Universität
Zürich^UZH

Matching Effective Theories Efficiently

MATCHETE

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In collaboration with

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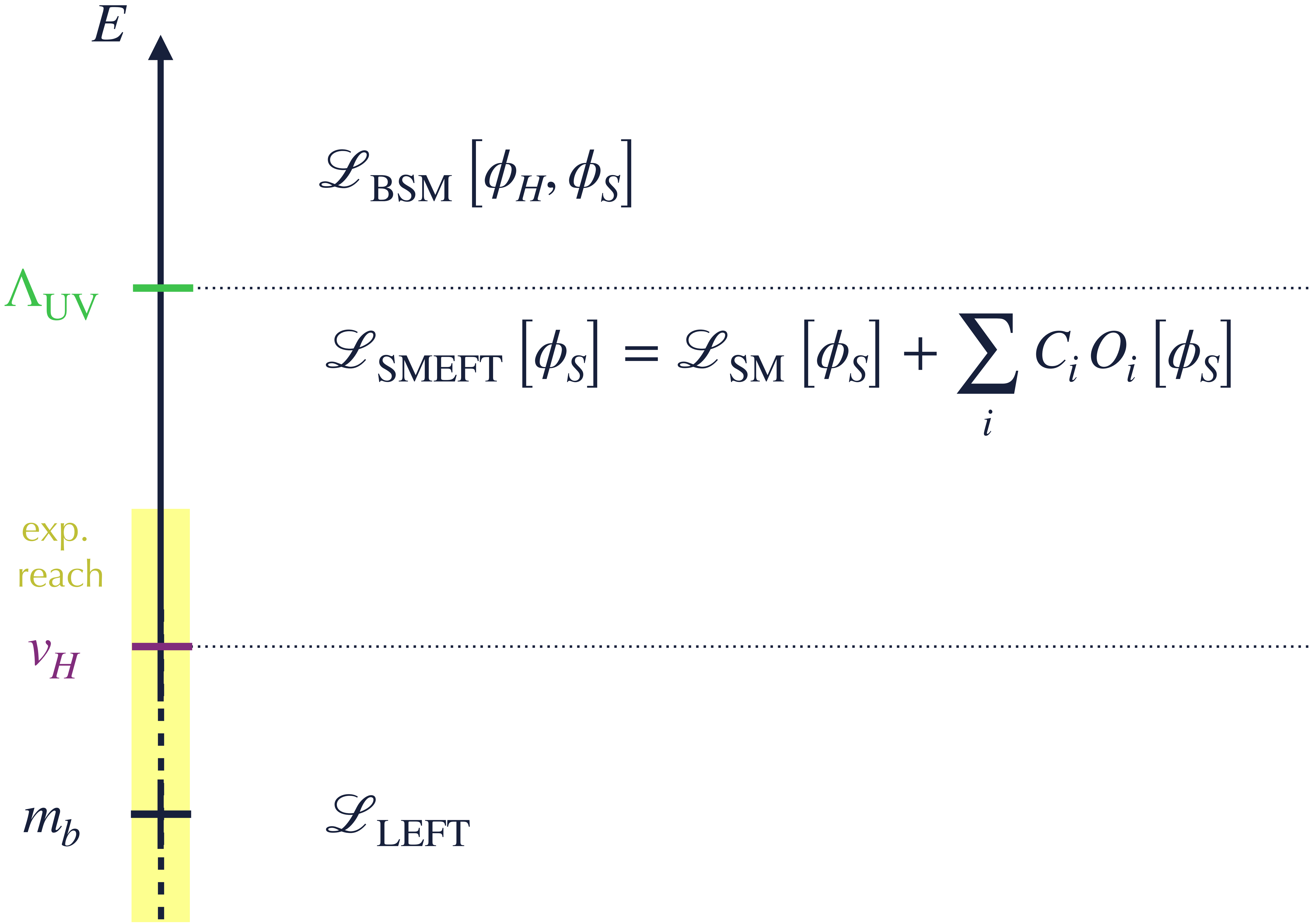
7 July 2022

