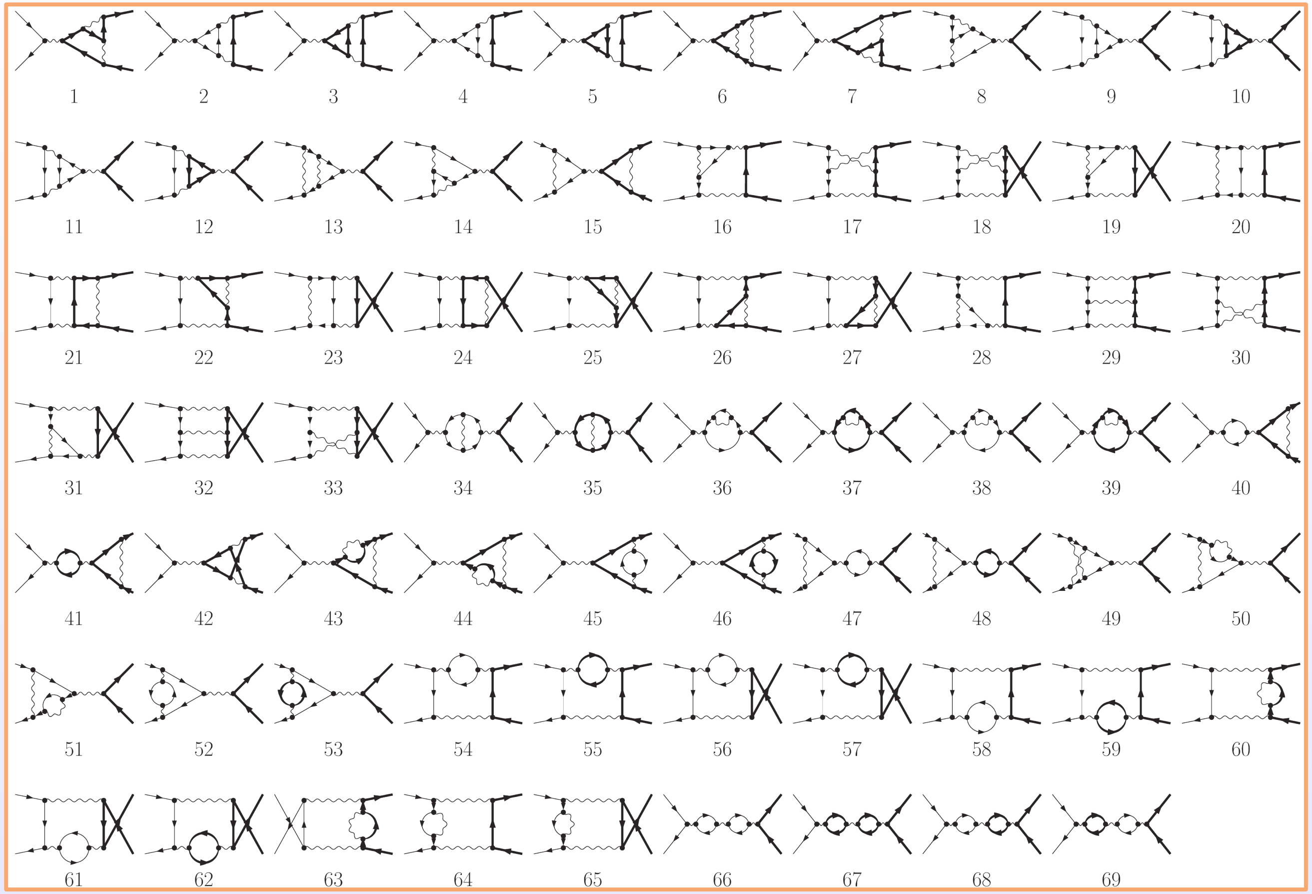
Di-muon production in QED: Feynman Diagrams

$$\mathcal{M}_b^{(1)} = A^{(1)} + n_l B_l^{(1)} + n_h C_h^{(1)}$$

$$\mathcal{M}_b^{(2)} = A^{(2)} + n_l B_l^{(2)} + n_h C_h^{(2)} + n_l^2 D_l^{(2)} + n_l n_h D_{lh}^{(2)} + n_h^2 F_h^{(2)}$$







[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Schubert, Torres Bobadilla, Tramontano, JR (2021)]

