

Hadron spectroscopy using holographic QCD and 't Hooft equation

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Overview

- 1 Light-front wavefunction
- 2 Supersymmetric formulation of LFH
- 3 Longitudinal dynamics
- 4 Predicting Hadron spectrum

$$H_{\text{QCD}}^{\text{LF}} |\Psi(P)\rangle = M^2 |\Psi(P)\rangle$$

where $H_{\text{QCD}}^{\text{LF}} = P^+ P^- - P_\perp^2$ is the LF QCD Hamiltonian and M is the hadron mass. At equal light-front time ($x^+ = 0$) and in the light-front gauge $A^+ = 0$, the hadron state $|\Psi(P)\rangle$ admits a Fock expansion, i.e.

$$|\Psi(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n, h_i} \int [dx_i] [d^2 \mathbf{k}_{\perp i}] \frac{1}{\sqrt{x_i}} \Psi_n(x_i, \mathbf{k}_{\perp i}, h_i) |n : x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, h_i\rangle$$

where $\Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)$ is the LFWF of the Fock state with n constituents and the integration measures are given by

$$[dx_i] \equiv \prod_i^n dx_i \delta(1 - \sum_{j=1}^n x_j) \quad [d^2 \mathbf{k}_{\perp i}] \equiv \prod_{i=1}^n \frac{d^2 \mathbf{k}_{\perp i}}{2(2\pi)^3} 16\pi^3 \delta^2(\sum_{j=1}^n \mathbf{k}_{\perp j}) .$$

$(k_i^+, k_i^-, \mathbf{k}_{\perp i})$ and h_i are the momentum and helicity of the i^{th} constituent and $x_i = k_i^+/P^+$.

The valence meson LFWF

For $n = 2$,

$$\mathbf{k}_{\perp 1} = -\mathbf{k}_{\perp 2} = \mathbf{k}_{\perp}$$

$$x_1 = 1 - x_2 = x$$

The position-space conjugate of \mathbf{k}_{\perp} , denoted by $\mathbf{b}_{\perp} = b_{\perp} e^{i\varphi}$, is the transverse separation between the quark and the antiquark.

Introduce a new light-front variable $\zeta = \sqrt{x(1-x)}\mathbf{b}_{\perp} = \zeta e^{i\varphi}$ leads to the meson LFWF in the position-space:

$$\Psi(\zeta, x, \phi) \xrightarrow{\text{factorization}} \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\phi} X(x)$$

$\phi(\zeta)$ and $X(x)$ are referred to as the transverse and longitudinal modes.

Holographic Schrödinger equation

Brodsky, de Teramond (PRL, 09)

Brodsky, de Teramond, Dosch, Erlich (Phys. Rep. 15)

In the semi-classical limit, i.e. zero quark mass and no quantum loop, based on AdS/CFT, one can show that the transverse mode of LFWF of the valence ($n = 2$ for mesons) state can be obtained from a 1-dimensional Schrödinger-like wave equation for the:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U_{\perp}(\zeta) \right) \phi(\zeta) = M_{\perp}^2 \phi(\zeta)$$

the potential is uniquely determined from the conformal symmetry breaking mechanism and correspondence with weakly coupled string modes in AdS_5 space, which results in a light-front harmonic oscillator potential in physical spacetime with confinement scale κ :

$$U_{\perp}(\zeta, J) = \kappa^4 \zeta^2 + \kappa^2 (J - 1)$$

$J = L + S$ is the total meson angular momentum.

