# Automation of Antenna Subtraction in Colour Space

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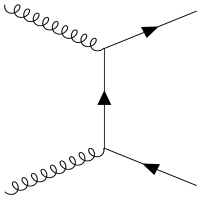
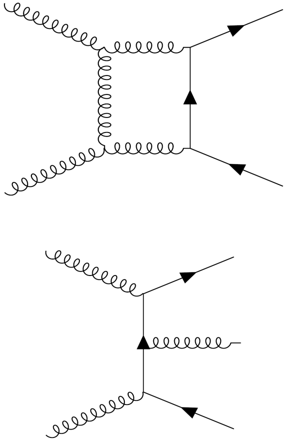
# Fixed Order Calculations

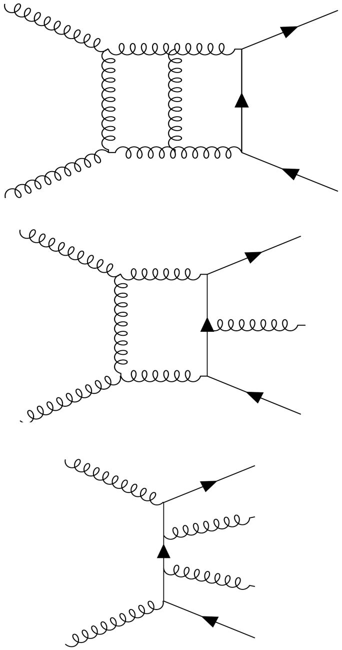
LO



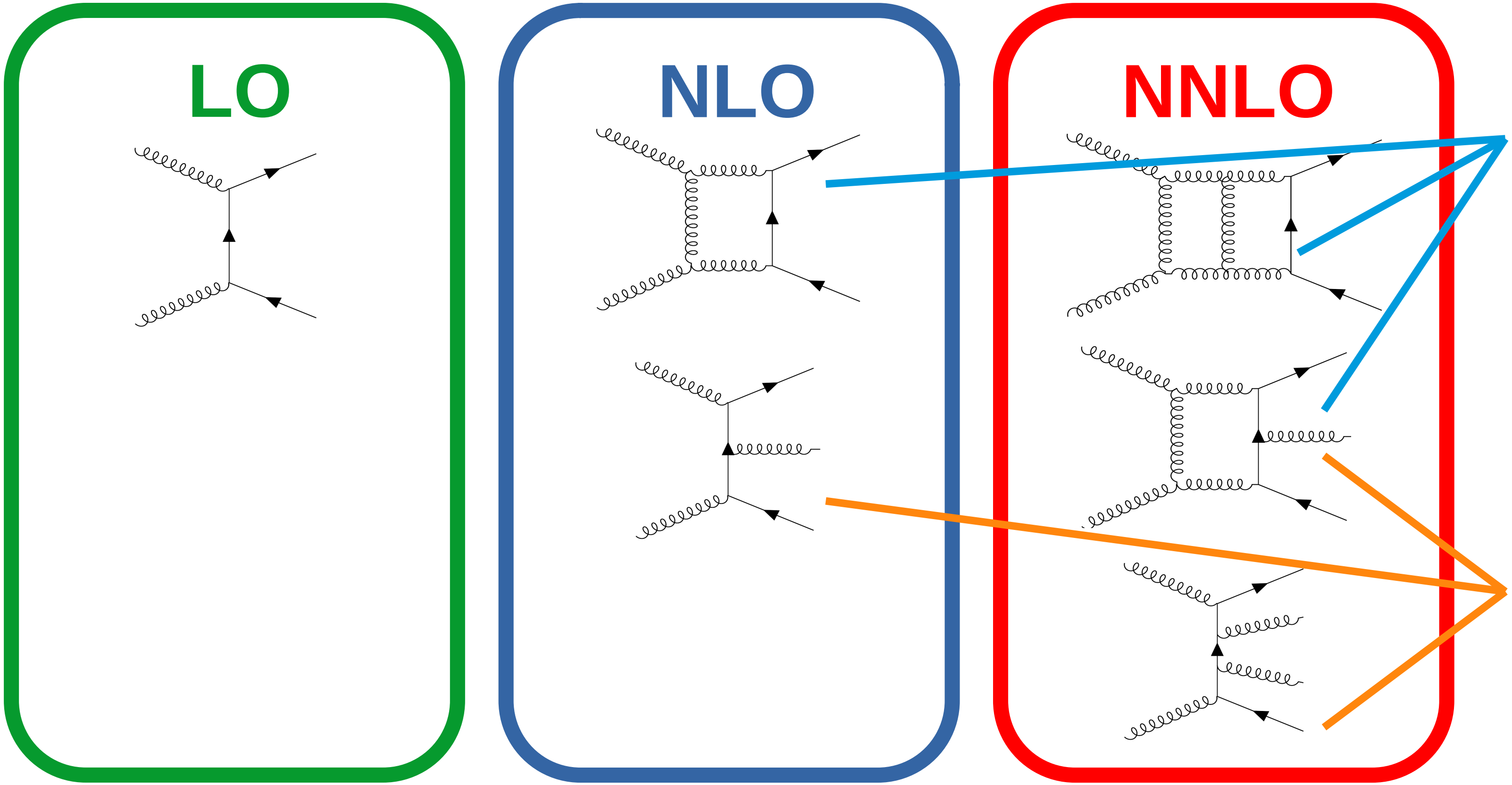
NLO



NNLO



# Fixed Order Calculations




**Virtual** corrections: explicit IR poles from the loops.

**The sum is finite (KLN theorem, IR safe obs)**

**Real** corrections: develop IR poles after phase space integration.

# Fixed Order Calculations

LO

NLO

NNLO

Virtual corrections:
...les
...s.

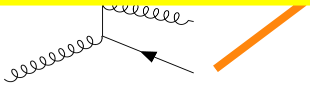
Strategy:

- Divergences regularization (**dim. reg.**);
- Divergences removal from virtual and real corrections **separately**;