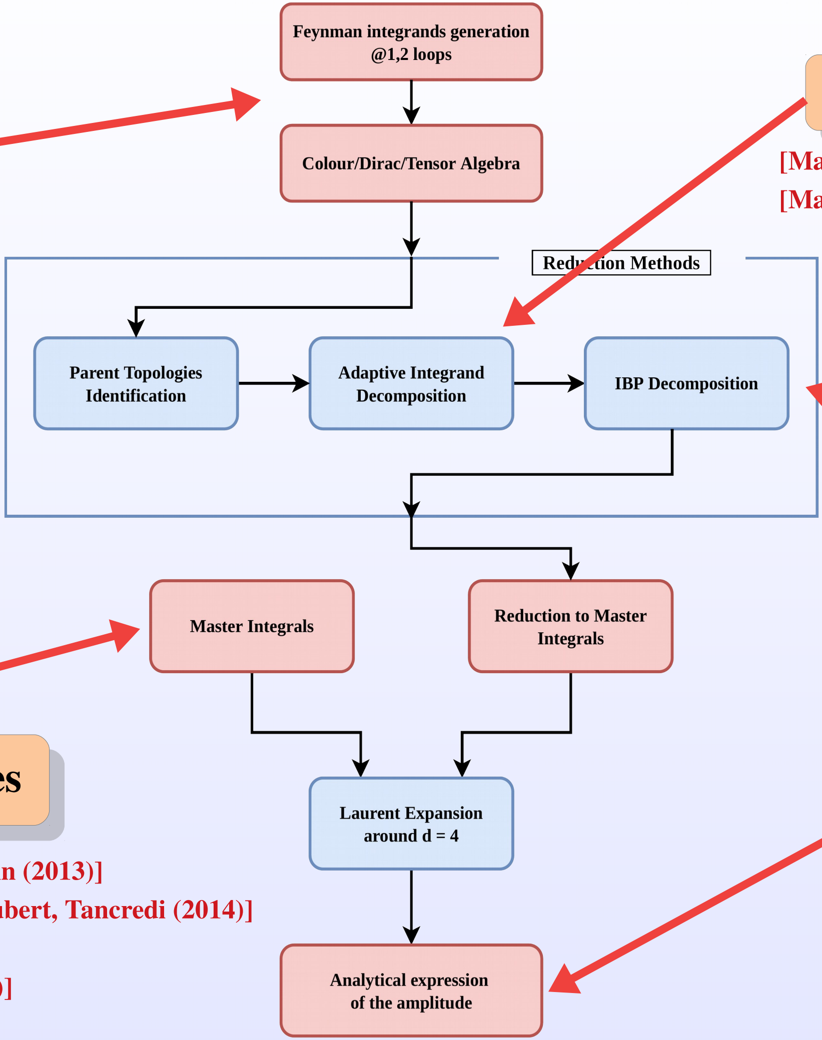
The AIDA framework

FeynCalc, FeynArts

[Mertig, Böhm, Denner (1991)]
[Shtabovenko, Mertig, Orellana (2016)]
[Hahn (2001)]

AIDA

[Mastrolia, Peraro, Primo, Torres (2016)]
[Mastrolia, Peraro, Primo, Torres, JR (To be published)]



Reduze, Kira

[Chetyrkin, Tkachev (1981)] [Laporta (2000)]
[von Manteuffel, Studerus (2012)]
[Maierhöfer, Usovitsch, Uwer (2018)]

In-house Mathematica routines

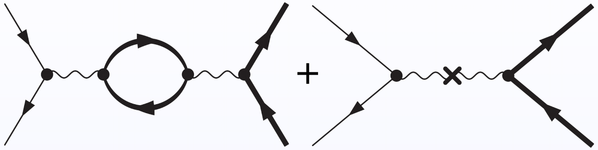
[Kotikov (1990)] [Gehrmann, Remiddi (1999)] [Henn (2013)]
[Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Schubert, Tancredi (2014)]
[Mastrolia, Passera, Primo, Schubert (2017)]
[Di Vita, Laporta, Mastrolia, Primo, Schubert (2018)]

HandyG, GiNaC, PolyLogTools

[Bauer, Frink, Kreckel (2000)]
[Naterop, Signer, Ulrich (2019)]
[Duhr, Dulat (2019)]

UV Renormalization

$\mathcal{M}_b^{(2)}$ is **UV divergent** $\xrightarrow{\text{Renormalisation}}$ $\mathcal{M}^{(2)}$



= UV finite

$$\mathcal{M} = Z_{2,e} Z_{2,\mu} \mathcal{M}_b(\alpha_b = \alpha_b(\alpha), M_b = M_b(M))$$

where

$$M_b(M) = Z_M M$$

$$\alpha_s S_\epsilon = \alpha(\mu^2)\mu^{2\epsilon} Z_\alpha$$

Renormalisation constants:

$$Z_j = 1 + \left(\frac{\alpha}{\pi}\right)\delta Z_j^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \delta Z_j^{(2)} + O(\alpha^3)$$

Renormalisation schemes

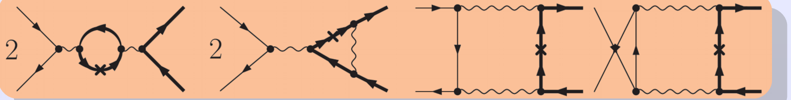
- **On-shell** renormalisation $Z_{2,e}, Z_{2,\mu}, Z_M$
- $\overline{\text{MS}}$ renormalisation Z_α

Renormalised interferences:

$$\mathcal{M}^{(0)} = \mathcal{M}_b^{(0)}$$

$$\mathcal{M}^{(1)} = \mathcal{M}_b^{(1)} + (\delta Z_{2,\mu}^{(1)} + Z_\alpha^{(1)})\mathcal{M}_b^{(0)}$$

$$\mathcal{M}^{(2)} = \mathcal{M}_b^{(2)} + (\delta Z_{2,\mu}^{(1)} + Z_\alpha^{(1)})\mathcal{M}_b^{(1)} + (\delta Z_{2,\mu}^{(2)} + \delta Z_{2,e}^{(2)} + Z_\alpha^{(2)} + \delta Z_{2,\mu}^{(1)} Z_\alpha^{(1)})\mathcal{M}_b^{(0)} + \delta Z_M^{(1)} \mathcal{M}_{\text{massCT}}^{(1)}$$