Solutions to holographic Schrödinger equation

With the confining potential specified, one can solve the holographic Schrödinger equation to obtain the meson mass spectrum,

$$M_\perp^2 = 4\kappa^2 \left(n_\perp + L + \frac{S}{2} \right)$$

which, as expected, predicts a massless pion. The corresponding normalized eigenfunctions are given by

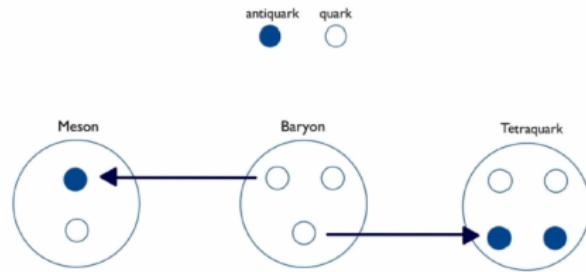
$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} \exp\left\{\left(\frac{-\kappa^2 \zeta^2}{2}\right)\right\} L_n^L(x^2 \zeta^2).$$

To completely specify the holographic meson wavefunction, we need the analytic form of the longitudinal mode $X(x)$. This is obtained by matching the expressions for the pion EM or gravitational form factor in physical spacetime and in AdS space. Either matching consistently results in $X(x) = \sqrt{x(1-x)}$

Supersymmetric LFH

H. G. Dosch, G. F. de Teramond, and S. J. Brodsky, PRD 92, 074010 (2015); PRD 95, 034016 (2017)

- A unified description of baryons and mesons/tetraquarks.
- Each baryon (viewed as a quark-diquark system) possesses two superpartners: a (quark-antiquark) meson and a (diquark-antidiquark) tetraquark.
- This supersymmetric connection stems from the fact that a diquark can be in the same $SU_c(3)$ representation as an antiquark, and an antidiquark can be in the same $SU_c(3)$ representation as a quark.



Supersymmetric Holographic Equation

$$H |\phi\rangle = M_{\perp}^2 |\phi\rangle$$

where

$$H = \begin{pmatrix} -\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + U_M(\zeta) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + U_B(\zeta) \end{pmatrix}$$

with

$$U_M(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L_M + S_M - 1),$$

$$U_B(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L_B + S_D)$$

where S_M is the total quark-antiquark spin and S_D is the diquark spin.

Supersymmetric Holographic Equation (Continue)

The 4-plet, $|\phi\rangle$, is given by

$$|\phi\rangle = \begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi^-(L_B + 1) \\ \psi^+(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$

where ψ^+ and ψ^- are the two components of the baryon wavefunction and $\phi_{M/T}$ is the meson/tetraquark wavefunction. The eigenvalues given by

$$M_{\perp,M}^2 = 4\kappa^2 \left(n_\perp + L_M + \frac{S_M}{2} \right) ,$$

$$M_{\perp,B}^2 = 4\kappa^2 \left(n_\perp + L_B + \frac{S_D}{2} + 1 \right) ,$$

and

$$M_{\perp,T}^2 = 4\kappa^2 \left(n_\perp + L_T + \frac{S_T}{2} + 1 \right) ,$$

where S_T is total diquark-antidiquark spin.

Superpartners Conditions

- Baryons with quantum numbers, $L_B = L_M - 1$ and $S_D = S_M$, are superpartners to mesons with quantum numbers, L_M and S_M , and tetraquarks with quantum numbers $S_T = S_D$ and $L_T = L_B$.
- Lowest lying mesons with $n_\perp = L_M = 0$ do not have a baryonic superpartner.
- pseudoscalar mesons with $n_\perp = L_M = S_M = 0$, like the pion and the kaon, are predicted to be massless, just as expected in the chiral limit of QCD.

The 't Hooft Equation

G. 't Hooft, A Two-Dimensional Model for Mesons, Nucl. Phys. B 75 (1974) 461470

In an earlier approach, 't Hooft derived a Schrödinger-like equation for the longitudinal mode, starting from the QCD Lagrangian in (1 + 1)-dim in the $N_c \gg 1$ approximation. This Lagrangian now contains two mass scales: the quark mass and the gauge coupling. The resulting 't Hooft Equation is:

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right) \chi(x) + U_L(x) \chi(x) = M_L^2 \chi(x)$$

with

$$U_L(x) \chi(x) = \frac{g^2}{\pi} \mathcal{P} \int dy \frac{\chi(x) - \chi(y)}{(x-y)^2}$$

The longitudinal mode $\rightarrow X(x) = \sqrt{x(1-x)} \chi(x)$, $M^2 = M_\perp^2 + M_L^2$