- Evaluation of **finite quantities** via Monte Carlo integration;

**Subtraction scheme**

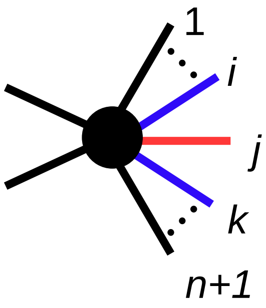
**is finite**
**...eorem)!**

...ns:
poles
after phase space integration.



# Antenna Subtraction (NLO)

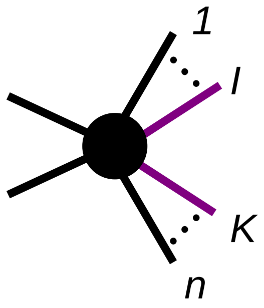
**Reals:**




parton j soft or collinear to i or k

factorization properties of QCD

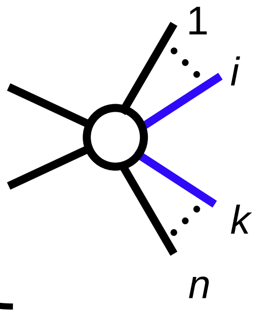
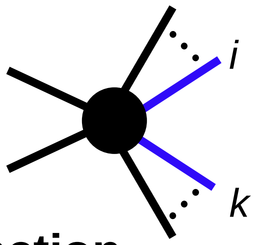
$X_3^0(i,j,k)\,\cdot$




**Antenna function (universal)**

Analytical integration over PS of the unresolved radiation

**Virtuals:**

Poles [  ] = Poles [ $\mathcal{X}_3^0(s_{ik})\,\cdot$  ]




**Integrated antenna function**

# Antenna Subtraction

Many succesful applications at NNLO in the past decade: Zj, Hj, Wj, VBF, VH, ɣ, ɣj, ɣɣ, jj, VHj.

