Particle Physics Theory Group
Department of Physics
National Technical University of Athens

Phenomenology of $N = 1$ split-like model resulting from the dimensional reduction of an $N = 1$, $10D$ E_8 gauge theory over a modified flag manifold

Gregory Patellis

G. Manolakos, GP, G. Zoupanos

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Motivation

Unified description of Nature

- Extra Gauge Symmetry (i.e. GUTs)
- Supersymmetry
- Extra Dimensions
  - Unify gauge and Higgs sectors
  - Also unify fermion interactions with the above sectors
  - SUSY can unify all the above in one vector supermultiplet
  - Less free parameters

Coset Space Dimensional Reduction

1. Compactification

   B - compact space

   $\dim B = D - 4 = d$

   D dims $\longrightarrow$ 4 dims

$$
\begin{array}{ccccc}
M^D & \rightarrow & M^4 & \times & B \\
| & & | & & | \\
x^M & & x^\mu & & y^a
\end{array}
$$

2. Dimensional Reduction

   $\mathcal{L}$ independent of the extra coordinates y^a:

   - "Naive" way: Discard the field dependence on y^a coordinates
   - Elegant way: Allow field dependence on y^a
     $\rightarrow$ compensated by a symmetry of the Lagrangian $\rightarrow$ Gauge Symmetry

3. Coset Space Dimensional Reduction

   Witten (1977); Forgacs, Manton (1980); Chapline, Slansky (1982); Kapetanakis, Zoupanos - Phys.Rept. (1992) Kubyshin, Mourao, Rudolph, Volobujev - Book (1989)

   - $B = S/R$
   - Allow a non-trivial dependence on y^a
   - impose the condition that a symmetry transformation by an element of the isometry group S of B is compensated by a gauge transformation

   $\rightarrow$ Gauge invariant $\mathcal{L} \rightarrow \mathcal{L}$ independent of y^a !

Reduction of a D-dimensional Yang-Mills Lagrangian

Consider a Yang-Mills-Dirac theory in D dims based on group G defined on $M^D \to M^4 \times S/R$, $D = 4 + d$

$$S = \int d^4x d^d y \sqrt{-g} \left[-\frac{1}{4} Tr(F_{MN} F_{K\Lambda}) g^{MK} g^{N\Lambda} + \frac{i}{2} \overline{\psi} \Gamma^M D_M \psi \right]$$

Demand: any transformation by an element of S acting on S/R is compensated by gauge transformations.

$\to$ Constraints on the fields of the theory A_α and ψ

Solution of constraints:

- The 4D gauge group: $H = C_G(R_G)$ (i.e. $G \supset R_G \times H$)
- 4D (surviving) fields $D = 4n + 2$ Weyl + Majorana fermions in vector-like rep $\to$ 4D chiral theory.
- Scalar Potential

The 4D Theory

Integrate out the extra coordinates (+ take into account constraints):

$$S = C \int d^4x \, \mathrm{tr} \left[-\frac{1}{8} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (D_\mu \phi_a)(D^\mu \phi^a) \right] + V(\phi) + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^a D_a \psi$$

where

$$V(\phi) = -\frac{1}{8} g^{ac} g^{bd} \mathrm{Tr} \left\{ (f_{ab}^C \phi_C - ig[\phi_a, \phi_b])(f_{cd}^D \phi_D - ig[\phi_c, \phi_d]) \right\}$$

$V(\phi)$ still only formal since ϕ_a must satisfy one more constraint.

- If $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$:

$$\begin{array}{ccccl} G \supset & S & \times & K & \leftarrow \text{ gauge group after SSB} \\ & \cup & & \cap & \\ G \supset & R & \times & H & \leftarrow \text{ gauge group in 4 dims} \end{array}$$

Harnad, Shnider, Tafel (1980)

Reduction of 10D, $N = 1$ E_8 over $S/R = SU(3)/U(1) \times U(1)$

Manousselis, Zoupanos (2001-2004)

The non-symmetric (nearly-Kähler) coset space $SU(3)/U(1) \times U(1)$:

- admits torsion and may have different radii
- naturally produces soft supersymmetry breaking terms
- preserves the supersymmetric multiplets

We use the decomposition

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

and choose $R = U(1)_A \times U(1)_B$

$$\rightarrow H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$$

- $N = 1,\ E_6 \times U(1)_A \times U(1)_B$ gauge group
- Three chiral supermultiplets $A^i : 27_{(3,1/2)}$, $B^i : 27_{(-3,1/2)}$, $C^i : 27_{(0,-1)}$
- Three chiral supermultiplets $A : 1_{(3,1/2)}$, $B : 1_{(-3,1/2)}$, $C : 1_{(0,-1)}$
- Gaugino mass $M = (1 + 3\tau)\frac{R_1^2+R_2^2+R_3^2}{8\sqrt{R_1^2R_2^2R_3^2}}$

The Wilson Flux Breaking

Hosotani (1983); Witten (1985); Zoupanos (1988); Kozimirov, Kuzmin, Tkachev (1989); Kapetanakis, Zoupanos (1989)

$$M^4 \times B_0 \rightarrow M^4 \times B, \qquad B = B_0/F^{S/R}$$

– $F^{S/R}$ is a freely acting discrete symmetry of B_0.

