Juan Ramos Cabezas (Ariel University, Israel)

Title of the presentation:

A Large family of solvable lattice models (Interaction round to face) based on WZW models

The talk is partially based on the paper: "The crossing multipliers of solvable lattice models", arxiv. 2110.09798. V. Belavin, D. Gepner, J. Ramos.

July 8, 2022
41st International Conference on High Energy Physics, Bologna, Italy.

Content:

Known models:

[Jimbo, Miwa and Okado, *Nucl.Phys.B* 300 (1988);

Commun. Math. Phys. 116(1988)]

- Definition of the Interaction Round-to-Face 2D lattice models.
- Solution of the Models.
- Physical quantities we can define and find.

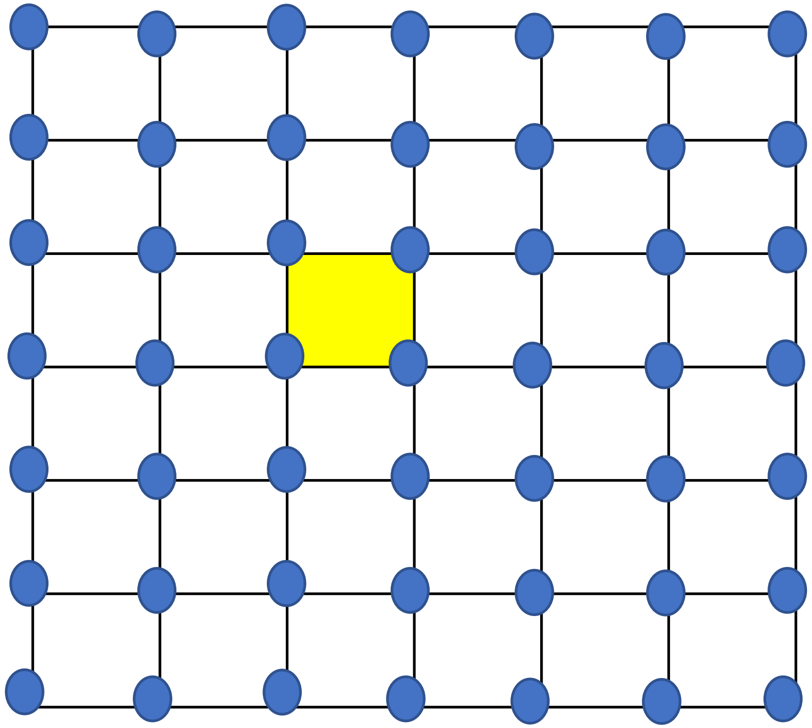
New models:

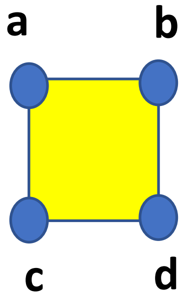
- Generalization of known models.
- A general method to solve them (Proposal).
- Application (connection with other fields of physics).

[Gepner, 1990, arxiv, 9211100]

Interaction Round-to-Face Lattice models (IRF)

Definition of the model: N





$$\omega \begin{pmatrix} a & b \\ c & d \end{pmatrix} = e^{-E(a,b,c,d)/kT}$$

$$Z = \sum_{\text{configurations}} \prod_{\text{faces}} \omega \left(\begin{matrix} a & b \\ c & d \end{matrix} \middle| u \right)$$

Interaction Round-to-Face Lattice models (IRF)

Affine Lie algebras: $A_n^1, \quad B_n^1, \quad C_n^1, \quad D_n^1.$

$$\mathfrak{su}(n+1)_k$$

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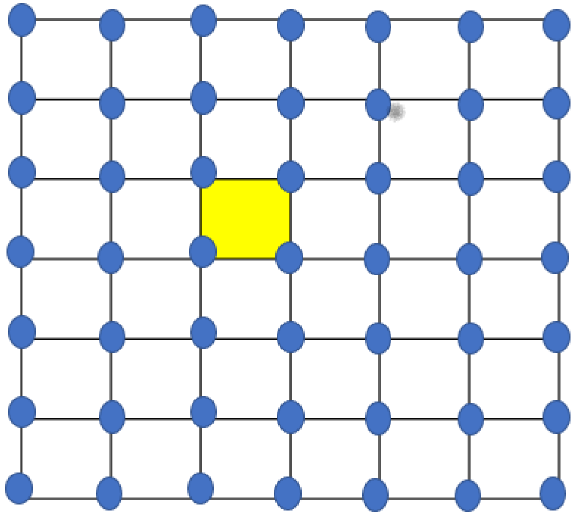

