A new mechanism for symmetry breaking from nilmanifolds

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Introduction and motivation

Exploring negative curvature compact geometries as a tool for model building at intermediate scale (relevant for unification and EW scales)

Important differences wrt flat compact extra dimensions

- Spectrum of the modes
- Symmetry breaking
- Scalar potential

Example: Gauge-Higgs unification models

- Standard approach uses tori as compact space (here negative curvature)
- Masses for the scalars are generated at loop level for the torus (here at tree level)

Nilmanifold

A Lie algebra is nilpotent if its lower central series terminates in the zero sub-algebra

$$[\mathfrak{g}, [\mathfrak{g}, \ldots, [\mathfrak{g}, [\mathfrak{g}, \mathfrak{g}]] \ldots] = 0$$

Nilmanifold is a differentiable manifold diffeomorphic to the quotient space N/H, the quotient of a nilpotent Lie group N modulo a closed subgroup H.

Nilmanifold

nilpotent Lie algebra ex. Heisenberg algebra:

$$[V_1, V_2] = -\mathtt{f} V_3 \;,\; [V_1, V_3] = [V_2, V_3] = 0$$

with f $\in \mathbb{R}$ and coordinate system

$$de^3 = \mathtt{f} e^1 \wedge e^2 \;;\; de^1 = 0 \;;\; de^2 = 0$$

$$e^1 = r^1 dx^1 \;;\quad e^2 = r^2 dx^2 \;;\; e^3 = r^3 \left(dx^3 + N x^1 dx^2\right)$$

$$\text{where} \quad N = \frac{r^1 r^2}{r^3} \mathtt{f} \in \mathbb{N} \;.$$

Why nilmanifold?

Heisenberg manifold ⇔ 2-torus with twisted circle fiber
Calculable spectrum of the Laplace operator $\Delta f = \lambda f$
Eigenfunctions form a complete set on the space:

$$f(x) = \sum_i c_i U_i(x)$$

$$\Delta B_m = \lambda B_m$$

Eigenscalars and one-forms have analytical expressions
To make the manifold compact:

$$x^1 \sim x^1 + n^1 \;;\; x^2 \sim x^2 + n^2 \;;\; x^3 \sim x^3 + n^3 - n^1 N x^2$$
$$n^1, n^2, n^3 \in \{0, 1\} \;.$$

From 7D Yang-Mills to 4D

The effective action is computed from the 7D YM action :

$$\mathcal{L}_{4D} = \int dy^3 \mathcal{L}_{7D} \ ; \ \mathcal{L}_{7D} = \frac{1}{2}\mathsf{Tr}\left(F_{MN}F^{MN}\right)$$

$$\begin{aligned} \mathcal{A}^a = \mathcal{A}^a_M(x^M)\mathrm{d}x^M &= \mathcal{A}^a_\mu(x^M)\mathrm{d}x^\mu + \mathcal{A}^a_m(x^M)\mathrm{d}x^m \\ &= \sum_I U_I(x^m)A^{aI}_\mu(x^\mu)\mathrm{d}x^\mu + \phi^{aI}(x^\mu)B_{Im}(x^m)\mathrm{d}x^m \end{aligned}$$

where U_I and B_I are respectively 3d eigenscalars and 3d eigen-one-forms of the Laplacian on the nilmanifold, while A^{aI} and ϕ^{aI} are a 4d one-form and a 4d scalar.

From 7D Yang-Mills to 4D

The sum over I is an infinite multi-index sum over the basis of 3d eigenforms, the geometrical limit ("large base, small fiber" limit)

$$|\mathbf{f}| \ll \frac{1}{r^i} \ , \ i = 1, 2, 3 \quad \Rightarrow \quad r^3 \ll r^{1,2}$$

separates the low-lying masses from the rest of the tower

The resulting 4D action "generates" a scalar part:

$$S = \int \mathrm{d}x^4 \mathrm{Tr}\Big(-\tfrac{1}{2} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{3} D_\mu \phi^i D^\mu \phi^i - M^2 (\phi^3)^2 - \mathcal{U} \Big)$$

Lowest modes

Scalars:

$$U_{I=1} = \frac{1}{\sqrt{V}} \; ; \quad \lambda_{U_1} = 0$$

one-forms:

$$B_{I=1} = \frac{1}{\sqrt{V}} e^1 \; ; \quad \lambda_{B_1} = 0$$

$$B_{I=2} = \frac{1}{\sqrt{V}} e^2 \; ; \quad \lambda_{B_2} = 0$$

$$B_{I=3} = \frac{1}{\sqrt{V}} e^3 \; ; \quad \lambda_{B_3} = \mathtt{f}^2$$

other modes:

$$(m_{tower})^2 \sim \frac{1}{(r^i)^2}$$

Scalar potential

$$\mathcal{U} = \mathsf{Tr}\Big(-2ig M[\phi^1, \phi^2]\phi^3 + \tfrac{1}{2}g^2 \sum_{i,j=1}^{3} [\phi^i, \phi^j][\phi^i, \phi^j] \Big)$$

with M = |f| and $g = g_7/\sqrt{V}$; scalars in the adjoint representation. We have to minimise the potential (mass + interaction part)

$$\frac{\mathcal{V}}{M^2} = \mathrm{Tr}(\phi^3)^2 + \frac{\mathcal{U}}{M^2}$$

computing $\delta\mathscr{V}/M^2$ and the masses from $\delta^2\mathscr{V}/M^2$

Vacuum condition : $\phi_3 = 0$; $[\phi_1, \phi_2] = 0$

$\Rightarrow$ Pick $\phi_1, \phi_2 \in$ Cartan sub-algebra.