B becomes multiply connected $\rightarrow$ breaking of H to $H' = C_H(T^H)$

T^H is the image of the homomorphism of $F^{S/R}$ into H

In our case

- $F^{S/R} = \mathbb{Z}_3 \rightarrow B = SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$
- $H = E_6 \times U(1)_A \times U(1)_B$
- $H' = SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$, still $N = 1$

Matter fields invariant under $F^{S/R} \oplus T^H$ survive

$\rightarrow \gamma_3 = \text{diag}(\mathbf{1}, \omega\mathbf{1}, \omega^2\mathbf{1}), \quad \omega = e^{2i\pi/3} \in \mathbb{Z}_3$

Irges, Zoupanos (2011)

The surviving matter fields are given by:

- $A^i = \gamma_3 A^i, \quad B^i = \omega\gamma_3 B^i, \quad C^i = \omega^2\gamma_3 C^i$
- $A = A, \quad B = \omega B, \quad C = \omega^2 C$

$$E_6 \supset SU(3)_c \times SU(3)_L \times SU(3)_R \qquad 27 = (1,3,\bar{3}) \oplus (\bar{3},1,3) \oplus (3,\bar{3},1)$$

Surviving matter content of the projected theory:

$$A_1 \equiv L \sim (1,3,\bar{3})_{(3,\frac{1}{2})}, \quad B_2 \equiv q^c \sim (\bar{3},1,3)_{(-3,\frac{1}{2})},$$
$$C_3 \equiv Q \sim (3,\bar{3},1)_{(0,-1)}, \quad A \equiv \theta \sim (1,1,1)_{(3,\frac{1}{2})}$$

Non-trivial monopole charges in $R \rightarrow$ three generations: $L^{(l)}, q^{c(l)}, \; Q^{(l)}, \; \theta^{(l)}$

Dolan (2003)

$$L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}_{(3,1/2)}, \quad q^c = \begin{pmatrix} d_R^{1c} & u_R^{1c} & D_R^{1c} \\ d_R^{2c} & u_R^{2c} & D_R^{2c} \\ d_R^{3c} & u_R^{3c} & D_R^{3c} \end{pmatrix}_{(-3,1/2)}, \quad Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}_{(0,-1)}$$

$$V = \frac{g^2}{5}\left(\frac{1}{R_1^4} + \frac{1}{R_2^4} + \frac{1}{R_3^4}\right) + V_F + V_D + V_{soft}$$

For one generation:

$$\frac{2}{g^2} V_D = \frac{1}{2} D^\alpha D^\alpha + \frac{1}{2} D_1 D_1 + \frac{1}{2} D_2 D_2$$

$$D^A = \frac{1}{\sqrt{3}}(\langle L|G^A|L\rangle + \langle q^c|G^A|q^c\rangle + \langle Q|G^A|Q\rangle)$$

$$D_1 = 3\sqrt{\frac{10}{3}}(\langle L|L\rangle - \langle q^c|q^c\rangle + |\theta^{(i)}|^2)$$

$$D_2 = \sqrt{\frac{10}{3}}(\langle L|L\rangle + \langle q^c|q^c\rangle - 2\langle Q|Q\rangle + 2|\theta|^2)$$

$$\frac{2}{g^2} V_F = 360\mathrm{tr}(\hat{L}^\dagger \hat{L} + \hat{q}^{c\dagger} \hat{q}^c + \hat{Q}^\dagger \hat{Q}) \qquad F_s = \frac{\partial \mathcal{W}}{\partial s} \qquad \mathcal{W} = \sqrt{40} d_{ijk} A^i B^j C^k + \sqrt{40} ABC$$

$$\begin{aligned}
\frac{2}{g^2} V_{soft} &= \left(\frac{4R_1^2}{R_2^2 R_3^2} - \frac{8}{R_1^2}\right)\left(\langle L|L\rangle + |\theta|^2\right) + \left(\frac{4R_2^2}{R_1^2 R_3^2} - \frac{8}{R_2^2}\right)\langle q^c|q^c\rangle \\
&+ \left(\frac{4R_3^2}{R_1^2 R_2^2} - \frac{8}{R_3^2}\right) Q|Q \\
&+ 80\sqrt{2}\left(\frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_1 R_2}\right)(d_{abc} L^a q^{cb} Q^c + h.c) \\
&= m_1^2\left(\langle L|L\rangle + |\theta|^2\right) + m_2^2 \langle q^c|q^c\rangle + m_3^2 \langle Q|Q\rangle + (\alpha_{abc} L^a q^{cb} Q^c + h.c)
\end{aligned}$$