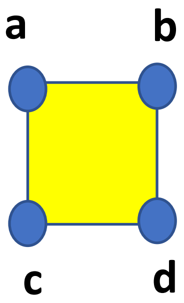

Basis: $(\Lambda_0, \Lambda_1, \ldots, \Lambda_n)$

$$\Lambda_i.\Lambda_j = F_{ij}, \quad \Lambda_0.\Lambda_i = 0$$

$$V = \sum_{i=0}^{n} a_i \Lambda_i \qquad \sum_i a_i = k = level, \qquad a_i \in \mathbf{Z}_+$$

"Rules of the game ": $a - b = \bar{\mu} \quad \bar{\mu} = \Lambda_{\mu+1} - \Lambda_\mu \qquad \mu = 0, 1, \ldots, n.$

How to find the Boltzmann weights of this model?

Yang-Baxter equation:

$$\omega\begin{pmatrix} a & b \\ c & d \end{pmatrix}\bigg| u \Bigg) \longrightarrow \sum_g \omega\begin{pmatrix} a & b \\ f & g \end{pmatrix}\bigg| u+v\Bigg)\, \omega\begin{pmatrix} f & g \\ e & d \end{pmatrix}\bigg| u\Bigg)\, \omega\begin{pmatrix} b & c \\ g & d \end{pmatrix}\bigg| v\Bigg) = \sum_g \omega\begin{pmatrix} a & g \\ f & e \end{pmatrix}\bigg| v\Bigg)\, \omega\begin{pmatrix} a & b \\ g & c \end{pmatrix}\bigg| u\Bigg)\, \omega\begin{pmatrix} g & c \\ e & d \end{pmatrix}\bigg| u+v\Bigg)$$

Interaction Round-to-Face Lattice models (IRF)

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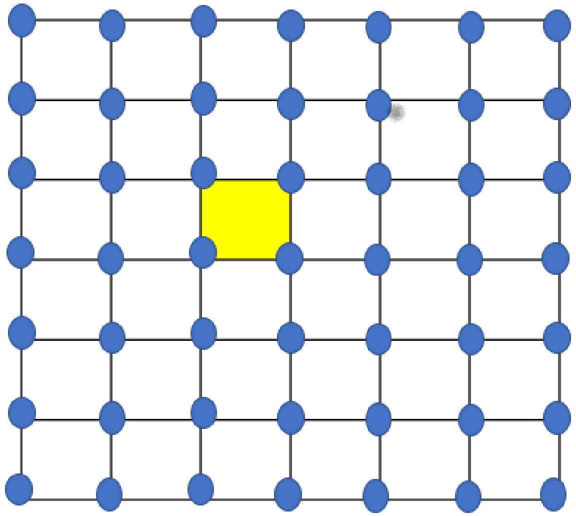

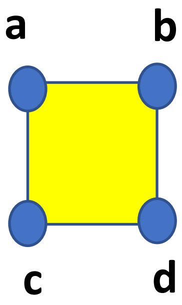

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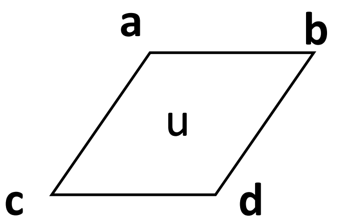
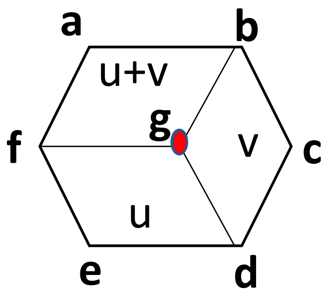
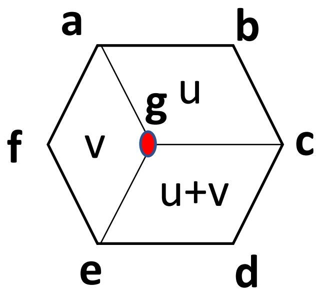
$$\omega \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Bigg| u \Bigg) =$$



$\sum_g$



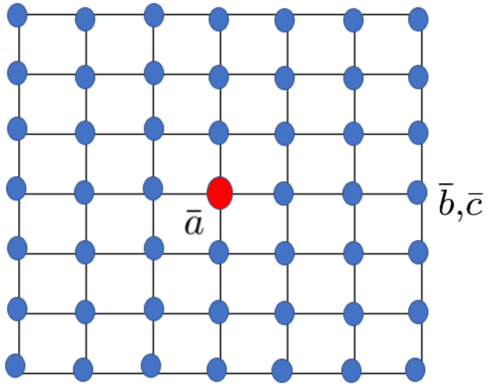
$= \sum_g$



Solutions of Jimbo-Miwa-Okado models

$$\omega\begin{pmatrix} a & a+\bar{\mu} \\ a+\bar{\mu} & a+2\bar{\mu} \end{pmatrix}\bigg| u \Bigg) = \frac{[1+u]}{[1]}$$

$$[u] = \theta_1(u,q) = 2|q|^{1/2}\sin\left(\frac{\pi u}{n+k+1}\right)\prod_{m=1}^{\infty}\left(1-2q^m cos\left(\frac{\pi u}{n+k+1}\right)+q^{2m}\right)$$