The mass matrix is block diagonal in root space

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Once the mass matrix is diagonalized, the masses for a given root Eα are :

$$0\ ,\ (m_\alpha^\pm)^2 = \frac{1}{2}M^2\left(1+2((b_1^\alpha)^2+(b_2^\alpha)^2)) \pm \sqrt{1+4((b_1^\alpha)^2+(b_2^\alpha)^2))}\right)$$

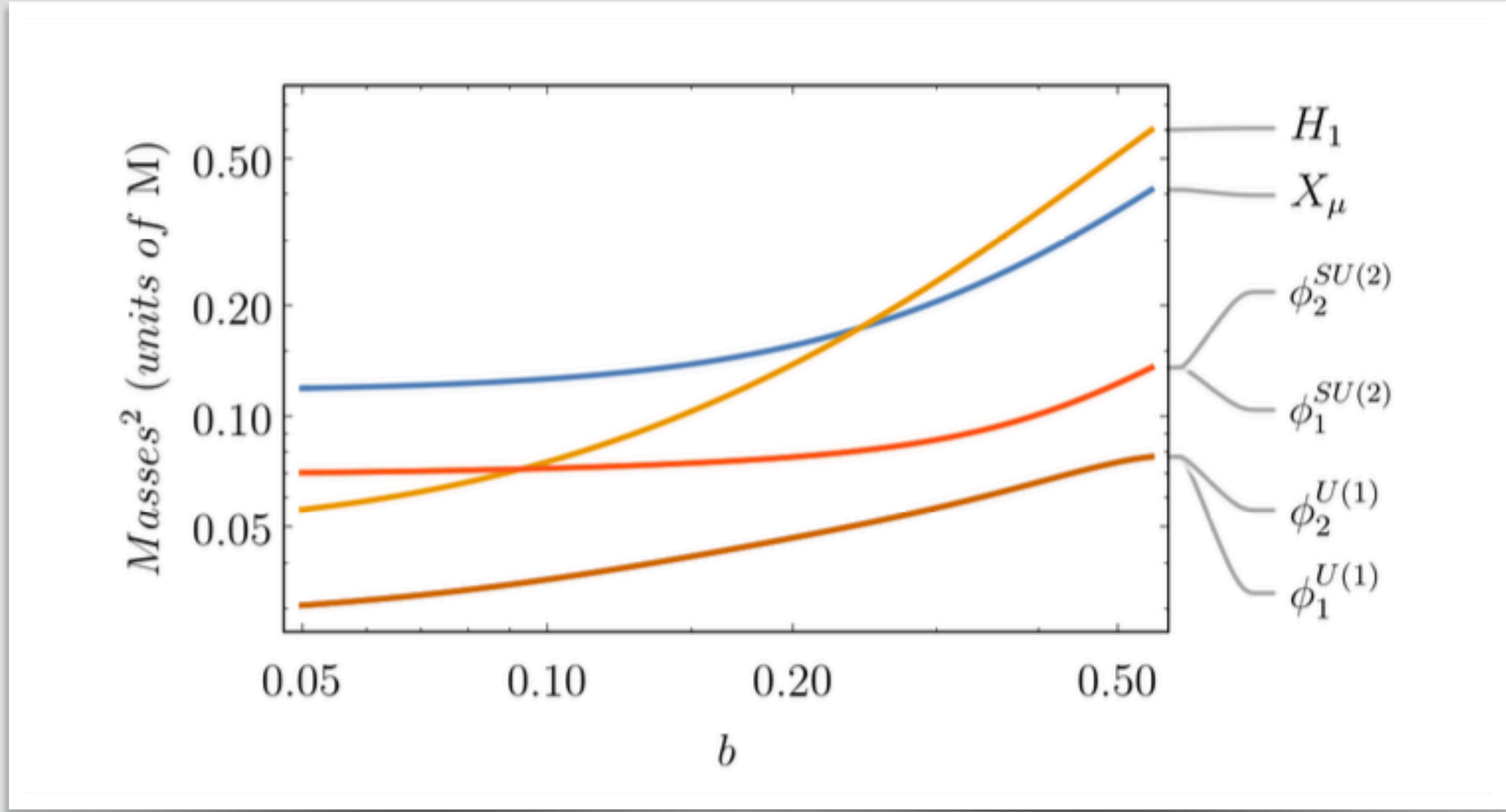
lifted by loop corrections

gauge boson masses

$$\begin{aligned} m^2_{\alpha,gauge} &= \mathrm{g}^2 \sum_{I=1}^{2} \phi_0^{Ii} \alpha_i \\ &= M^2\left((b_1^\alpha)^2+(b_2^\alpha)^2\right) \end{aligned}$$

In this convention the vacuum parameters b_i are dimensionless, so that b_i = Mg $\tilde{b}_i$, and $\tilde{b}_i$ has mass dimension one. In the following $b_1=b_2=b$

SU(3) -> SU(2)xU(1)



One-loop renormalized masses of the low-mass scalars for the SU(3) breaking pattern. H_1 and X_μ are in the fundamental representation of SU(2) × U(1), while $\phi_{SU(2)}$ is the adjoint of SU(2) and $\phi_{U(1)}$ in the adjoint of U(1).

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Conclusion

- Twist f ⇔ Mass at tree level M
- Potential allows for various symmetry breaking
- Model is rigid (Yang-Mills in 7D + nilmanifold)
- Analytical results all the way for any gauge group G (but G may be constrained by physical motivations)
- Moduli of the metric on the Heisenberg manifold computed
- Laplacian spectrum for scalars/vectors with arbitrary metric solved for the lowest modes
- Dirac operator with arbitrary metric solved, fermions can be considered both in 7D (bulk) and 4D (localised)