Top-down Effective Field Theories



$$\mathscr{L}_{\text{BSM}}\left[\phi_H, \phi_S\right]$$

$$\mathscr{L}_{\text{SMEFT}}\left[\phi_S\right] = \mathscr{L}_{\text{SM}}\left[\phi_S\right] + \sum_i C_i\, O_i\left[\phi_S\right]$$

$$\mathscr{L}_{\text{LEFT}}$$

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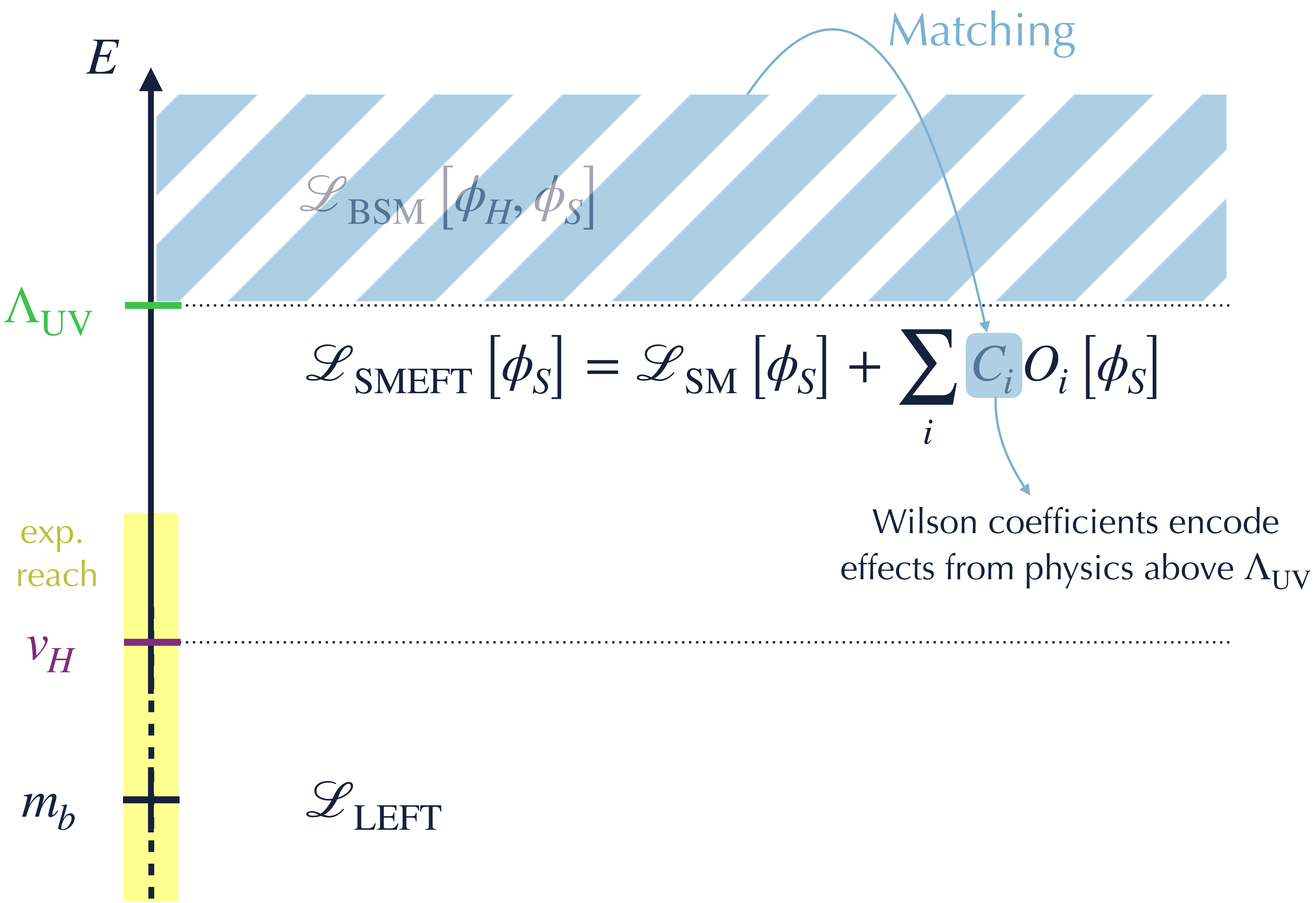

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Wilson coefficients encode effects from physics above Λ_{UV}

