Renormalization Group beta function and anomalous dimensions

Oliver Witzel

Universität Siegen

Center for Particle Physics Siegen

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Motivation

- Study properties of strongly coupled gauge-fermion systems
- Characterize nature of such systems
  $\rightarrow$ Where is the onset of the conformal window?
- Determine properties such as anomalous dimensions
  $\rightarrow$ Important for BSM model building

Renormalization Group β function

$$\beta(g^2) = \mu^2 \frac{dg^2}{d\mu^2}$$

- Encodes dependence of coupling g^2 on the energy scale μ^2
- β has no explicit dependence on μ^2, only implicit through $g^2(\mu)$
- Known perturbatively up to 5-loop order in the $\overline{\text{MS}}$ scheme (1- and 2-loop are universal)
  [Baikov et al. PRL118(2017)082002] [Ryttov and Shrock PRD94(2016)105015]
- Perturbative predictions reliable at weak coupling,
  nonperturbative methods needed for strong coupling

Step-Scaling β function

- Discretized β function determined using numerical lattice field theory calculations [Lüscher et al. NPB359(1991)221]
  - → Choose symmetric L^4 setup where the size L of the lattice is the only scale
  - → Determine β function by calculating scale change $L \to s \cdot L$
- Gradient flow [Narayanan and Neuberger JHEP 0603 (2006) 064] [Lüscher CMP 293 (2010) 899][JHEP 1008 (2010) 071]
  - → Continuous smearing transformation which can be used to define a renormalized coupling

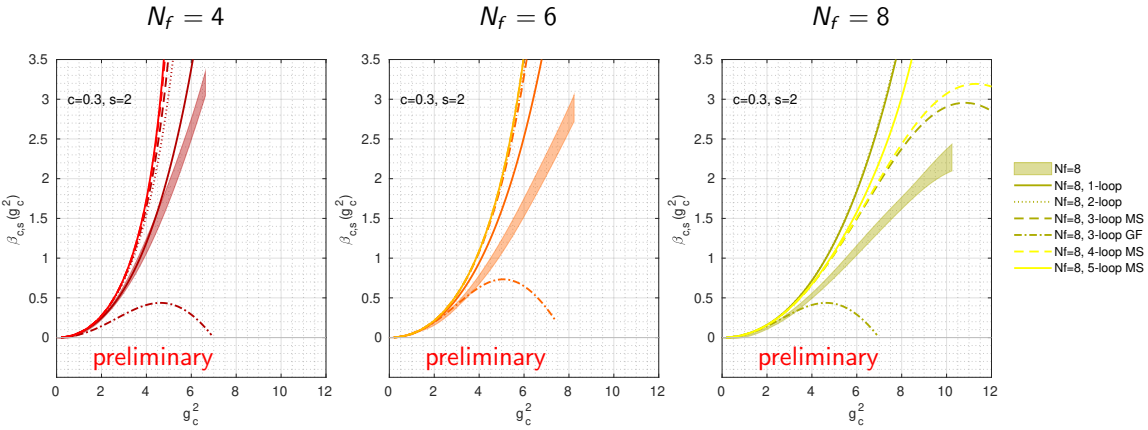
$$g_c^2(L) = \frac{128\pi^2}{3(N_c^2-1)} \frac{1}{C(c,L)} t^2 \langle E(t) \rangle$$

  - → Relate flow time t to scale L: $\sqrt{8t} = c \cdot L$ [Fodor et al. JHEP11(2012)007][JHEP09(2014)018]
  - → Calculate scale difference

$$\beta_s^c(g_c^2; L) = \frac{g_c^2(sL) - g_c^2(L)}{\log(s^2)}$$

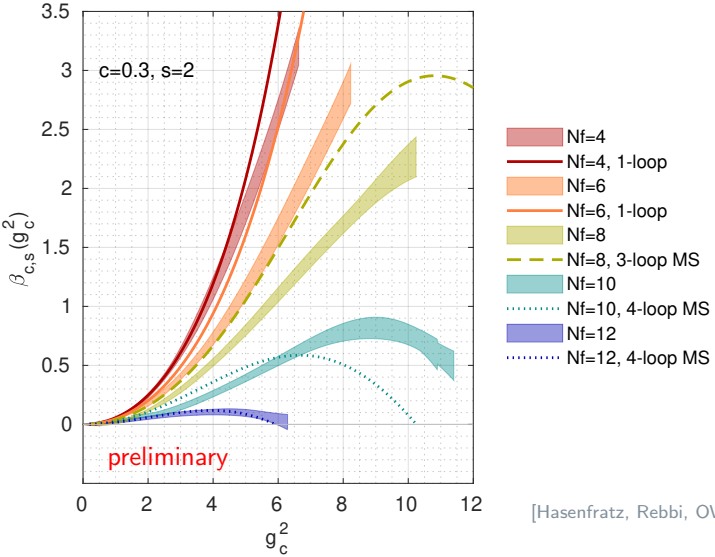
  - → Extrapolate $L \to \infty$ to remove discretization effects and take the continuum limit