Limitations:

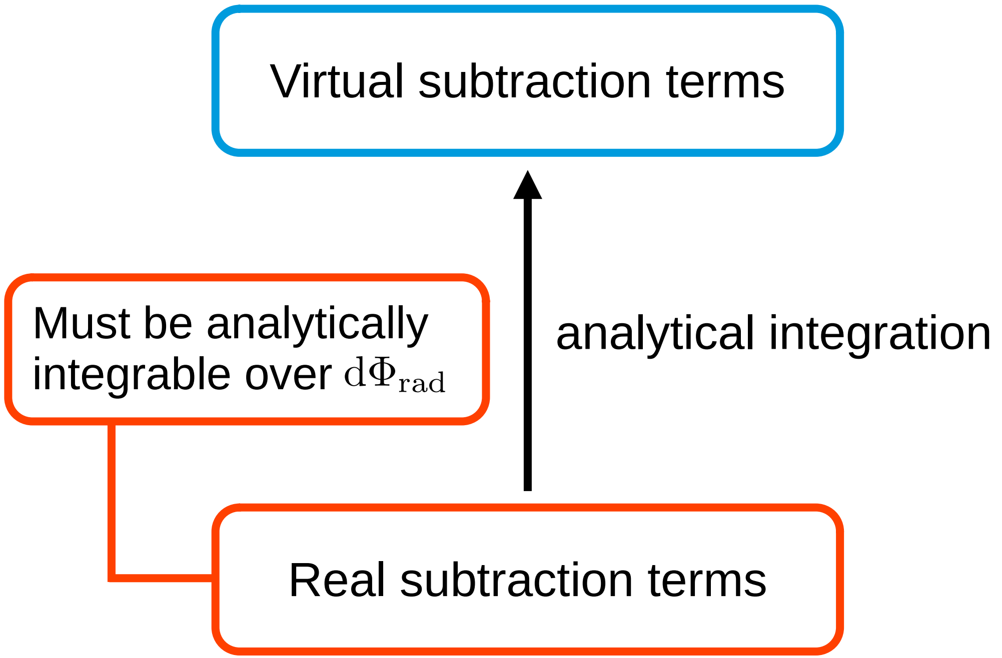
- Poor scaling with the number of external partons $n_p$.
- Highly non-trivial construction of subtraction terms for $n_p \geq 4$.

A **new formulation** is required:

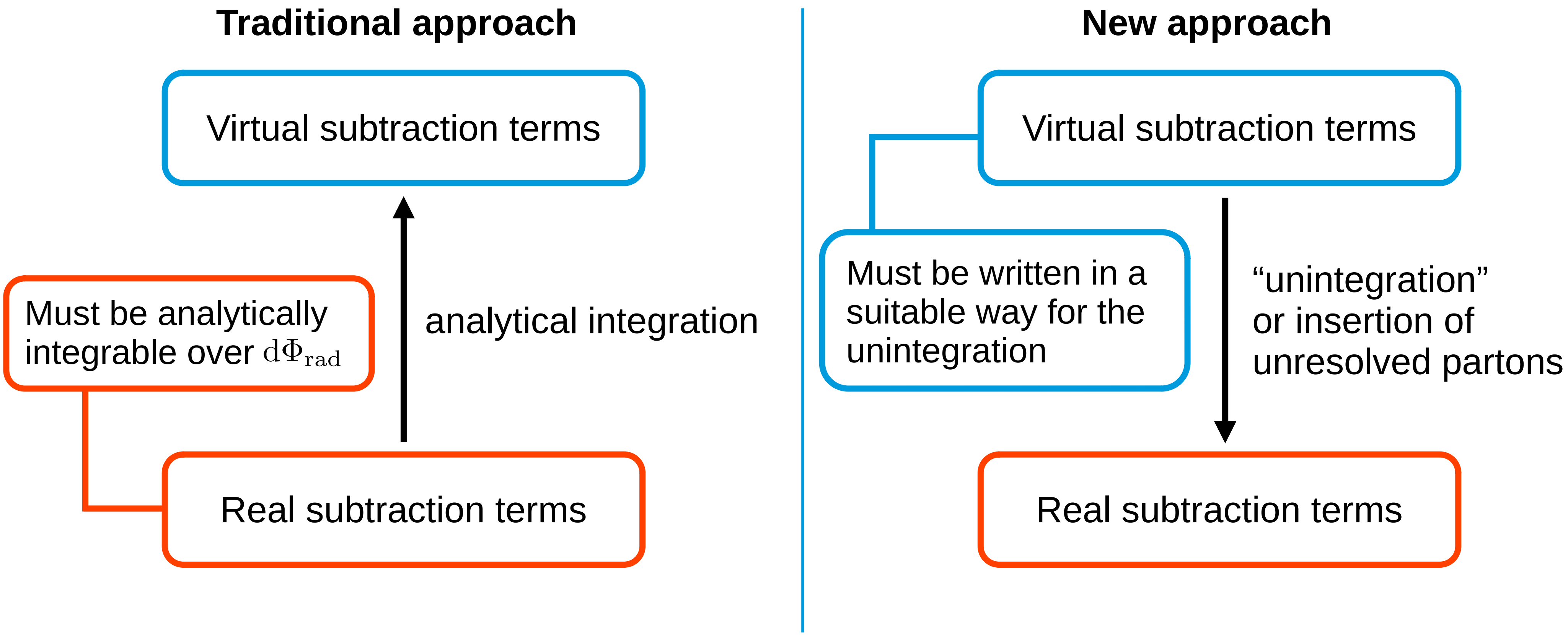
- automation and efficiency;
- improved understanding/organization of the subtraction infrastructure;
- go beyond $n_p = 4$ .