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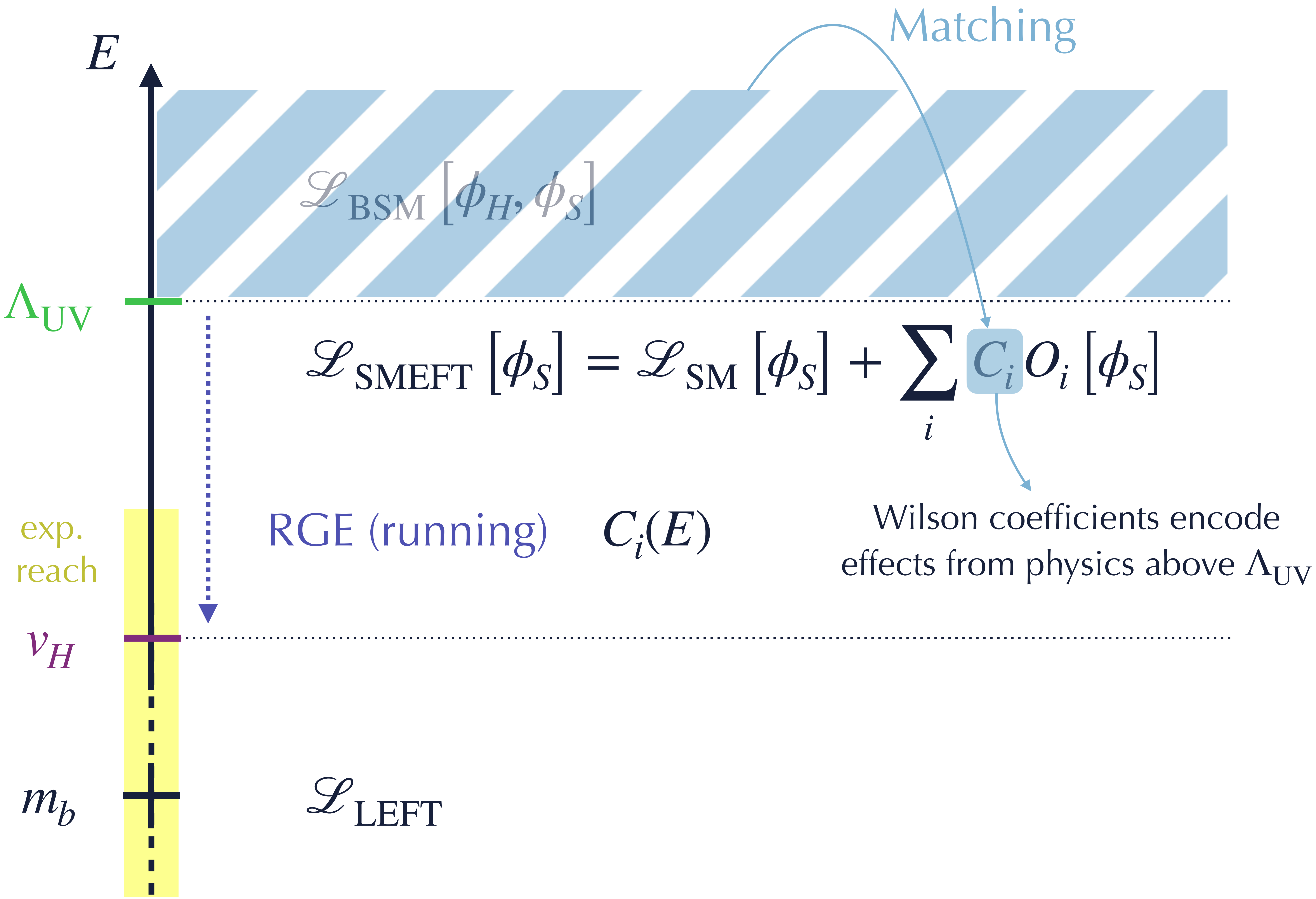

E

$\mathscr{L}_{\text{BSM}}\left[\phi_H, \phi_S\right]$

Matching

Λ_{UV}

$$\mathscr{L}_{\text{SMEFT}}\left[\phi_S\right] = \mathscr{L}_{\text{SM}}\left[\phi_S\right] + \sum_i \boxed{C_i} O_i \left[\phi_S\right]$$

