The QCD topological susceptibility at high temperatures via staggered fermions spectral projectors

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Axion cosmology and QCD topology

The **Peccei–Quinn Axion**, introduced to solve the strong CP problem ($|\theta_{\mathrm{exp}}| \lesssim 10^{-9}$), is also a promising Dark Matter candidate.
$\rightarrow$ Axion physics at **early times** of the Universe evolution (i.e., high temperatures T) interesting for cosmology.

Axion couples to QCD topological charge $\implies$ great interest around theoretical computation of QCD topological properties at high-T for axion cosmology:

$$Q = \frac{1}{64\pi^2}\epsilon^{\mu\nu\rho\sigma}\int F^a_{\mu\nu}F^a_{\rho\sigma}\, d^4x \quad \rightarrow \quad \text{QCD } \textbf{Topological Charge} \in \mathbb{Z}$$

$$\chi(T) = \frac{\langle Q^2 \rangle_T}{V} \quad \rightarrow \quad \text{QCD } \textbf{Topological Susceptibility}$$

$$m_a^2(T) = \frac{\chi(T)}{f_a^2} \quad \rightarrow \quad \textbf{Axion effective mass} \text{ related to } \chi(T)$$

Non-perturbative computation of $\chi(T)$ from Lattice QCD fundamental input for f_a and cosmological observables (e.g., axion relic abundance)
$\rightarrow$ utmost importance for current and future experimental axion searches!

Non-chiral fermions and would-be-zero modes

In the QCD path-integral, field configurations are weighted with the determinant of the massive Dirac operator:

$$\text{weight} \propto \det\{\not{D} + m_{\text{quark}}\} = \prod_{\lambda \in \mathbb{R}} (i\lambda + m_{\text{quark}}) \, .$$

The Index Theorem relates the presence of zero-modes ($\lambda = 0$) in the spectrum of $\not{D}$ to the topological charge of the gluon field:

$$Q = n_+ - n_- .$$

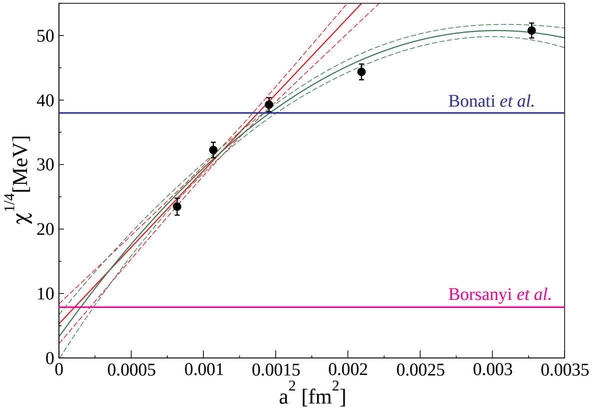
If gluon field has $Q \neq 0$, lowest eigenvalues: $\lambda_{\text{min}} = m_{\text{quark}}$.

On the lattice, however, some fermionic discretizations (e.g., staggered) do not have exact zero-modes. $\implies$ The determinant fails to efficiently suppress non-zero charge configurations.

$$\lambda_{\text{min}} = m_{\text{quark}} \longrightarrow m_{\text{quark}} + i\lambda_0, \qquad \lambda_0 \underset{a\to 0}{\to} 0.$$

Non-chiral fermions and large lattice artifacts

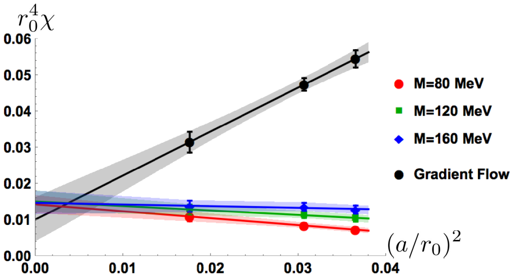
Bad suppression of non-zero charge configurations $\Longrightarrow$ large discretization corrections $\Longrightarrow$ continuum extrapolation not under control (Bonati et al., 2018):



In (Borsanyi et al., 2016) lattice artifacts affecting χ at high-T have been suppressed a posteriori by reweighting configurations with the corresponding continuum lowest eigenvalues of $\not{D}$.