Predicting hadron spectrum-input parameters

$$M^2(n_L, n_T, J, L) = M_T^2(n_T, J, L) + M_L^2(n_L)$$

Hadron	g	$m_{u/d}$	m_s	m_c	m_b
Light	0.128	0.046	0.357	-	-
Heavy-light	0.410	0.330	0.500	1.370	4.640
Heavy-heavy	0.523	-	-	1.370	4.640

- $\kappa = 0.523 \text{ GeV}$
- Parity: $P = (-1)^{L_M+1} = (-1)^{L_B} = (-1)^{L_T}$
The charge conjugation of mesons/tetraquarks
 $C = (-1)^{n_L+L_M+S_M} = (-1)^{n_L+L_T+S_T-1}$
- Finding: $n_L \geq n_T + L$, i.e. in any hadron, an orbital and radial excitations in the transverse dynamics is always accompanied by an excitation in the longitudinal dynamics.

Predicting hadron spectrum: light-light

Meson				Baryon				Tetraquark							
$J^P(C)$	Name	$M_{ }$	M_{\perp}	M	$J^P(C)$	Name	$M_{ }$	M_{\perp}	M	$J^P(C)$	Name	$M_{ }$	M_{\perp}	M	
0^{-+}	$\pi(140)$	140	0	140	-		-	-	-	-		-	-	-	-
1^{+-}	$h_1(1170)$	335	1046	1098	$(1/2)^+$	$N(940)$	188	1046	1063	0^{++}	$f_0(500)$	335	1046	1098	
2^{-+}	$\eta_2(1645)$	460	1479	1549	$(3/2)^-$	$N(1520)$	296	1479	1508	1^{-+}	-	408	1479	1534	
1^{--}	$\rho(770), \omega(780)$	140	740	753	-		-	-	-	-		-	-	-	-
2^{++}	$a_2(1320), f_2(1270)$	335	1281	1324	$(3/2)^+$	$\Delta(1232)$	188	1281	1295	1^{++}	$a_1(1260)$	235	1281	1302	
3^{--}	$\rho_3(1690), \omega_3(1670)$	460	1654	1717	$(3/2)^-$	$\Delta(1700)$	296	1654	1680	1^{-+}	$\pi_1(1600)$	335	1654	1688	
4^{++}	$a_4(1970), f_4(2050)$	559	1957	2035	$(7/2)^+$	$\Delta(1950)$	372	1957	1992	-	-	-	-	-	-
0^-	$K(495)$	456	0	456	-		-	-	-	-		-	-	-	-
1^+	$\bar{K}_1(1270)$	550	1046	1182	$(1/2)^+$	$\Lambda(1115)$	500	1046	1159	0^+	$K_0^*(1430)$	546	1046	1180	
2^-	$K_2(1770)$	617	1479	1603	$(3/2)^-$	$\Lambda(1520)$	588	1479	1592	1^-	-	633	1479	1609	
0^-	$K(495)$	456	0	456	-		-	-	-	-		-	-	-	-
1^+	$\bar{K}_1(1270)$	550	1046	1182	$(1/2)^+$	$\Sigma(1190)$	500	1046	1159	0^{++}	$a_0(980)$	920	1046	1393	
$\eta'(958)$	759	0	759	-	-	-	-	-	-	-		-	-	-	-
$h_1(1380)$	883	1046	1369	$(1/2)^+$	$\Xi(1320)$	805	1046	1320	0^{++}	$f_0(1370)$	920	1046	1393		
$\eta_2(1870)$	968	1479	1768	$(3/2)^-$	$\Xi(1620)$	876	1479	1719	1^{-+}	$a_0(1450)$	-	969	1479	1768	
$\Phi(1020)$	759	740	1060	-	-	-	-	-	-	-		-	-	-	-
$f_2'(1525)$	883	1281	1556	$(3/2)^+$	$\Xi^*(1530)$	805	1281	1513	1^{++}	$f_1(1420)$	850	1281	1537		
$\Phi_3(1850)$	968	1654	1916	$(3/2)^-$	$\Xi(1820)$	876	1654	1872	-	$a_1(1420)$	-	-	-	-	-
$f_2(1640)$	883	1281	1556	$(3/2)^+$	$\Omega(1672)$	1114	1281	1698	1^+	$K_1(1650)$	1159	1281	1728		