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exp. reach

RGE (running) $C_i(E)$

v_H

m_b

$\mathscr{L}_{\text{LEFT}}$

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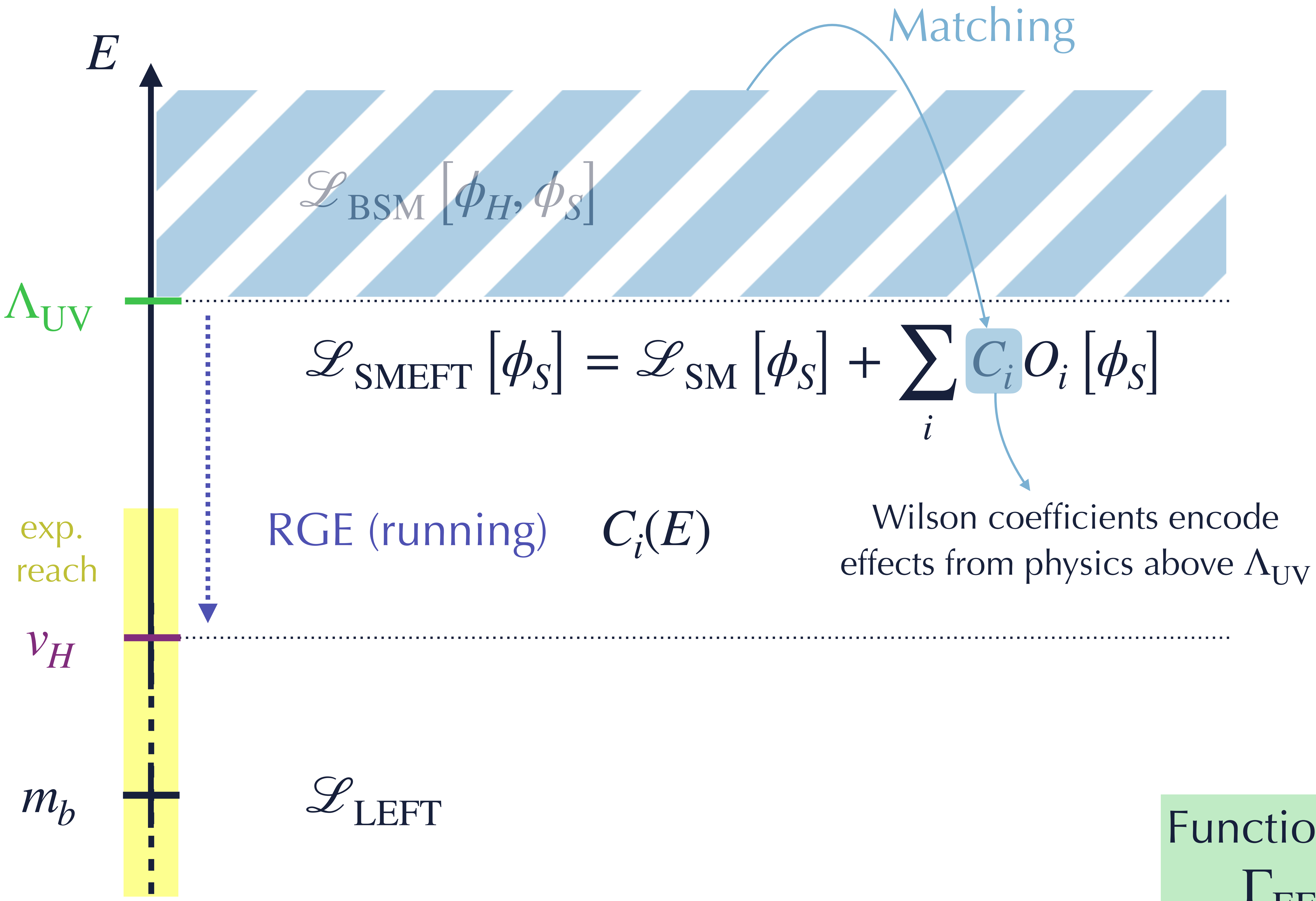

Matching

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Wilson coefficients encode effects from physics above Λ_{UV}

$\mathscr{L}_{\text{LEFT}}$

Why use EFTs instead of the full UV theory?

- SMEFT coefficients can directly be compared to experimental results with programs like `smelli`.
- Isolate relevant degrees of freedom at a specific scale by expanding in inverse powers of the heavy scale.
- Resum large logarithms from widely separated scales.

Two ways of matching:

Functional methods

$\Gamma_{\text{EFT}} \approx \Gamma_{\text{UV}}$

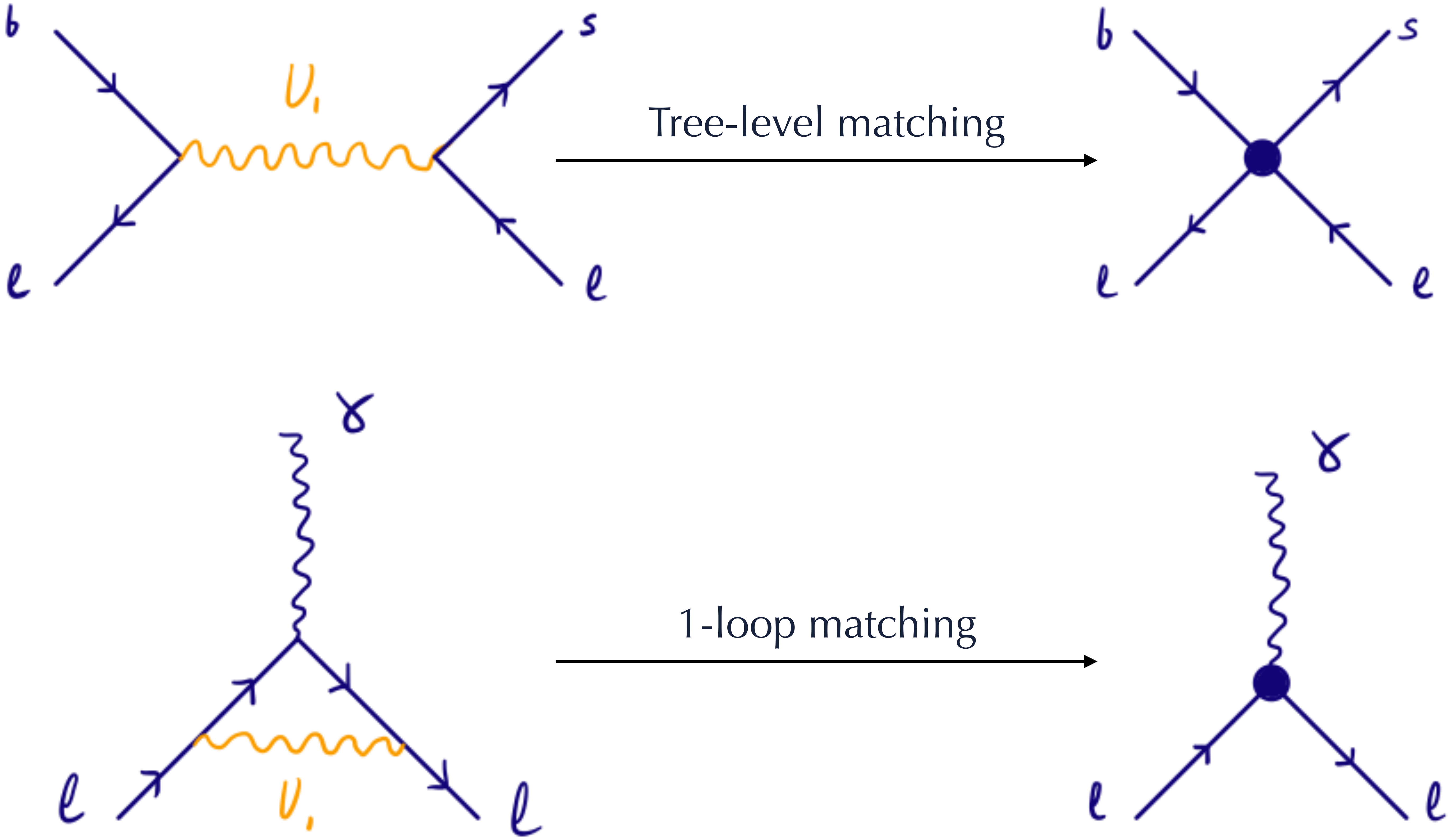
Diagrammatically

$\mathscr{A}_{\text{EFT}} \approx \mathscr{A}_{\text{UV}}$

One-loop matching

Some effects only appear at one-loop:

$U_1 \sim (3,1)_{2/3}$



Tree-level matching

1-loop matching

Functional matching procedure

Goal: match $\mathscr{L}[\eta_H, \eta_L]$ to $\mathscr{L}_{\text{EFT}}[\eta_L]$, $m_H \gg m_L$ by equating the effective action $\Gamma_{\text{UV}} \approx \Gamma_{\text{EFT}}$

heavy d.o.f light d.o.f

defined by $e^{i\Gamma_{\text{UV}}[\hat{\eta}]} = \int \mathscr{D}\eta \exp\left(i \int d^d x\, \mathscr{L}_{\text{UV}}[\hat{\eta} + \eta]\right)$

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<u>Method:</u>

1. Split fields $\eta \to \hat{\eta} + \eta$ quantum fluctuation

background field

Functional matching procedure

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2. Expand Lagrangian $\mathscr{L}_{\text{UV}}[\hat{\eta}+\eta] = \mathscr{L}_{\text{UV}}[\hat{\eta}] + \frac{1}{2}\bar{\eta}_i \left.\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta\eta_i\,\delta\eta_j}\right|_{\eta=\hat{\eta}} \eta_j + \mathscr{O}(\eta^3)$

tree level

one-loop

Functional matching procedure

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background field

2. Expand Lagrangian

$$\mathscr{L}_{\text{UV}}[\hat{\eta} + \eta] = \underbrace{\mathscr{L}_{\text{UV}}[\hat{\eta}]}_{\text{tree level}} + \frac{1}{2}\bar{\eta}_i \underbrace{\left.\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta\eta_i\, \delta\eta_j}\right|_{\eta=\hat{\eta}}}_{\text{one-loop}} \eta_j + \mathscr{O}(\eta^3)$$