Fermionic topological charge

Another possible solution, which does not require further assumptions, could be to switch, through the Index Theorem, from gluonic to **fermionic** definitions of Q. Using the same "bad" operator to weight configurations and to count eigenmodes to measure Q may introduce smaller lattice artifacts.



Idea supported by results at $T = 0$ (Alexandrou et al., 2017): twisted mass Wilson fermions employed for the MC evolution and for the measure of χ through spectral projectors $\longrightarrow$ improved scaling of χ towards the continuum!

Goal: use staggered fermions spectral projectors definition (CB et al., 2019) to study χ at high-T from full QCD simulations with staggered fermions.

Spectral projectors with staggered fermions

In the continuum, only zero-modes contribute to Q. On the lattice with staggered fermions, no exact zero-mode:

$$Q = \sum_{\lambda=0} u_\lambda^\dagger \gamma_5 u_\lambda \quad \longrightarrow \quad \sum_{|\lambda|\leq M} u_\lambda^\dagger \gamma_5 u_\lambda = \mathrm{Tr}\left\{\gamma_5 \mathbb{P}_M\right\},$$

$$\mathbb{P}_M = \sum_{|\lambda|\leq M}, u_\lambda u_\lambda^\dagger, \qquad i\not{D}_{stag} u_\lambda = \lambda u_\lambda, \quad \lambda \in \mathbb{R}.$$

To avoid a mode over-counting, taste degeneration has to be considered (number of *tastes*: $n_t = 4$):

$$Q_{\mathrm{SP}}^{(\mathrm{stag})} = n_t^{-1}\, \mathrm{Tr}\{\gamma_5 \mathbb{P}_M\}.$$

Lattice charge gets a renormalization $Z_Q^{(\mathrm{stag})}$, which can be derived from Ward identities for the flavor-singlet axial current:

$$Q_{\mathrm{SP},R}^{(\mathrm{stag})} = Z_Q^{(\mathrm{stag})} Q_{\mathrm{SP}}^{(\mathrm{stag})}, \qquad Z_Q^{(\mathrm{stag})} = \sqrt{\frac{\langle \mathrm{Tr}\{\mathbb{P}_M\}\rangle}{\langle \mathrm{Tr}\{\gamma_5 \mathbb{P}_M \gamma_5 \mathbb{P}_M\}\rangle}}.$$

Choice of the cut-off mass M

The choice of the cut-off mass M is irrelevant in the continuum limit. Its renormalized value M_R must be kept constant as $a \to 0$ to guarantee $O(a^2)$ corrections:

$$\chi_{\mathrm{SP}}(a, M_R) = \chi + c_{\mathrm{SP}}(M_R)a^2 + o(a^2).$$

For staggered fermions, M renormalizes as a quark mass, meaning that the ratio:

$$M/m_{\mathrm{quark}} = M_R/m_{\mathrm{quark}}^{(R)}$$

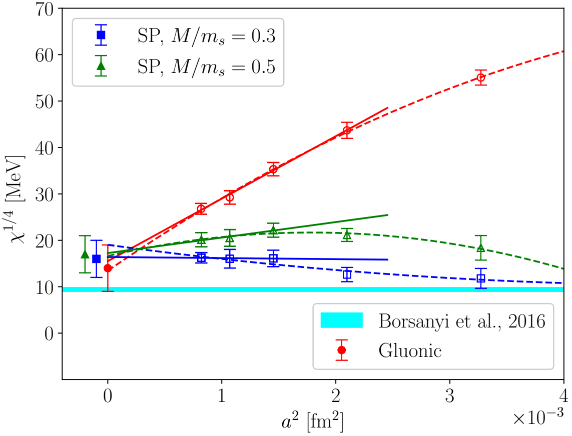
is a physical renormalized quantity.