Checks

$$\mathcal{M}_b^{(1)} = A^{(1)} + n_l B_l^{(1)} + n_h C_h^{(1)}$$

$$\mathcal{M}_b^{(2)} = A^{(2)} + n_l B_l^{(2)} + n_h C_h^{(2)} + n_l^2 D_l^{(2)} + n_l n_h D_{lh}^{(2)} + n_h^2 F_h^{(2)}$$

Evaluating the interferences with **HandyG** and **GiNaC**

$$\frac{s}{M^2} = 5, \quad \frac{t}{M^2} = -\frac{5}{4}, \quad \mu = M$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}	ϵ
$\mathcal{M}^{(0)}$	-	-	-	-	$\frac{181}{100}$	-2
$A^{(1)}$	-	-	$-\frac{181}{100}$	1.99877525	22.0079572	−11.7311017
$B_l^{(1)}$	-	-	-	-	−0.069056030	4.94328573
$C_h^{(1)}$	-	-	-	-	−2.24934027	2.54943566
$A^{(2)}$	$\frac{181}{400}$	−0.499387626	−35.4922919	19.4997261	49.0559119	-
$B_l^{(2)}$	-	$-\frac{181}{400}$	0.785712779	−16.1576674	−3.75247701	-
$C_h^{(2)}$	-	-	1.12467013	−9.50785825	−25.8771503	-
$D_l^{(2)}$	-	-	-	-	−3.96845688	-
$E_{hl}^{(2)}$	-	-	-	-	−4.88512563	-
$F_h^{(2)}$	-	-	-	-	−0.158490810	-

[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Schubert, Torres Bobadilla, Tramontano, JR (2021)]

How do we check these terms?

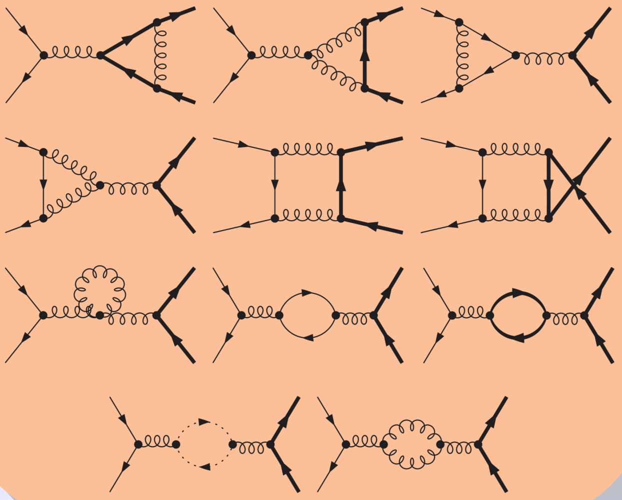
Checks :: Literature

$q\bar{q} \to t\bar{t}$ **in QCD** ⟹ $e^-e^+ \to \mu^-\mu^+$ **in QED**

- Top-pair production admit a color decomposition like $\mathcal{M}^{(n)}$
- 1-loop and 2-loop corrections already known in literature
- Abelian part get contributions from *QED-like diagrams* only

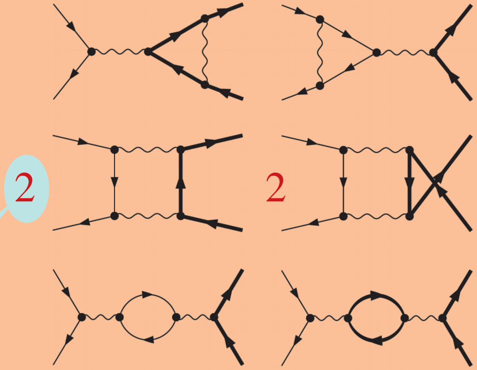
[Czakon(2008)]
[Bonciani, Ferroglia, Gehrmann, Maitre, Studerus (2008)]
[Bärnreuther, Czakon, Fiedler (2014)]

$q\bar{q} \to t\bar{t}$ @one-loop in QCD



Abelian, Color-stripped ⟹

$e^-e^+ \to \mu^-\mu^+$@one-loop in QED



Remainder of color factors

✔ **Full agreement** with the abelian part of top-pair production

Checks :: IR structure

Two-loop IR poles from **one-loop and tree** (renormalised) **contributions**

$$\mathcal{M}^{(1)}\Big|_{\text{poles}} = \frac{1}{2} Z_1^{\text{IR}}\, \mathcal{M}^{(0)}\Big|_{\text{poles}}$$

$$\mathcal{M}^{(2)}\Big|_{\text{poles}} = \frac{1}{8}\left[\left(Z_2^{\text{IR}} - \left(Z_1^{\text{IR}}\right)^2\right)\mathcal{M}^{(0)} + 2\, Z_1^{\text{IR}}\, \mathcal{M}^{(1)}\right]\Big|_{\text{poles}}$$

[Becher, Neubert (2009)]
[Hill (2017)]

IR Renormalisation Factor

Beta function

$$\ln Z_{\text{IR}} = \frac{\alpha}{4\pi}\left(\frac{\Gamma_0'}{4\epsilon^2} + \frac{\Gamma_0}{2\epsilon}\right) + \left(\frac{\alpha}{4\pi}\right)^2\left(-\frac{3\beta_0\Gamma_0'}{16\epsilon^3} + \frac{\Gamma_1' - 4\beta_0\Gamma_0}{16\epsilon^2} + \frac{\Gamma_1}{4\epsilon}\right) + \mathcal{O}\left(\alpha^3\right)$$

$$\gamma_i = \sum_{j=0}^{n}\left(\frac{\alpha}{\pi}\right)^{j+1}\gamma_i^{(j)} + O\left(\alpha^{n+1}\right)$$