Further Gauge Breaking of $SU(3)^3$

Babu, He, Pakvasa (1986); Ma, Mondragon, Zoupanos (2004); Leontaris, Rizos (2006); Sayre, Wiesenfeldt, Willenbrock (2006)

Two generations of L acquire vevs that break the GUT:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}, \quad \langle L_s^{(2)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}$$

each one alone is not enough to produce the (MS)SM gauge group:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$$
$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)'_R \times U(1)'$$

Their combination gives the desired breaking:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

electroweak breaking then proceeds by:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} \upsilon_d & 0 & 0 \\ 0 & \upsilon_u & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Choice of Radii

Manolakos, Patellis, Zoupanos (2020)

— Soft trilinear terms $\sim \frac{1}{R_i}$

— Soft squared scalar masses $\sim \frac{1}{R_i^2}$

Two main possible directions:

- Large $R_i \rightarrow$ calculation of the Kaluza-Klein contributions of the 4D theory

  $\times$ Eigenvalues of the Dirac and Laplace operators unknown.

- Small $R_i \rightarrow$ high scale SUSY breaking
- Small $R_i \sim \frac{1}{M_{GUT}}$ with R_3 slightly different in a specific configuration

$$\rightarrow \quad m_1^2 \sim -\mathcal{O}(TeV^2), \quad m_{2,3}^2 \sim -\mathcal{O}(M_{GUT}^2), \quad a_{abc} \gtrsim M_{GUT}$$

— supermassive squarks

— TeV-scale sleptons

Reminder: in this scenario $M_C = M_{GUT}$

Lepton Yukawas and μ terms

At the GUT level, the singlet scalars also acquire vevs

$\langle\theta^{(1,2)}\rangle \sim \mathcal{O}(M_{GUT})\,, \langle\theta^{(3)}\rangle \sim \mathcal{O}(TeV)\,,$

thus breaking the extra abelian $U(1)$s, which however remain in the theory as global symmetries.

- The two global $U(1)$s forbid Yukawa terms for leptons

  $\rightarrow$ introduce higher-dimensional operators: $L\bar{e}H_d\left(\frac{\overline{K}}{M}\right)^3$

- μ terms for each generation of Higgs doublets are absent

  $\rightarrow$ solution through higher-dimensional operators: $H_u^{(i)}H_d^{(i)}\overline{\theta}^{(i)}\,\frac{\overline{K}}{M}$

— $\overline{K}$ is the vev of the conjugate scalar component of either $S^{(i)}$, $\nu_R^{(i)}$ or $\theta^{(i)}$, or any combination of them

Approximate Scale of Parameters

Parameter	Scale
soft trilinear couplings	$\mathcal{O}(GUT)$
squark masses	$\mathcal{O}(GUT)$
slepton masses	$\mathcal{O}(TeV)$
$\mu^{(3)}$	$\mathcal{O}(TeV)$
$\mu^{(1,2)}$	$\mathcal{O}(GUT)$
unified gaugino mass M_U	$\mathcal{O}(TeV)$

Gauge Unification

Since many SUSY masses are superheavy, we consider these particles to decouple at an intermediate scale M_{int}.

The 1-loop gauge β-functions are:

$$2\pi\beta_i = b_i\alpha_i^2$$

— b_i depends on the particle content

Scale	b_1	b_2	b_3
M_{EW}-M_{TeV}	$\frac{21}{5}$	-3	-7
M_{TeV}-M_{int}	$\frac{11}{2}$	$-\frac{1}{2}$	-5
M_{int}-M_{GUT}	$\frac{39}{5}$	3	-3

- $\alpha_{1,2}$ are used as input to determine M_{GUT}

$\rightarrow$ α_3 is found within 2σ of the experimental value

$$a_s(M_Z) = 0.1218$$

$$a_s^{EXP}(M_Z) = 0.1187 \pm 0.0016$$

Scale	GeV
M_{GUT}	$\sim 1.7 \times 10^{15}$
M_{int}	$\sim 9 \times 10^{13}$
M_{TeV}	~ 1500

✓ The two global $U(1)$s forbid any proton decay inducing channel.

Conclusions / Work in Progress

- Special choice of coset radii for split-like SUSY SM
- $M_{GUT} \sim 10^{15}$ GeV — no proton decay
- Promising preliminary 1-loop analysis
- LSP $\gtrsim$ 1500 GeV

- Comprehensive 2-loop analysis
- Full (light) SUSY spectrum
- *o* Application of B-physics constraints
- *o* Calculation of CDM relic density
- *o* Investigation of discovery potential at existing and future colliders

- Examination of high energy potential $\rightarrow$ test agreement with observed value of cosmological constant

Thank you for your attention!