WESTFÄLISCHE
WILHELMS-UNIVERSITÄT
MÜNSTER

Classical and quantum gravitational scattering with Generalized Wilson Lines

Domenico Bonocore

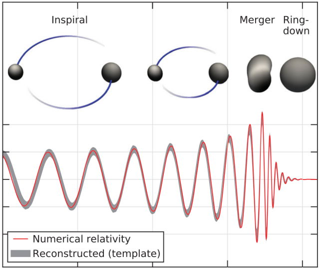
University of Münster

based on JHEP 03 (2022) **2112.02009** and ongoing work
in collaboration with **Anna Kulesza** and **Johannes Pirsch**

ICHEP Bologna
7 July 2022

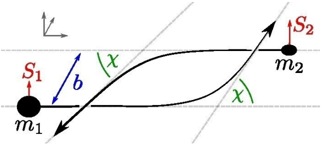
THE GRAVITATIONAL PROBLEM

- **BH and neutron star mergers**



Highly noisy signal $\to$ demand for precise theoretical predictions $\to$ numerical relativity is costly $\to$ need for analytic methods

- **Classical scattering in GR**



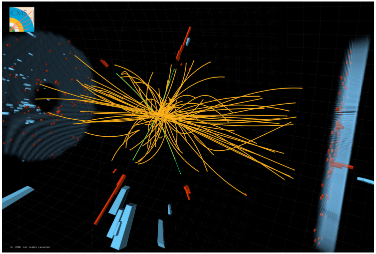
can be mapped to the bound state problem [Buonanno Damour 1999, Vines 2020,...]

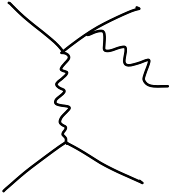
[Image credits: Abbott et al. PRL 116, 061102 (2016), Antonelli et al. PRL 125, 011103]

The QFT road to GR

General idea:

- exploit huge toolbox developed in **particle physics** (QFT, Feynman diagrams, amplitudes, loops, ...)





- isolate terms suriving the **classical limit $\hbar \to 0$** asap in the calculation
- perturbation theory in G (Post-Minkowskian expansion)

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \,, \qquad \kappa^2 = 32\pi G$$

[Image credits: CMS (cds.cern.ch/record/1406073)]

A recent explosion of interest

- EFT methods [Goldberger and Rothstein, 2006; Levi and Steinhoff, 2016; Kälin, Liu, and Porto, 2020; Huber, Brandhuber, De Angelis, Travaglini,2020; ...]
- eikonal methods [Amati, Ciafaloni, Veneziano 1990; Ciafaloni, Colferai, and Veneziano, 2019; Bjerrum-Bohr, Damgaard, Festuccia, Planté, and Vanhove, 2018; Di Vecchia, Heissenberg, Russo, Veneziano, 2021; ...]
- amplitudes/unitarity methods [Bern, Cheung, Roiban, Shen, Solon, and Zeng, 2019; Kosower, Maybee, and O'Connell, 2019; Arkani-Hamed, Huang, O'Connell 2019; Parra-Martinez, Ruf and Zeng, 2020; Cristofoli, Gonzo, Kosower, O'Connell 2021; ...]
- soft theorems [Guevara, Ochirov, Vines 2019; Di Vecchia, Heissenberg, Russo, Veneziano 2021; ...]
- double copy methods [Monteiro, O'Connell, White 2014, Chester 2018, Shen 2018, Plefka, Steinhoff, Wormsbecher 2018, Almeida, Foffa, Sturani 2020; ...]
- worldline methods [Jakobsen, Mogull, Plefka, and Steinhoff, 2021, Bastianelli, Combierati, de la Cruz 2021, Riva Vernizzi 2021; ...]

(Disclaimer: highly incomplete list!)

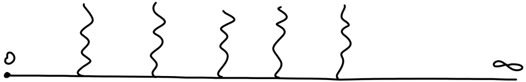
A SPECIFIC QCD-INSPIRED METHOD: THE GWL

- **Wilson Line** on a straight trajectory is well-known:

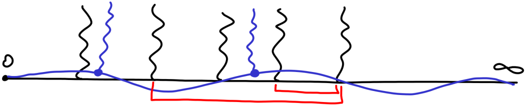
$$\text{QCD: } W_p(0,\infty) = \exp\left\{ ig \int_0^\infty dt\, p^\mu A_\mu(pt) \right\} = e^{ig \int \frac{d^dk}{(2\pi)^d} \frac{p_\mu}{p\cdot k} \tilde{A}^\mu(k)}$$

$$\text{grav: } W_p(0,\infty) = \exp\left\{ \frac{-i\kappa}{2} \int_0^\infty dt\, p_\mu p_\nu\, h^{\mu\nu}(pt) \right\} = e^{\frac{-\kappa}{2} \int \frac{d^dk}{(2\pi)^d} \frac{p_\mu p_\nu}{p\cdot k} \tilde{h}^{\mu\nu}(k)}$$

  WL generates an infinite number of soft emissions along direction p^μ (soft resummation)



- **Generalized Wilson Line** $\widetilde{W}_p(0,\infty)$ [Laenen, Stavenga, White 2008, White 2011, DB 2020]: generalize this procedure to subleading powers in k (i.e. include fluctuations and correlations) .