Predicting Meson spectrum: Heavy-light

Meson				Baryon				Tetraquark				
$J^P(C)$	Name	$M_{ }$	M_{\perp}	$J^P(C)$	Name	$M_{ }$	M_{\perp}	$J^P(C)$	Name	$M_{ }$	M_{\perp}	
0 ⁻	$D(1870)$	1861	0	1861	-	-	-	-	-	-	-	
1 ⁺	$D_1(2420)$	2135	1046	2377	(1/2) ⁺	$\Lambda_c(2290)$	2191	1046	2428	0 ⁺	$\bar{D}_0^*(2400)$	2510
2 ⁻	$D_J(2600)$	2326	1479	2756	(3/2) ⁻	$\Lambda_c(2625)$	2460	1479	2870	1 ⁻	-	2751
0 ⁻	$D(1870)$	1861	0	1861	-	-	-	-	-	-	-	
1 ⁺	$\bar{D}_1(2420)$	2135	1046	2377	(1/2) ⁺	$\Sigma_c(2455)$	2191	1046	2428	0 ⁺	$D_0^*(2400)$	2510
1 ⁻	$D^*(2010)$	1861	740	2003	-	-	-	-	-	-	-	
2 ⁺	$D_s^*(2460)$	2135	1281	2490	(3/2) ⁺	$\Sigma_c^*(2520)$	2191	1281	2538	1 ⁺	$D(2550)$	2510
3 ⁻	$D_s^*(2750)$	2326	1654	2854	(3/2) ⁻	$\Sigma_c(2800)$	2460	1654	2964	-	-	-
0 ⁻	$D_s(1968)$	2025	0	2025	-	-	-	-	-	-	-	
1 ⁺	$D_{s1}(2460)$	2283	1046	2511	(1/2) ⁺	$\Xi_c(2470)$	2348	1046	2570	0 ⁺	$\bar{D}_{s0}^*(2317)$	2676
2 ⁻	$D_{s2}?$	2464	1479	2874	(3/2) ⁻	$\Xi_c(2815)$	2586	1479	2979	1 ⁻	-	2908
1 ⁻	$D_s^*(2110)$	2025	740	2156	-	-	-	-	-	-	-	
2 ⁺	$D_{s2}^*(2573)$	2283	1281	2618	(3/2) ⁺	$\Xi_c^*(2645)$	2348	1281	2675	1 ⁺	$D_{s1}(2536)$	2676
3 ⁻	$D_{s3}^*(2860)$	2464	1654	2968	-	-	-	-	-	-	-	
1 ⁺	$D_{s1}?$	2283	1046	2511	(1/2) ⁺	$\Omega_c(2695)$	2524	1046	2732	0 ⁺	-	2845
2 ⁺	$D_{s2}^*?$	2283	1281	2618	(3/2) ⁺	$\Omega_c(2770)$	2524	1281	2830	1 ⁺	-	3012
0 ⁻	$\bar{B}(5280)$	5130	0	5130	-	-	-	-	-	-	-	
1 ⁺	$\bar{B}_1(5720)$	5385	1046	5486	(1/2) ⁺	$\Lambda_b(5620)$	5460	1046	5559	0 ⁺	$B_J(5732)$	5775
2 ⁻	$B_J(5970)$	5560	1479	5753	(3/2) ⁻	$\Lambda_b(5920)$	5714	1479	5902	1 ⁻	-	5999
0 ⁻	$B(5280)$	5130	0	5130	-	-	-	-	-	-	-	
1 ⁺	$B_1(5720)$	5385	1046	5486	(1/2) ⁺	$\Sigma_b(5815)$	5460	1046	5559	0 ⁺	$\bar{B}_J(5732)$	5775
1 ⁻	$B^*(5325)$	5130	740	5183	-	-	-	-	-	-	-	
2 ⁺	$B_s^*(5747)$	5385	1281	5535	(3/2) ⁺	$\Sigma_b^*(5835)$	5460	1281	5608	1 ⁺	$B_J(5840)$	5775
0 ⁻	$B_s(5365)$	5292	0	5292	-	-	-	-	-	-	-	
1 ⁺	$B_{s1}(5830)$	5528	1046	5626	(1/2) ⁺	$\Xi_b(5790)$	5610	1046	5707	0 ⁺	$\bar{B}_{s0}^*?$	5936
1 ⁻	$B_s^*(5415)$	5292	740	5343	-	-	-	-	-	-	-	
2 ⁺	$B_{s2}^*(5840)$	5528	1281	5674	(3/2) ⁺	$\Xi_b^*(5950)$	5610	1281	5754	1 ⁺	$B_{s1}?$	5936
1 ⁺	$B_{s1}?$	5528	1046	5626	(1/2) ⁺	$\Omega_b(6045)$	5791	1046	5885	0 ⁺	-	6110