Fluctuation operator

$$\mathscr{O}_{ij} = \delta_{ij}\Delta_i^{-1} - X_{ij}$$

particle interactions

inverse propagator

Functional matching procedure

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tree level

one-loop

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inverse propagator

3. Perform Gaussian integral and use expansion by regions to identify $\mathscr{L}_{\text{EFT}}$:

$e^{i\Gamma^{(1)}_{\text{UV}}} = (\text{SDet}\,\mathcal{O})^{-\frac{1}{2}} \Rightarrow \Gamma^{(1)}_{UV} = \frac{i}{2}\text{STr}\ln\mathcal{O}$ and Taylor expand in power counting as $\Delta X \sim m_H^{-1}$

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tree level one-loop

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tree level	$\mathscr{L}^{(0)}_{\text{EFT}} = \mathscr{L}_{\text{UV}}[\hat{\eta}_L, \hat{\eta}_H(\hat{\eta}_L)]$ where we replace $\hat{\eta}_H$ by its EOM
one-loop	$\int d^d x\, \mathscr{L}^{(1)}_{\text{EFT}} = \Gamma^{(1)}_{\text{UV}}\Big\vert_{\text{hard}} = \frac{i}{2}\text{STr}\ln\Delta^{-1}\Big\vert_{\text{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\text{STr}\left[(\Delta X)^n\right]\Big\vert_{\text{hard}}$

Functional matching procedure

$$\int d^dx\,\mathscr{L}^{(1)}_{\text{EFT}} = \boxed{\frac{i}{2}\text{STr}\ln\Delta^{-1}}\Big|_{\text{hard}} - \boxed{\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\text{STr}\left[(\Delta X)^n\right]}\Big|_{\text{hard}}$$

log-type supertrace

- model-independent
- depend on propagator type

power-type supertrace

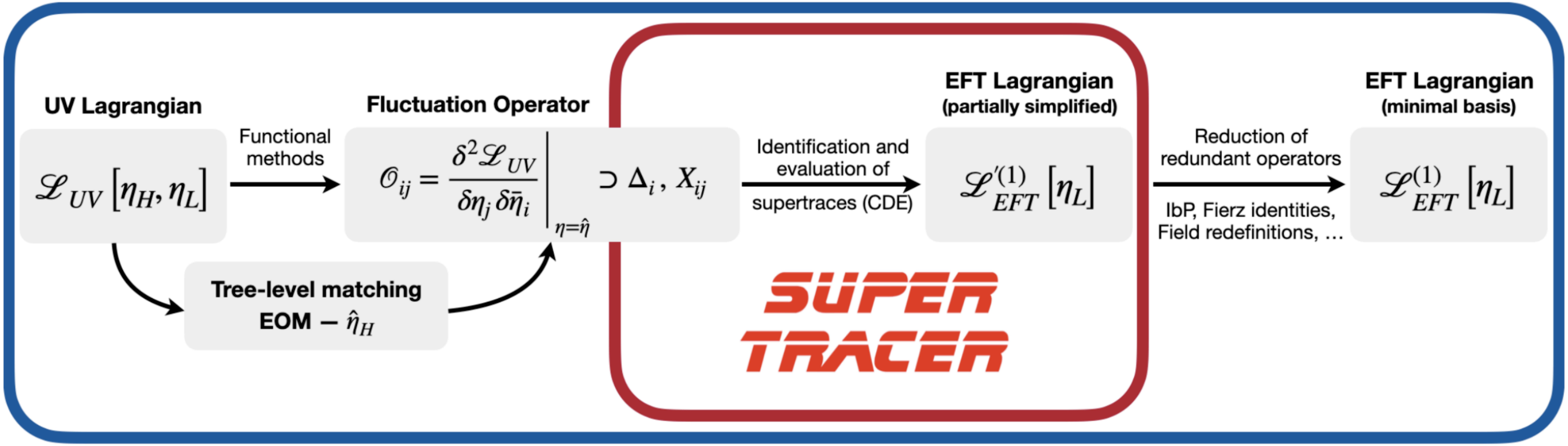
- depend on interaction terms

Advantages of functional matching:

- EFT operators are automatically generated → no prerequisite knowledge of the EFT basis needed
- very systematic method → well suited for an algorithmic approach
- manifestly gauge covariant computations

Presentation of the program

Mathematica program



UV Lagrangian

$\mathscr{L}_{UV}\left[\eta_H, \eta_L\right]$

Functional methods

Fluctuation Operator

$\mathcal{O}_{ij} = \left.\frac{\delta^2 \mathscr{L}_{UV}}{\delta\eta_j\, \delta\bar{\eta}_i}\right|_{\eta=\hat{\eta}} \supset \Delta_i\,,\, X_{ij}$

Tree-level matching
EOM — $\hat{\eta}_H$

Identification and evaluation of supertraces (CDE)

EFT Lagrangian
(partially simplified)