How?

The worldline formalism

Schwinger's method (1951): inverse scalar propagator interpreted as a Hamiltonian governing evolution in **proper time** T. In the free case:

$$\frac{i}{p^2 - m^2 + i\epsilon} = \int_0^\infty dT\, e^{\,i\,(\overbrace{p^2 - m^2}^{H(x,p)} + i\epsilon)T}$$

$\rightarrow$ 1-dim QFT in terms of fields $x^\mu(t)$ and $p^\mu(t)$

$\rightarrow$ Path integral repr. for the (LSZ truncated) **dressed propagator**:

$$(p_f^2 - m^2 + i\epsilon)\,\left\langle p_f\right| (2i(H - i\epsilon))^{-1} \left|x_i\right\rangle$$

$$= e^{ip_f x_i} \int\limits_{\tilde{x}(0)=0} \mathcal{D}\tilde{x}\mathcal{D}a\mathcal{D}b\mathcal{D}c \exp\left(i \int\limits_0^\infty dt\, e^{-\epsilon t}\, L[\tilde{x}, a, b, c] \right)$$

$$L[\tilde{x}, a, b, c] = -\frac{1}{2}\left((\dot{\tilde{x}}^\mu \dot{\tilde{x}}^\nu + a^\mu a^\nu + b^\mu c^\nu) g_{\mu\nu} + i(\dot{\tilde{x}} + p_f)^\mu g_{\mu\nu} V^\nu - \frac{1}{4} V^\mu g_{\mu\nu} V^\nu \right)$$

Ghost fields a, b, c remove UV div of the 1-dim QFT due to $g_{\mu\nu}(x)$

The gravitational (scalar) GWL

The path integral can be solved order by order in the soft limit. At **next-to-soft** (or next-to-eikonal) level, we get:

$$
\begin{aligned}
&\widetilde{W}_p(0,\infty) \\
&= \exp\left\{ \frac{i\kappa}{2} \int_0^\infty dt \left[-p_\mu p_\nu + i p_\nu \partial_\mu - \frac{i}{2}\eta_{\mu\nu} p^\alpha \partial_\alpha + \frac{i}{2} t p_\mu p_\nu \partial^2 \right] h^{\mu\nu}(pt) \right. \\
&\quad + \frac{i\kappa^2}{2} \int_0^\infty dt \int_0^\infty ds \left[\frac{p^\mu p^\nu p^\rho p^\sigma}{4} \min(t,s)\, \partial_\alpha h_{\mu\nu}(pt) \partial^\alpha h_{\rho\sigma}(ps) \right. \\
&\quad \left.\left. + p^\mu p^\nu p^\rho\, \theta(t-s)\, h_{\rho\sigma}(ps) \partial_\sigma h_{\mu\nu}(pt) + p^\nu p^\sigma\, \delta(t-s)\, h^\mu{}_\sigma(ps) h_{\mu\nu}(pt) \right] \right\}
\end{aligned}
$$

- consistent derivation in the worldline formalism of the GWL [DB 2020, DB, Kulesza, Pirsch 2021]
- nice relation between soft expansion and PM expansion
- But: is the GWL useful to isolate classical contributions?

Classical VS quantum

By $\hbar$ power counting

$$L[\tilde{x},a,b,c] = -\frac{1}{2}\left((\dot{\tilde{x}}^{\mu}\dot{\tilde{x}}^{\nu} + a^{\mu}a^{\nu} + b^{\mu}c^{\nu})g_{\mu\nu} + i(\dot{\tilde{x}} + p_f)^{\mu} g_{\mu\nu} V^{\nu} - \frac{1}{4} V^{\mu} g_{\mu\nu} V^{\nu} \right)$$

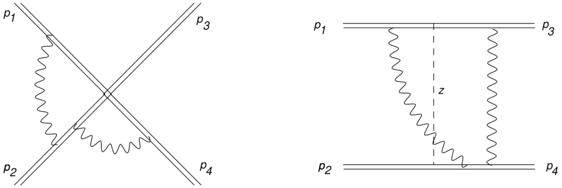
and neglecting quantum loop, we get

$$\widetilde{W}_p(0,\infty)$$
$$= \exp\left\{ \frac{i\kappa}{2}\int_0^{\infty} dt\, \left[-p_{\mu}p_{\nu} + ip_{\nu}\partial_{\mu} - \frac{i}{2}\eta_{\mu\nu}p^{\alpha}\partial_{\alpha} + \frac{i}{2}tp_{\mu}p_{\nu}\partial^2 \right] h^{\mu\nu}(pt) \right.$$
$$+ \frac{i\kappa^2}{2}\int_0^{\infty} dt \int_0^{\infty} ds \left[\frac{p^{\mu}p^{\nu}p^{\rho}p^{\sigma}}{4} \min(t,s)\, \partial_{\alpha}h_{\mu\nu}(pt)\partial^{\alpha}h_{\rho\sigma}(ps) \right.$$
$$\left.\left. + p^{\mu}p^{\nu}p^{\rho}\, \theta(t-s)\, h_{\rho\sigma}(ps)\partial_{\sigma}h_{\mu\nu}(pt) \, + \, p^{\nu}p^{\sigma}\, \delta(t-s)\, h^{\mu}{}_{\sigma}(ps)h_{\mu\nu}(pt) \right] \right\}$$