Anomalous dimension

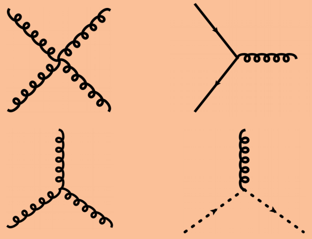
$$\Gamma = \gamma_{\text{cusp}}(\alpha)\ln\left(-\frac{s}{\mu^2}\right) + 2\gamma_{\text{cusp}}(\alpha)\ln\left(\frac{t-M^2}{u-M^2}\right) + \gamma_{\text{cusp,M}}(\alpha, s) + 2\gamma_h(\alpha) + 2\gamma_\psi(\alpha)$$

☑ **Full agreement** with the direct calculation of the two-loop contribution

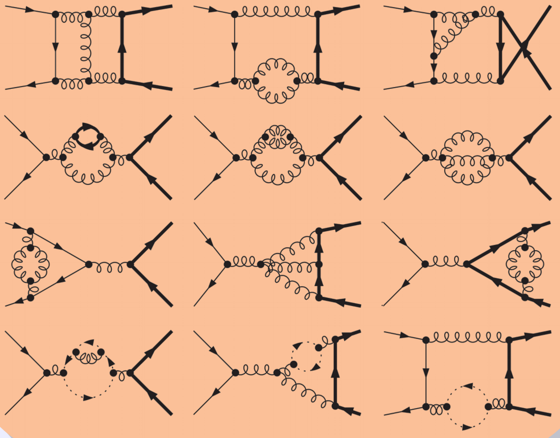
Extension to two-loop top-pair production @ NNLO QCD

Full two-loop $q\bar{q} \to t\bar{t}$ @NNLO QCD
calculation available only **numerical**

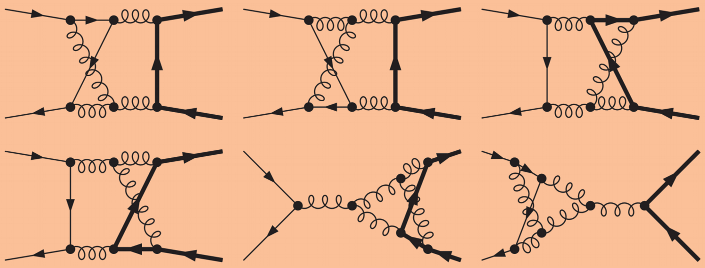
More particles and interactions



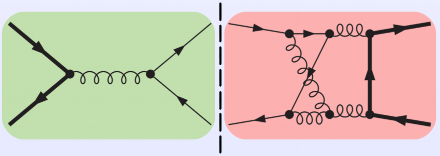
More diagrams (~220)



New topologies occur



BUT



$\propto T^a_{ij} T^a_{lk} \, f^{abc} (T^d T^c)_{kl} (T^b T^d T^a)_{ji} = 0$ **!**

No additional Master Integrals are required

Extension to Two-loop top-pair production @ NNLO QCD

Full two-loop $q\bar{q} \to t\bar{t}$ @NNLO QCD

calculation available only **numerical**

PREPARED FOR SUBMISSION TO JHEP — MPP-2022-38

Two-loop scattering amplitude for heavy-quark pair production through light-quark annihilation in QCD

Manoj K. Mandal,[a] Pierpaolo Mastrolia,[a,b] Jonathan Ronca,[c] and William J. Torres Bobadilla[d]

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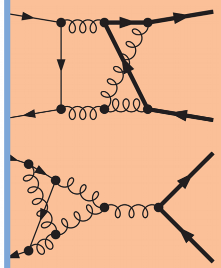

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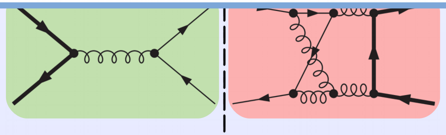


$\propto T^a_{ij} T^a_{lk} \, f^{abc} (T^d T^c)_{kl} (T^b T^d T^a)_{ji} = 0$ **!**

No additional Master Integrals are required

Extension to two-loop top-pair production @ NNLO QCD

$$\mathcal{M}^{(1)} = 2(N_c^2-1)\Big(N_c A^{(1)} + B^{(1)} + n_l C_l^{(1)} + n_h C_h^{(1)}\Big)$$

$$\mathcal{M}^{(2)} = 2(N_c^2-1)\Big(N_c^2 A^{(2)} + B^{(2)} + \frac{C^{(2)}}{N_c^2} + n_l N_c D_l^{(2)} + n_h N_c D_h^{(2)} + n_l \frac{E_l^{(2)}}{N_c} + n_h \frac{E_h^{(2)}}{N_c} + n_l^2 F_l^{(2)} + n_l n_h F_{lh}^{(2)} + n_h^2 F_h^{(2)}\Big)$$