# Colourful Antenna Subtraction

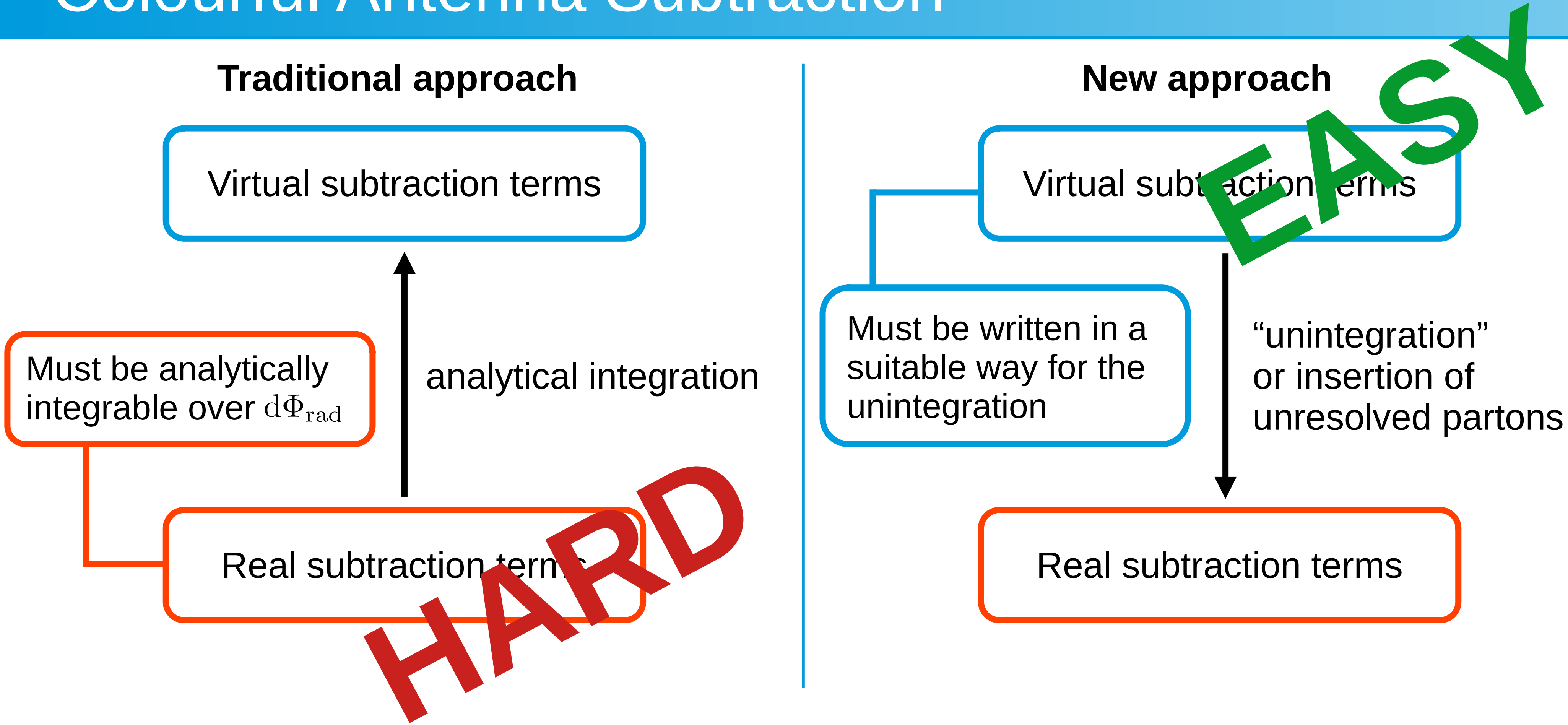
**Traditional approach**



# Colourful Antenna Subtraction



# Colourful Antenna Subtraction



# Colour space

The IR singularity structure of loop amplitudes in QCD is best described in **colour space.**

An n-parton ℓ-loop QCD amplitude can be written as:

$$|\mathcal{A}_n^\ell(\{p\}_n)\rangle = \sum_{i \in I^\ell} \boldsymbol{\mathcal{C}}_{n,i}^\ell \cdot A_{n,i}^\ell(\{p\}_n)$$

colour basis,
vectors in colour space

colour-ordered
partial amplitudes

# Colour space

IR singularity structure of (renormalised) one- and two-loop amplitudes:

$$|\mathcal{A}_n^1\rangle = \boldsymbol{I}^{(1)}(\epsilon,\mu_r^2)|\mathcal{A}_n^0\rangle + \text{ finite terms}$$

$$|\mathcal{A}_n^2\rangle = \boldsymbol{I}^{(1)}(\epsilon,\mu_r^2)|\mathcal{A}_n^1\rangle + \boldsymbol{I}^{(2)}(\epsilon,\mu_r^2)|\mathcal{A}_n^0\rangle + \text{ finite terms}$$

[Catani '98]
[Bern, De Freitas, Dixon '03]
[Becher, Neubert '09]

$\boldsymbol{I}^{(1)}$ and $\boldsymbol{I}^{(2)}$ are **infrared insertion operators** in colour space:

$$\boldsymbol{I}^{(1)}(\epsilon,\mu_r^2) = \sum_{(i,j)} (\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{I}_{ij}^{(1)}(\epsilon,\mu_r^2)$$