$\mathscr{L}^{\prime(1)}_{EFT}\left[\eta_L\right]$

Reduction of redundant operators

IbP, Fierz identities, Field redefinitions, …

EFT Lagrangian
(minimal basis)

$\mathscr{L}^{(1)}_{EFT}\left[\eta_L\right]$

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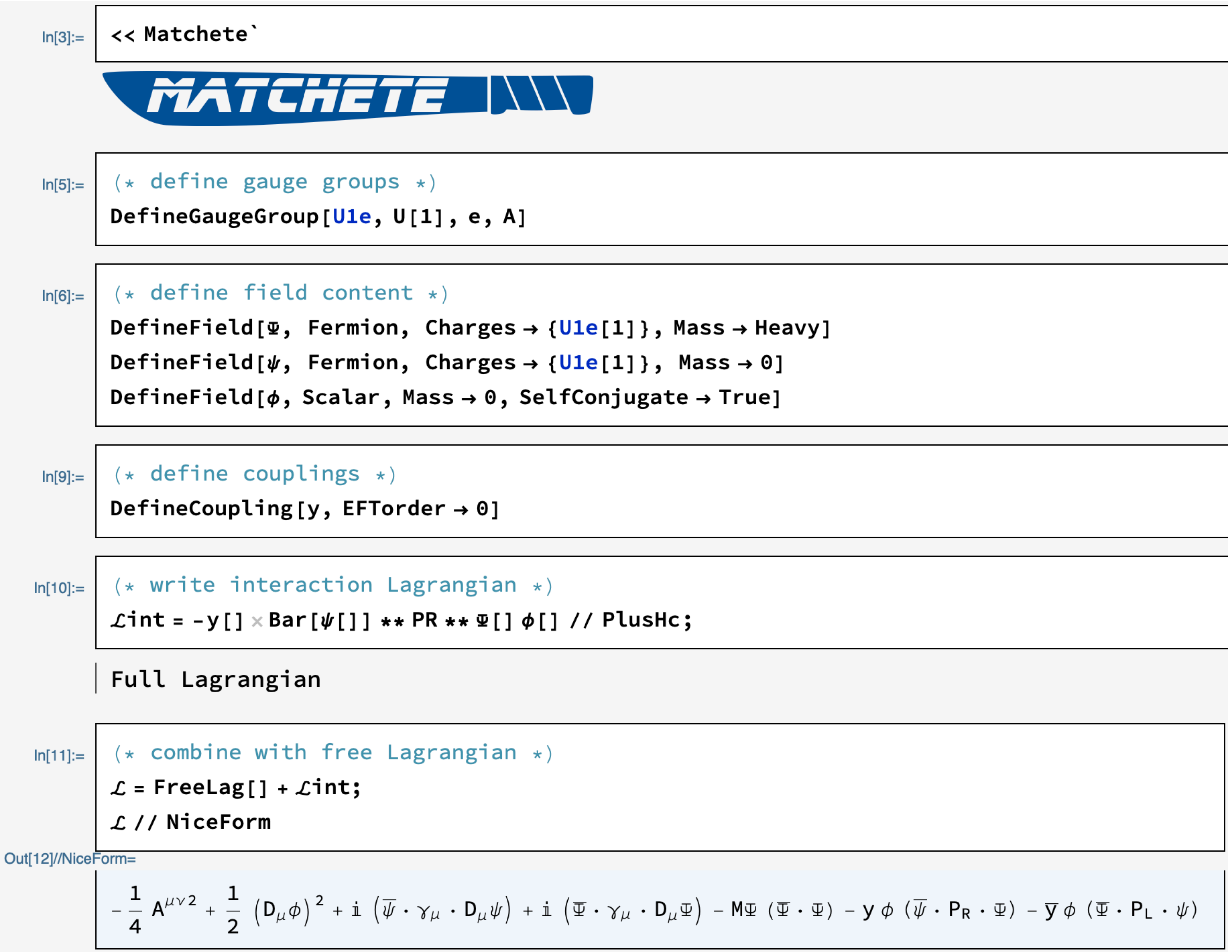
User-friendly input ⟶ Supertraces ⟶ Reduction to a basis