Evaluating the interferences with **HandyG** and **GiNaC**

$$\frac{s}{M^2} = 5, \quad \frac{t}{M^2} = -\frac{5}{4}, \quad \mu = M$$

	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}	ϵ^{1}
$A^{(0)}$	-	-	-	-	$\frac{181}{100}$	-2
$A^{(1)}$	-	-	$-\frac{181}{400}$	0.1026418456757775	1.356145770566065	2.230403451742140
$B^{(1)}$	-	-	$\frac{181}{400}$	-0.3180868339485723	-5.763132746701004	2.913169881363488
$C_l^{(1)}$	-	-	0	0	-0.01726400752682416	1.235821434465827
$C_h^{(1)}$	-	-	0	0	-0.5623350683773134	0.6373589172648111
$A^{(2)}$	$\frac{181}{800}$	1.391733154324222	-2.298174307221209	-4.145752448999165	17.37136598564062	-
$B^{(2)}$	$-\frac{181}{400}$	-1.323646320375650	8.507455541210568	6.035611156200398	-35.12861106350758	-
$C^{(2)}$	$\frac{181}{800}$	-0.06808683394857230	-18.00716652035224	6.302454931016090	3.524044912826756	-
$D_l^{(2)}$	0	- $\frac{181}{800}$	0.2605057338631945	-0.7250180282219092	-1.935417246635768	-
$D_h^{(2)}$	0	0	0.5623350683773134	0.1045606449242690	-1.704747997587188	-
$E_l^{(2)}$	0	$\frac{181}{800}$	-0.3323207299541260	7.904121951420471	2.848697836597635	-
$E_h^{(2)}$	0	0	-0.5623350683773134	4.528240788258799	12.73232424278180	-
$F_l^{(2)}$	0	0	0	0	-1.984228442234312	-
$F_{lh}^{(2)}$	0	0	0	0	-2.442562819239786	-
$F_h^{(2)}$	0	0	0	0	-0.07924540546146283	-

[Mandal, Mastrolia, Torres Bobadilla, JR (2022)]

First fully-analytical calculation

Full agreement with the literature

[Czakon(2008)]
[Bonciani, Ferroglia, Gehrmann, Maitre, Studerus (2008)]
[Bärnreuther, Czakon, Fiedler (2014)]
[Badger, Hartanto, Zoia (2021)]

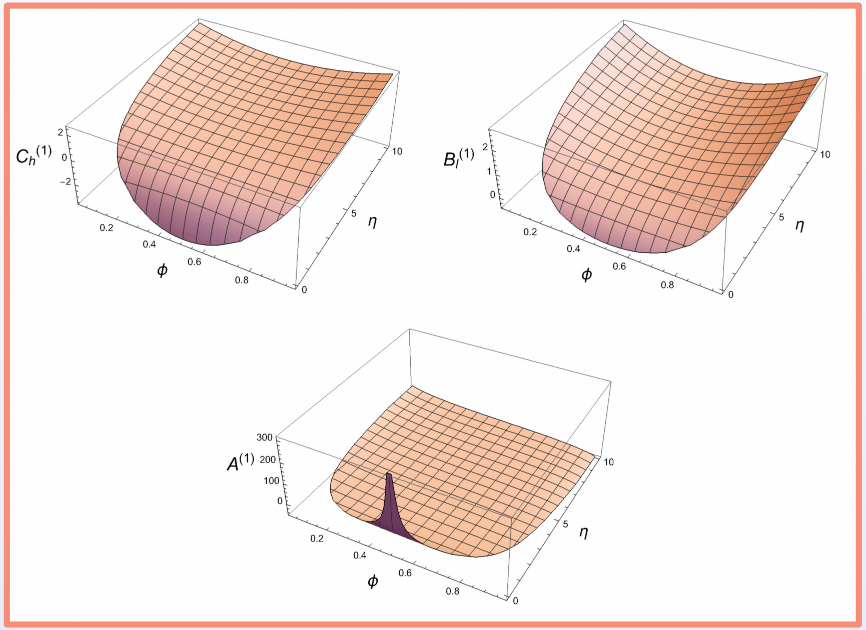
Results: One-loop di-muon production @ NLO QED

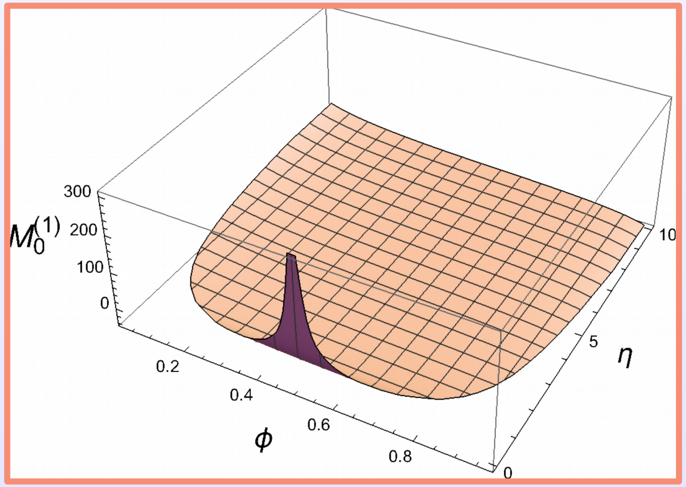
$$\mathcal{M}_0^{(1)} = A^{(1)} + n_l B_l^{(1)} + n_h C_h^{(1)} \Big|_{\text{finite}}$$

$$\eta = \frac{s}{4M^2} - 1, \quad \phi = -\frac{(t - M^2)}{s}$$

Production region

$$\eta > 0, \quad \frac{1}{2}\left(1 - \frac{\eta}{1+\eta}\right) < \phi < \frac{1}{2}\left(1 + \frac{\eta}{1+\eta}\right)$$