$$\boldsymbol{I}^{(2)}(\epsilon,\mu_r^2) = -\frac{1}{2}\sum_{(i,j)}\sum_{(k,l)} (\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,(\boldsymbol{T}_k\cdot\boldsymbol{T}_l)\,\mathcal{I}_{ij}^{(1)}(\epsilon,\mu_r^2)\mathcal{I}_{kl}^{(1)}(\epsilon,\mu_r^2)$$
$$-\frac{b_0 N_c}{\epsilon}\sum_{(i,j)} (\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{I}_{ij}^{(1)}(\epsilon,\mu_r^2) + \sum_{(i,j)} (\boldsymbol{T}_i\cdot\boldsymbol{T}_j)\,\mathcal{I}_{ij}^{(2)}(\epsilon,\mu_r^2)$$

- Colour charge dipole structure;
- Retain full colour correlations;
- Universal;

# Colourful Antenna Subtraction

We exploit this to write down the IR singularities of loop matrix elements as:

$$Poles\left\{|\mathcal{M}_n^1|^2\right\} = Poles\left\{2\mathrm{Re}\langle\mathcal{A}_n^1|\mathcal{A}_n^0\rangle\right\} = 2Poles\left\{\langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle\right\}$$

$$Poles\left\{|\mathcal{M}_n^2|^2\right\} = Poles\left\{2\mathrm{Re}\langle\mathcal{A}_n^2|\mathcal{A}_n^0\rangle + \langle\mathcal{A}_n^1|\mathcal{A}_n^1\rangle\right\} = 2Poles\Big\{2\mathrm{Re}\langle\mathcal{A}_n^1|\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle - \langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(1)}\otimes\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle - \frac{\beta_0 N_c}{\epsilon}\langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(1)}|\mathcal{A}_n^0\rangle + \langle\mathcal{A}_n^0|\boldsymbol{\mathcal{J}}^{(2)}|\mathcal{A}_n^0\rangle\Big\}$$