SU(3) with N_f fundamental flavors



[Hasenfratz, Rebbi, OW in preparation]

SU(3) with N_f fundamental flavors



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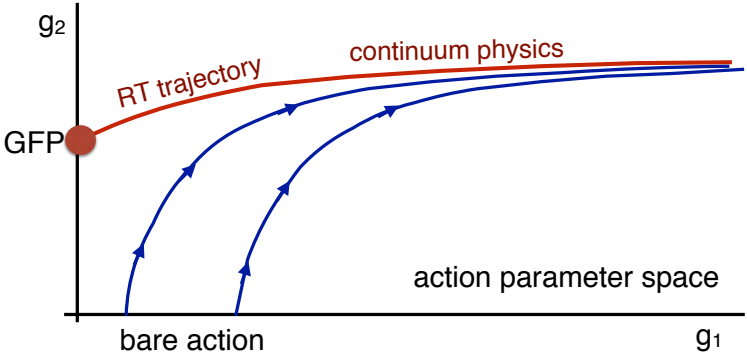
Beyond Step-Scaling: real-space Renormalization Group (RG) flow



Beyond Step-Scaling: real-space Renormalization Group (RG) flow

- RG flow: change of (bare) parameters and coarse graining (blocking)

- Gradient flow is a continuous transformation
  → Define real-space RG blocked quantities by incorporating coarse graining as part of calculating expectation values [Carosso, Hasenfratz, Neil PRL 121 (2018) 201601]

- Relate GF time t/a^2 to RG scale change $b \propto \sqrt{t/a^2}$
  → Quantities at flow time t/a^2 describe physical quantities at energy scale $\mu \propto 1/\sqrt{t}$
  → Local operator with non-vanishing expectation value can be used to define running coupling
  ⇝ Simplest choice: $t^2\langle E(t)\rangle$ [Lüscher JHEP 1008 (2010) 071]

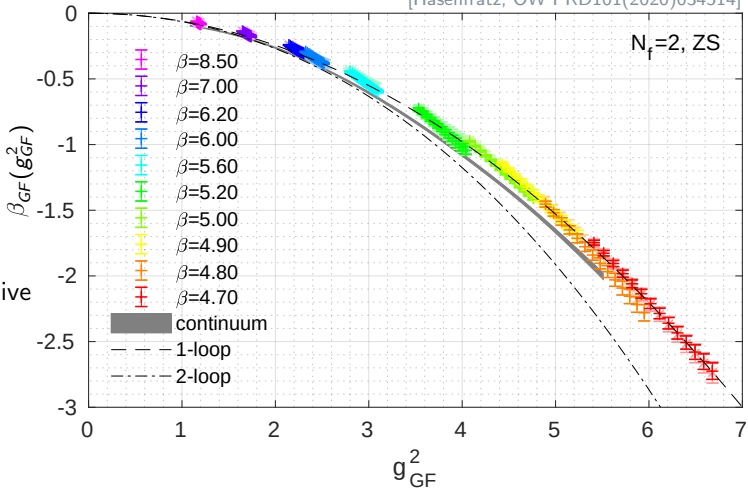
- Continuous RG β function

$$\beta(g_{GF}^2) = \mu^2 \frac{dg_{GF}^2}{d\mu^2} = -t\frac{dg_{GF}^2}{dt}$$

Example: QCD

- QCD: SU(3) gauge theory with 2 light flavors in the fundamental representation
- Fast "running" coupling
  $\rightsquigarrow$ Confinement
- Plot: Comparison of non-perturbative and perturbative determinations