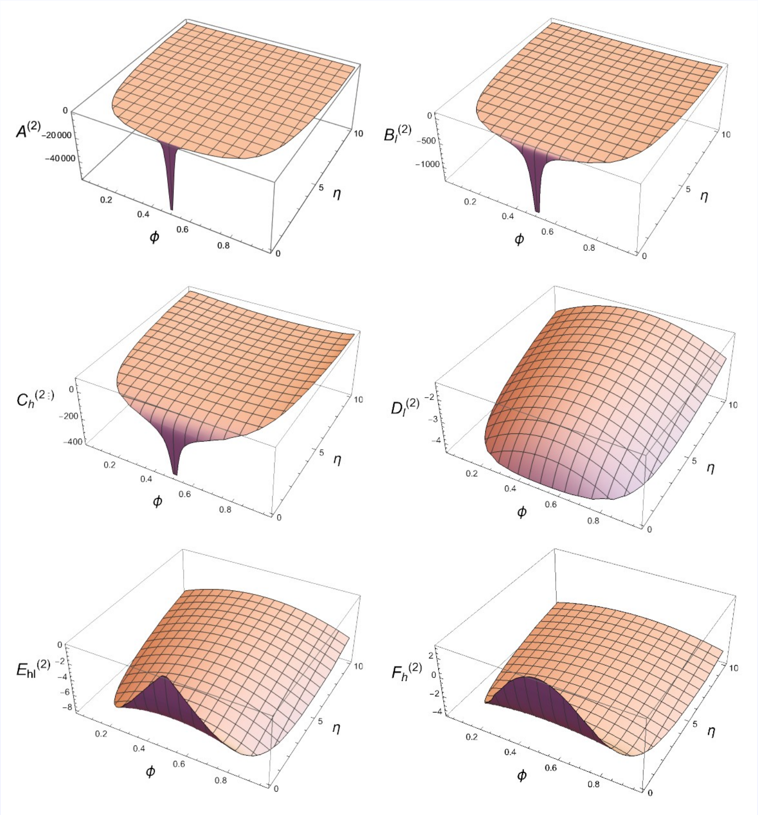




[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Schubert, Torres Bobadilla, Tramontano, JR (2021)]

Results: Two-loop di-muon production @ NNLO QED

$$\mathcal{M}_0^{(2)} = A^{(2)} + n_l B_l^{(2)} + n_h C_h^{(2)} + n_l^2 D_l^{(2)} + n_l n_h E_{lh}^{(2)} + n_h^2 F_h^{(2)} \Big|_{\text{finite}}$$



[Bonciani, Broggio, Di Vita, Ferroglia, Mandal, Mastrolia, Mattiazzi, Primo, Schubert, Torres Bobadilla, Tramontano, JR (2021)]

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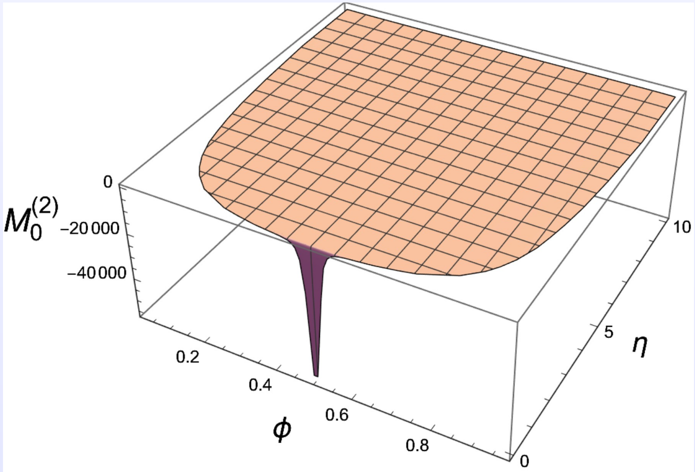


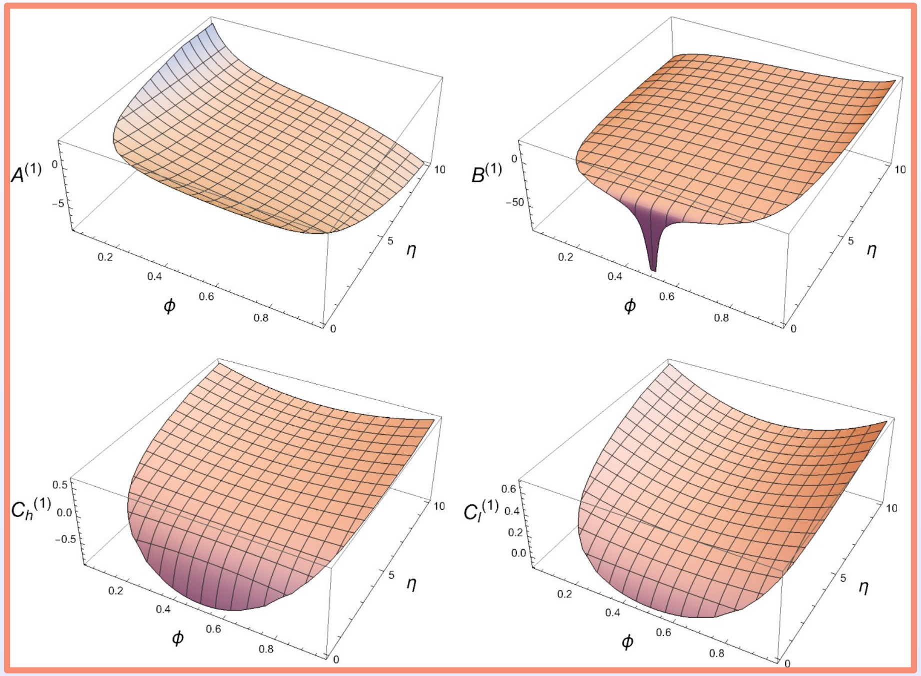
Results: One-loop top-pair production @ NLO QCD