$\boldsymbol{\mathcal{J}}^{(1)}$ and $\boldsymbol{\mathcal{J}}^{(2)}$ are analogous to $\boldsymbol{I}^{(1)}$ and $\boldsymbol{I}^{(2)}$, but are constructed using **integrated antenna functions**:

- exact extraction of virtual IR poles;
- explicit connection with real IR divergences via the **correspondence of integrated and unintegrated antenna functions**;

# Colourful Antenna Subtraction

The structure of the IR divergences for real emissions is obtained from the previous expressions replacing integrated antenna functions with their unintegrated counterparts:

$$\mathcal{X}(s_{ij}) \quad \to \quad X(i,k,j)$$

# Colourful Antenna Subtraction

The structure of the IR divergences for real emissions is obtained from the previous expressions replacing integrated antenna functions with their unintegrated counterparts:

Insertion of the unresolved parton *k*

$$\mathcal{X}(s_{ij}) \quad \rightarrow \quad X(i,k,j)$$

Integrated antenna function: reproduces poles of **virtual** corrections

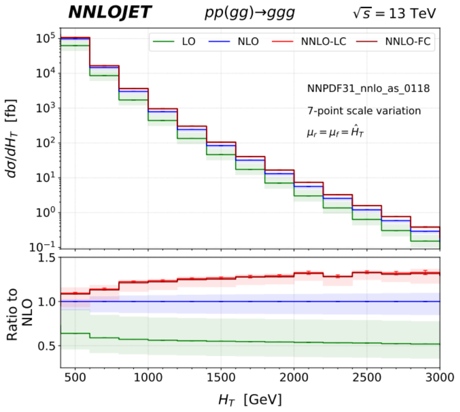
Unintegrated antenna function: reproduces divergent behaviour of **real** corrections in soft and collinear limits

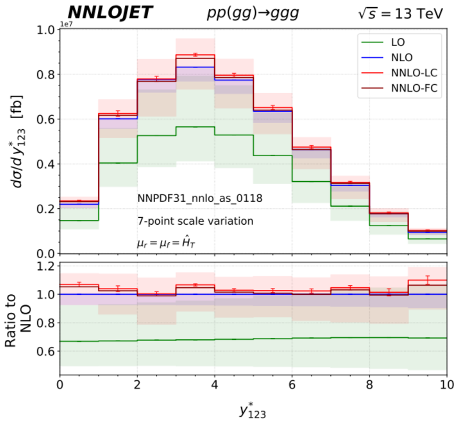
- Cancellation of IR divergences;
- Systematic generation of the subtraction infrastructure;
- Knowledge of all the antenna functions is crucial;

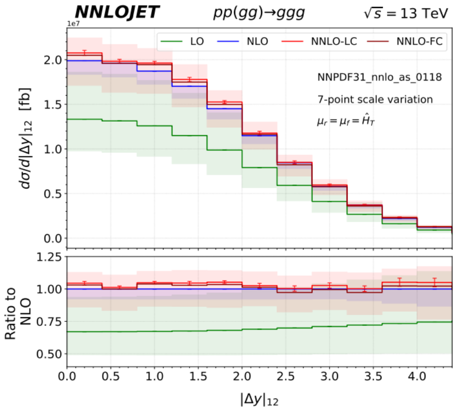
# Status

- Colorful antenna subtraction: a formalism to achieve a **systematic and automatable** extraction of IR singularities at NNLO for any number of external partons;

- Successful calculation of gg→ggg at NNLO in the **gluons-only** assumption (see 2203.13531);




- Work in progress towards full **3-jet production at NNLO**: complete establishment of this approach;

*Thanks for your attention!*