What other physical quantities we can compute?

$$Z = \sum_{\text{configurations}}\prod_{\text{faces}}\omega\left(\begin{matrix} a & b \\ c & d \end{matrix}\bigg| u\right)$$

$$\textbf{Local State Probability} = P_{\bar{a}} := \frac{Z_{\bar{a}}}{Z}$$

- To compute it we use: the CTM method of Baxter; Some simplifications; Inversion relation of BW

$$q = e^{-\epsilon}, \quad \epsilon, u \to 0 \quad e^{-u/\epsilon}$$

$$\sum_g \bar{\omega}\left(\begin{matrix} a & b \\ c & g \end{matrix}\bigg| u\right)\bar{\omega}\left(\begin{matrix} d & c \\ b & g \end{matrix}\bigg| -u-n-1\right) = \frac{[u][u-n-1]}{[1]^2}\delta_{ad}$$

Crossing multipliers

Principal specialized character a representation

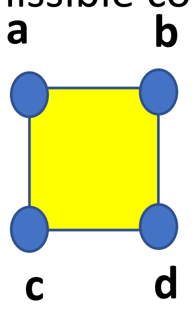
$$\bar{\omega}\left(\begin{matrix} a & b \\ c & d \end{matrix}\bigg| u\right) = \left(\frac{G_a G_d}{G_b G_C}\right)^{1/2}\omega\left(\begin{matrix} a & b \\ c & d \end{matrix}\bigg| u\right)$$

- Relation of LSP with the branching functions → $P_{\bar{a}} \propto B_{\bar{a},\bar{b},\bar{c}}$ $\qquad \dfrac{G_{\bar{a}}}{G_{\bar{b}}G_{\bar{c}}}$

Generalization and Gepner method

Generalization of the admissible conditions (rules of the game)

$\mathfrak{su}(n+1)_k$



$$a \times \Lambda_1 = \sum_i V_i$$

Affine Lie algebras: $A_n^1, \quad B_n^1, \quad C_n^1, \quad D_n^1.$

Basis: $(\Lambda_0, \Lambda_1, \ldots, \Lambda_n)$

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“Rules of the game ”: $a - b = \bar{\mu} \quad \bar{\mu} = \Lambda_{\mu+1} - \Lambda_\mu \qquad \mu = 0, 1, \ldots, n.$ $\qquad a \times \Lambda_1 = \sum_i V_i$

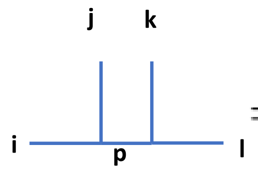
Generalization of the rules: $a \times \phi = \sum_i N_{a\phi}^i V_i$

Gepner’s method: With this method we can find Boltzmann weights of any IRF models based on any algebra once we have:

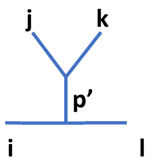
- Fusion rules of fluctuation variables: $\phi_i \times \phi_j = \sum_k N_{ij}^k \phi_k$
- Braiding matrices of the CFT related to the IRF model in question

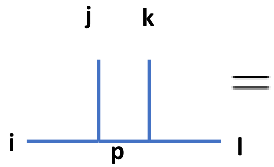
$$\phi_i \times \phi_j = C_{ij}\phi_p \qquad \phi_k \times \phi_l = C_{kl}\phi_p$$

$$\langle \phi_i \phi_j \phi_k \phi_l \rangle = C_{ijkl}^p \langle \phi_p \phi_p \rangle \Rightarrow \sum_p C_{ijkl}^p$$

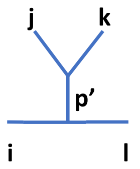


$$= \sum_{p'} C_{iljk}^{p'}$$





$$= \sum_{p'} B_{pp'} \begin{bmatrix} i & j \\ k & l \end{bmatrix}$$



Conclusions

- We expect that our conjecture about the LSP of other IRF models based on the other AL algebras holds.
- We can check this proposal for some specific new models (algebras).
- The method here discussed is a powerful method that can help us to solve new interesting IRF models. We plan to apply this method to solve new models.
- [M.Gaberdiel, Gopakumar, An AdS_3 Dual for Minimal Model CFTs] . Is the LSP related to some models which live in an AdS_3?

Thank you for your attention