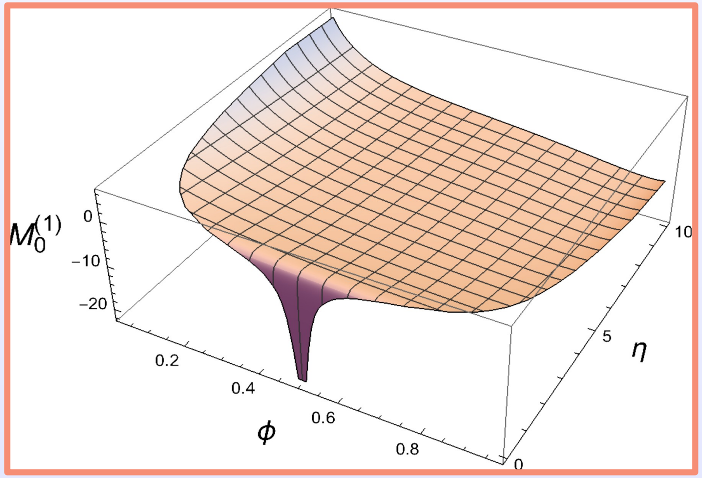
$$\mathcal{M}_0^{(1)} = 2(N_c^2 - 1)\left(N_c A^{(1)} + \frac{B^{(1)}}{N_c} + n_l C_l^{(1)} + n_h C_h^{(1)}\right)\Big|_{\text{finite}}$$

$$\eta = \frac{s}{4M^2} - 1, \quad \phi = -\frac{(t - M^2)}{s}$$

Production region

$$\eta > 0, \quad \frac{1}{2}\left(1 - \frac{\eta}{1+\eta}\right) < \phi < \frac{1}{2}\left(1 + \frac{\eta}{1+\eta}\right)$$

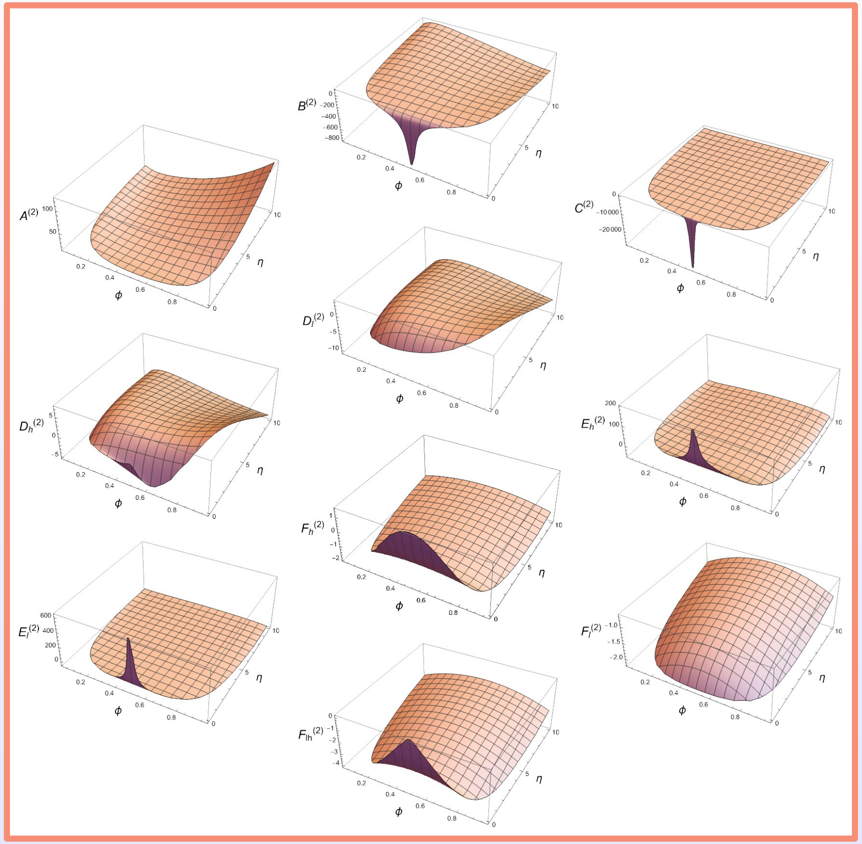




[Mandal, Mastrolia, Torres Bobadilla, JR (2022)]