Since a Line of Constant Physics (LCP) is known where m_π and m_{ud}/m_s are kept constant to the physical experimental values, it is sufficient to keep M/m_{quark} constant as $a \to 0$ following the LCP. Here we choose $m_{\mathrm{quark}} = m_s$.

Continuum limit of $\chi^{1/4}$ at finite T ($T = 430$ MeV)

Same lattice setup of the $T = 0$ case. Also in this case, we consider the following continuum-scaling function for Spectral Projectors (SP):

$$\chi^{1/4}_{\mathrm{SP}}(a, M/m_s) = \chi^{1/4} + c_{\mathrm{SP}}(M/m_s)a^2 + o(a^2).$$



Spectral lattice artifacts are suppressed compared to the gluonic case when M/m_s is chosen in the previously determined optimal interval:

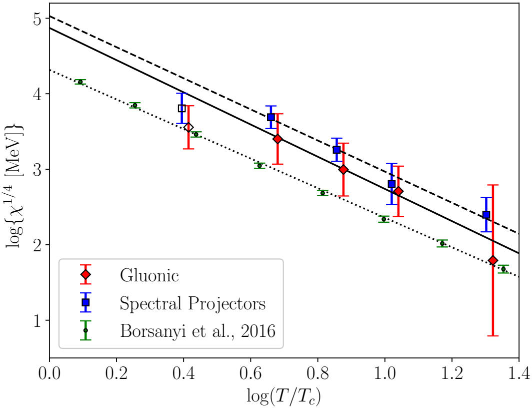
$c_{\mathrm{SP}}(0.3)/c_{\mathrm{gluo}} \sim 5 \cdot 10^{-2}$,

$c_{\mathrm{SP}}(0.5)/c_{\mathrm{gluo}} \sim 10^{-1}$.

$\chi(T)$ for $T > T_c$ from Spectral Projectors

The Dilute Instanton Gas Approximation (DIGA) predicts:

$$\chi^{1/4}(T) = A(T/T_c)^{-b}, \quad T \gg T_c, \qquad b_{\text{DIGA}} \simeq 2.$$



Our data for $T/T_c \gtrsim 2$ are in very good agreement with DIGA power-law:

$$b_{\text{SP}} = 2.06(41)$$
$$b_{\text{gluo}} = 2.1(1.1)$$

Compare also with result from Borsanyi at al., 2016:

$$b = 1.96(2), \text{ for } T/T_c \gtrsim 1.1.$$

Best fit lines are $\sim$ parallel, SP prefactor of $\chi^{1/4}$ is $\sim$ a factor of 2 larger compared to Borsanyi at al., 2016, i.e., **an order of magnitude for $\chi = m_a^2 f_a^2$.**

Conclusions

Summary of the talk

- Spectral Projectors (SP) provide a theoretically well-posed and numerically efficient method to define the topological susceptibility
- Systematics related to the continuum extrapolation are well under control through the choice of M
- Good agreement among SP data and DIGA prediction for $T/T_c \gtrsim 2$: $\chi_{\mathrm{SP}}^{1/4}(T) \sim T^{-b}$, $b_{\mathrm{SP}} = 2.06(41)$ VS $b_{\mathrm{DIGA}} \simeq 2$
- $\sim 2-3\ \sigma$ tension with previous results obtained by different methods, resulting in $\chi = m_a^2 f_a^2$ differing by about an order of magnitude

Future outlooks

- it would be interesting to explore **higher temperatures**, where Spectral Projectors are expected to provide major improvements
- to reach $T \gtrsim 1$ GeV, lattice spacings $a \lesssim 0.01$ fm needed $\implies$ severe Topological Critical Slowing Down with standard algorithms. Promising candidate for a viable solution: **Parallel Tempering on Boundary Conditions** (Hasenbusch, 2017; Berni, CB, D'Elia, 2019; CB, Bonati, D'Elia, 2021, CB, D'Elia, Lucini, Vadacchino, 2022)

Thank you for your attention!