Predicting Meson spectrum: Heavy-heavy

Meson				Baryon				Tetraquark							
$J^P(C)$	Name	$M_{ }$	M_{\perp}	M	$J^P(C)$	Name	$M_{ }$	M_{\perp}	M	$J^P(C)$	Name	$M_{ }$	M_{\perp}	M	
0 ⁻⁺	$\eta_c(2984)$	2927	0	2927	-	-	-	-	-	-	-	-	-	-	-
1 ⁺⁻	$h_c(3525)$	3440	1046	3596	(1/2) ⁺	$\Xi_{cc}^{SELEX}(3520)$ $\Xi_{cc}^{LHCb}(3620)$	3254	1046	3418	0 ⁺⁺	$\chi_{c0}(3415)$	3864	1046	4003	
1 ⁻⁻	$J/\psi(3096)$	2927	740	3019	-	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{c2}(3556)$	3440	1281	3671	(3/2) ⁺	$\Xi_{cc}^{LHCb}(3620)$	3254	1281	3497	1 ⁺⁺	$\chi_{c1}(3510)$	3580	1281	3802	
1 ⁻⁻	$\psi'(3686)$	3440	1281	3671	-	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{c2}(3927)$	3794	1654	4139	(3/2) ⁺	$\Xi_{cc}^*?$	3751	1654	4099	1 ⁺⁺	$X(3872)$	4062	1654	4386	
1 ⁺⁻	$Z_c(3900)$	4240	1654	4551						1 ⁺⁻	$Z_c(3900)$	4240	1654	4551	
0 ⁻⁺	$\eta_b(9400)$	9424	0	9424	-	-	-	-	-	-	-	-	-	-	-
1 ⁺⁻	$h_b(9900)$	9776	1046	9832	(1/2) ⁺	$\Xi_{bb}?$	9750	1046	9806	0 ⁺⁺	$\chi_{b0}(9860)$	10285	1046	10338	
1 ⁻⁻	$\Upsilon(9460)$	9424	740	9453	-	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{b2}(9910)$	9776	1281	9860	(3/2) ⁺	$\Xi_{bb}?$	9750	1281	9834	1 ⁺⁺	$\chi_{b1}(9893)$	10081	1281	10162	
1 ⁻⁻	$\Upsilon(2S)(10020)$	9776	1281	9860	-	-	-	-	-	-	-	-	-	-	-
2 ⁺⁺	$\chi_{b2}(10270)$	10024	1654	10160	(3/2) ⁺	$\Xi_{bb}?$	10100	1654	10234	1 ⁺⁺	$X_b?$	10425	1654	10555	

Conclusion

- 't Hooft Equation is complementary to the holographic Schrödinger Equation in predicting the full hadron spectrum.
- While the holographic Schrödinger Equation generates the hadronic mass in the chiral limit of QCD, the 't Hooft Equation generates the contribution to the hadronic mass due to non-zero quark masses and longitudinal confinement.
- Agreement with spectroscopic data spectra is generally good, and this is achieved with keeping the transverse confinement scale universal across the full spectrum.
- The disagreement for the tetraquark candidates, $f_0(500)$ and $f_0(980)(a_0(980))$, already present when neglecting longitudinal dynamics, persists.