Results: Two-loop top-pair production @ NNLO QCD

$$\mathcal{M}_0^{(2)} = 2(N_c^2-1)\left(N_c^2 A^{(2)} + B^{(2)} + \frac{C^{(2)}}{N_c^2} + n_l N_c D_l^{(2)} + n_h N_c D_h^{(2)} + n_l \frac{E_l^{(2)}}{N_c} + n_h \frac{E_h^{(2)}}{N_c} + n_l^2 F_l^{(2)} + n_l n_h F_{lh}^{(2)} + n_h^2 F_h^{(2)}\right)\Big|_{\text{finite}}$$



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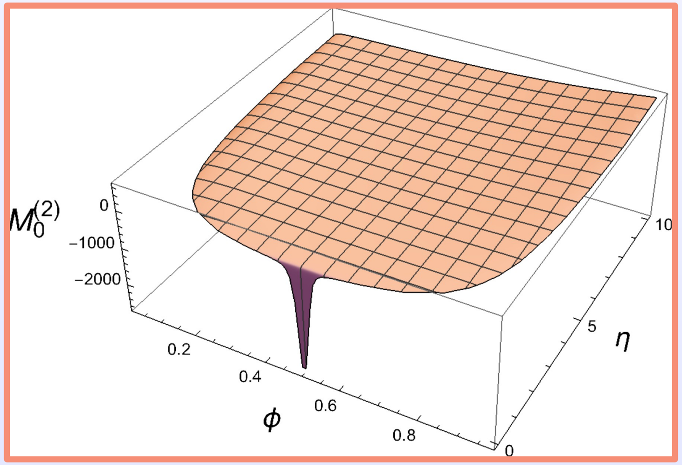

Conclusions & Outlook

Electron-muon elastic scattering @NNLO QED is a crucial input for the **MuonE experiment**

- ☑ **Crossing:** the two-loop contributions to **di-muon production** @NNLO QED via electron-positron annihilation
  - ☑ First QED **analytical two-loop** calculation for di-muon production process
  - ☑ Complete **automation** through the **AIDA framework**
  - ☑ **Cross-checked** against
    - Independent calculations
    - IR structure cross-checked against the SCET prediction
  - ☑ Grid of 10500 phase-space points has been generated

- ☑ Extension to **two-loop** contributions to **top-pair production via quark-antiquark annihilation** @NNLO QCD
- ☑ Recent development: **analytical one-loop squared** contributions to di-muon production @NNLO QED

- Inclusion of **non-zero** electron mass to electron-muon elastic scattering calculation: **massification**
- **Threshold expansion** for both di-muon and top-pair production @NNLO
- Inclusion of our contribution on MC generators

[Mitov, Moch (2006)]
[Becher, Melnikov (2007)]
[Engel, Gnendiger, Signer, Ulrich (2019)]
[Heller (2021)]

Thank you

Reduction of Feynman Integrals

$$\mathcal{M}_b^{(2)} = (S_\epsilon)^2 \int \prod_{i=1}^{2} \frac{d^d k_i}{(2\pi)^d} \sum_G \frac{N_G}{\prod_{\sigma \in G} D_\sigma}$$

Adaptive Integrand Decomposition

Idea: $d = d_{||} + d_{\perp}$

$k_i = k_{||i} + k_{\perp i}$

D_σ will not depend on transverse directions

⇓

Direct integration

⇓

$$\frac{N_G}{\prod_{\sigma \in G} D_\sigma} = \sum_{\tau \in P(G)} \frac{\Delta_\tau}{\prod_{j \in \tau} D_j}$$

Integration-by-parts Identities

$$\int \prod_{i=1}^{2} \frac{d^d k_i}{(2\pi)^d} \frac{\partial}{\partial k_l^\mu} \left(q^\mu \frac{\mathcal{N}}{\prod_{\sigma \in G} D_\sigma} \right) = 0$$

- IBPs generate a linear system of Eqs.
- Relation between integrals
- Coefficient depending on scales and d

# of independent Eqs. = # of **Master Integrals**

$$\mathcal{M}_b^{(2)} = \mathbb{C}^{(2)} \cdot \mathbf{I}^{(2)}$$

[Mastrolia, Peraro, Primo, Torres (2016)]
[Mastrolia, Peraro, Primo, Torres, JR (To be published)]

[Chetyrkin, Tkachev (1981)] [Laporta (2000)]

Master Integrals

Master Integrals for $\mu e \to \mu e$
are known in literature.

$$\mathbf{I}^{(2)}_{\mu e \to \mu e} = \mathbf{I}^{(2)}_{ee \to \mu\mu}\big|_{s \leftrightarrow t}$$

Representation through **Generalized PolyLogarithms**

$$I_j^{(2)}(s,t,M;d) = \sum_i C_i(\{w\}_{ji}, d) G(\{w\}_{ji}, 1)$$

where

$$G(w_n, \ldots, w_1; \tau) = \int_0^\tau \frac{dt}{t - w_n} G(w_{n-1}, \ldots, w_1; t)$$

Letters
$w_j = w_j(s,t,M)$

- O(4000) GPLs
- GPLs up to weight 4
- 18 letters

[Kotikov (1990)] [Gehrmann, Remiddi (1999)] [Henn (2013)]
[Argeri, Di Vita, Mastrolia, Mirabella, Schlenk, Schubert, Tancredi (2014)]
[Mastrolia, Passera, Primo, Schubert (2017)]
[Di Vita, Laporta, Mastrolia, Primo, Schubert (2018)]