$\rightarrow$ define a *classical* GWL $\widetilde{W}_p^{\mathrm{cl}}(0,\infty)$

Amplitude level: two exponentiations

Worldline exponentiation (via (G)WLs) returns two other exponentiations at the amplitude level (by taking VEV w.r.t. EH action): [Korchemskaya, Korchemsky 1996; Melville, Luna, Naculich, White 2016; Di Vecchia, Naculich, Russo, Veneziano, White 2020]



- **(next-to-) soft exponentiation**

$$\mathcal{S} = \langle 0|W_{p_1}(-\infty,0)W_{p_2}(-\infty,0)W_{p_3}(0,\infty)W_{p_4}(0,\infty)|0\rangle = \exp(i\mathcal{W})$$

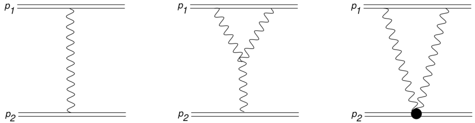
- **(next-to-) eikonal exponentiation** in the high energy limit $s \gg t$

$$\mathcal{A}_{\mathrm{E}} = \langle 0|W_{p_1}(0,-\infty,0)W_{p_2}(z,-\infty,0)W_{p_3}(0,0,\infty)W_{p_4}(z,0,\infty)|0\rangle$$

$$= \exp\left[K(z)\left(i\pi s + t\log\left(\frac{s}{-t}\right)\right)\right] = e^{i\chi_{\mathrm{E}}}\left(\frac{s}{-t}\right)^{K(z)t}$$

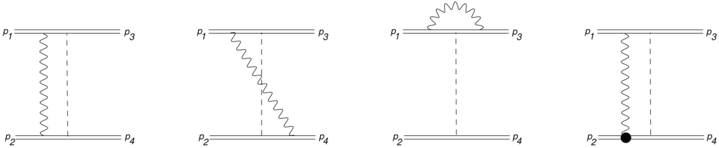
Amplitude level: high energy limit

- **classical** terms: deflection angle θ at $\mathcal{O}(\kappa^2)$ from the next-to-eikonal phase $\chi_{\rm NE}$



$$e^{i\chi_{\rm NE}} = \langle 0 | \widetilde{W}^{\rm cl}_{p_1}(0,-\infty,\infty)\widetilde{W}^{\rm cl}_{p_2}(z,-\infty,\infty)|0\rangle \,, \qquad \theta \sim \frac{\partial \chi_{\rm NE}}{\partial z}$$

- inclusion of **quantum** terms: graviton Reggeization from subleading terms (in t/s)



GENERALIZATIONS

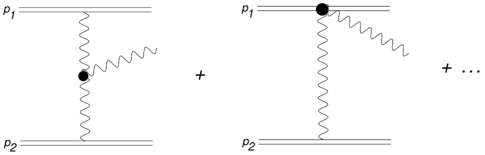
- **spin**: Supersymmetric worldline model (e.g. $\mathcal{N} = 1$ for spin $1/2$) [Barducci,Casalbuoni,Lusanna(1976), Brink,DiVecchia,Howe(1977), Gershun,Tkach(1979), Howe,Penati,Pernici,Townsend(1988-89), ...]

$$S = \int dt \left(p_\mu \dot{x}^\mu + \frac{i}{2}\psi^a \psi^b \eta_{ab} - \frac{i}{2}\psi^5 \dot{\psi}_5 - \frac{1}{2} e \overbrace{p_\mu p^\mu}^{H} - i\chi \overbrace{\psi_\mu p^\mu}^{Q} \right) ,$$

  Technically more involved, but again it is possible to derive GWL, and isolate classical contribution

- **waveforms**: insert graviton field in the VEV

$$\langle 0 | \, h_{\mu\nu} \, \widetilde{W}_{p_1} \widetilde{W}_{p_2} | 0 \rangle =$$



Conclusions

- High demand for (analytic) theory predictions in gravitational wave astronomy.
- Much recent progress in understanding GR perturbation theory via QFT (with $\hbar \to 0$).
- Among the QCD-inspired methods, GWL is a powerful tool, introduced few years ago. However, it was not exploited for the classical limit. Also, technical issue were present (quantization, ordering prescription, UV regularization).
- We have presented a derivation of the GWL within the worldline formalism, thus clarifying the technical issues, its virtue for the classical limit and the relation with similar methods in literature.
- Many possible extensions (spin, higher orders, waveforms, ...)