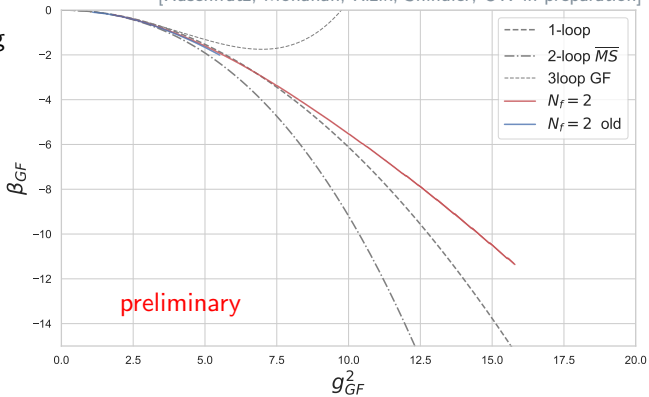
[Hasenfratz, OW PRD101(2020)034514]



Example: QCD

- Extended data to stronger coupling
  → Confining region, $g^2_{\mathrm{GF}} \sim 16$
- At strong coupling RG β function is highly linear
  → Nonperturbative phenomenon
- Integrate β function to obtain Λ_{GF}
  → g^2_m GF renormalized coupling at energy scale $\mu = 1/\sqrt{8t_0}$

[Hasenfratz, Monahan, Rizik, Shindler, OW in preparation]



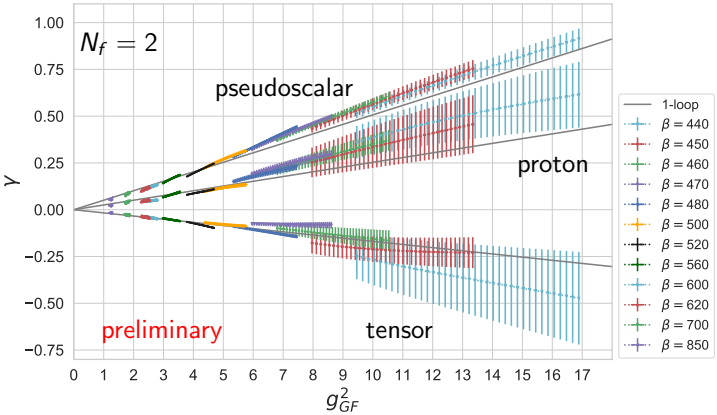
$$\Lambda_{GF} = \mu\left(b_0 g_m^2\right)^{-\frac{b_1}{2b_0^2}} \exp\left(-\frac{1}{2b_0 g_m^2}\right) \exp\left[-\int_0^{g_m^2} \mathrm{d}g^2 \left(\frac{1}{\beta(g^2)} + \frac{1}{b_0 g^4} - \frac{b_1}{b_0^2 g^2}\right)\right]$$

$\Rightarrow \Lambda^{\mathrm{prelim}}_{\overline{\mathrm{MS}}} = 351.4(9.5)$ MeV (stat. error only) compare to ALPHA: f_K: 310(20) MeV

[Fritzsch et al. NPB865(2012)397]

Mass anomalous dimension

[Hasenfratz, Monahan, Rizik, Shindler, OW in preparation]

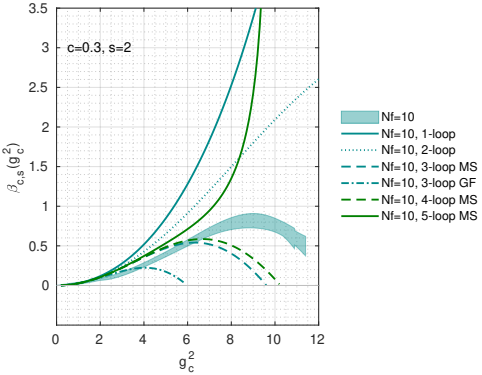


- No continuum limit or infinite volume extrapolation

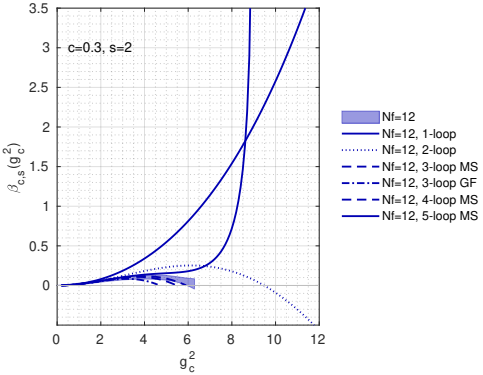
extra

SU(3) with $N_f = 10$, 12 fundamental flavors

$N_f = 10$



$N_f = 12$



[Hasenfratz, Rebbi, OW PRD 101(2020)114508] [Hasenfratz, Rebbi, OW PRD 100(2019)114508]

[Hasenfratz, Rebbi, OW PLB 798(2019)134937]