Light-front coordinates

Lorentz transformation mixes the components of the space-time 4-vector.
 $x^\mu \equiv (x^0, x^1, x^2, x^3)$.

However, one can define combinations of the 4-vector components which are the eigenstates of the Lorentz Transformation and so get scaled under Lorentz boost:

$$x^+ = x^0 + x^3, \quad x^- = x^0 - x^3, \quad x^\perp = x^1, x^2$$

$$x^2 = x^\mu x_\mu = x^+ x^- - x^\perp{}^2$$

x^+ → Light-front time x^- → Light-front distance

For energy momentum 4-vector $p^\mu \equiv (p^0, p^1, p^2, p^3)$

Light-front energy → $p^- = p^0 - p^3$

Light-front momentum → $p^+ = p^0 + p^3$

Meson holographic LFWF

The meson holographic LFWFs for massless quarks can thus be written in closed form:

$$\Psi_{nL}(\zeta, x, \phi) = e^{iL\phi} \sqrt{x(1-x)} (2\pi)^{-1/2} \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^L \exp\left\{\left(\frac{-\kappa^2 \zeta^2}{2}\right)\right\} L_n^L(x^2 \zeta^2)$$

For non-zero quark mass, Brodsky and de Teramond prescription is to shift the longitudinal mode:

$$X(x) = \sqrt{x(1-x)} \longrightarrow X_{\text{BdT}}(x) = \sqrt{x(1-x)} \exp\left(-\frac{(1-x)m_q^2 + xm_{\bar{q}}^2}{2\kappa^2 x(1-x)}\right)$$

Example: pion LFWF ($m_q = m_{\bar{q}}$)

$$\Psi^\pi(x, \zeta^2) = \mathcal{N} \sqrt{x(1-x)} \exp\left\{\left[-\frac{\kappa^2 \zeta^2}{2}\right]\right\} \exp\left\{\left[-\frac{m_q^2}{2\kappa^2 x(1-x)}\right]\right\}$$

Meson spectrum

The shift in meson mass when moving away from chiral limit:

$$\begin{aligned}\Delta M_{\text{BdT}}^2 &= \int \frac{dx}{x(1-x)} \\ &\times X_{\text{BdT}}^2(x) \left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x} \right)\end{aligned}$$

$$M^2 = M_\perp^2 + \Delta M_{\text{BdT}}^2$$

For pion $M_\perp = 0$ and the above prescription leads to $M_\pi^2 = \Delta M_{\text{BdT}}^2 \propto m_q^2$ which is not in agreement with Gell-Mann-Oakes-Renner (GMOR) relation $M_\pi^2 \propto m_q$.

Another problem with this prescription is that it is the same for ground state and excited states mesons.

Numerical solutions

Expand the longitudinal mode onto a Jacobi polynomial basis:

$$\chi(x) = \sum_n c_n f_n(x) \quad (1)$$

with

$$f_n(x) = N_n x^{\beta_1} (1-x)^{\beta_2} P_n^{(2\beta_2, 2\beta_1)}(2x-1), \quad (2)$$

where $P_n^{(2\beta_2, 2\beta_1)}$ are the Jacobi polynomials and

$$N_n = \sqrt{(2n + \tilde{\beta}_1 + \tilde{\beta}_2)} \\ \times \sqrt{\frac{n! \Gamma(n + \tilde{\beta}_1 + \tilde{\beta}_2)}{\Gamma(n + \tilde{\beta}_1 + 1) \Gamma(n + \tilde{\beta}_2)}} \quad (3)$$

Solve the eigenvalue problem for eigenvalues M_L^2 and eigenvectors $\{c_n\}$.