Toy-model: heavy vector-like fermion



```
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        DefineGaugeGroup[U1e, U[1], e, A]

In[6]:= (* define field content *)
        DefineField[Ψ, Fermion, Charges → {U1e[1]}, Mass → Heavy]
        DefineField[ψ, Fermion, Charges → {U1e[1]}, Mass → 0]
        DefineField[ϕ, Scalar, Mass → 0, SelfConjugate → True]

In[9]:= (* define couplings *)
        DefineCoupling[y, EFTorder → 0]

In[10]:= (* write interaction Lagrangian *)
         ℒint = -y[] × Bar[ψ[]] ** PR ** Ψ[] ϕ[] // PlusHc;

         Full Lagrangian

In[11]:= (* combine with free Lagrangian *)
         ℒ = FreeLag[] + ℒint;
         ℒ // NiceForm
Out[12]//NiceForm=
```

$$-\frac{1}{4} A^{\mu\nu 2} + \frac{1}{2} \left(D_\mu \phi\right)^2 + i \left(\overline{\psi} \cdot \gamma_\mu \cdot D_\mu \psi\right) + i \left(\overline{\Psi} \cdot \gamma_\mu \cdot D_\mu \Psi\right) - M\Psi \left(\overline{\Psi} \cdot \Psi\right) - y\,\phi \left(\overline{\psi} \cdot P_R \cdot \Psi\right) - \overline{y}\,\phi \left(\overline{\Psi} \cdot P_L \cdot \psi\right)$$

Define model:

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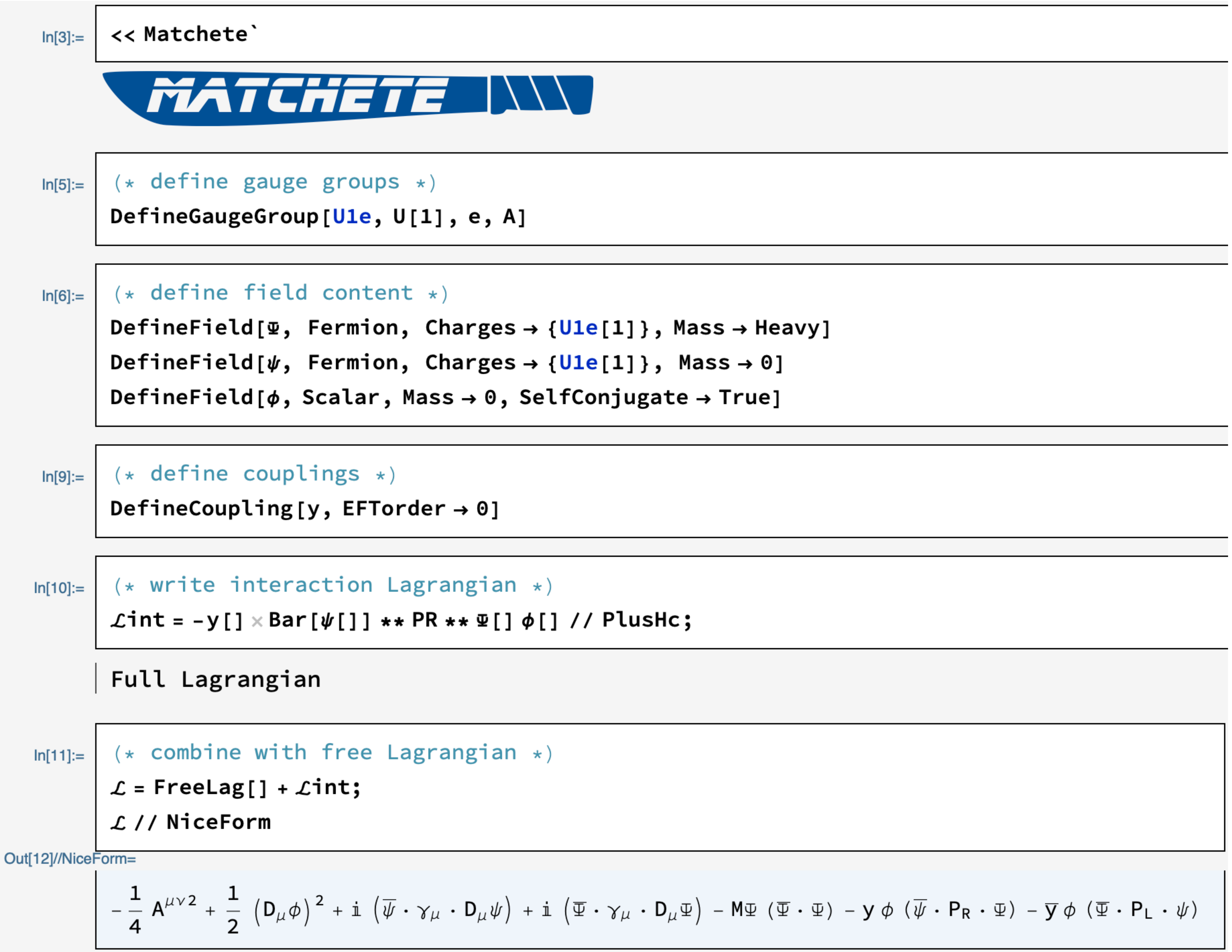

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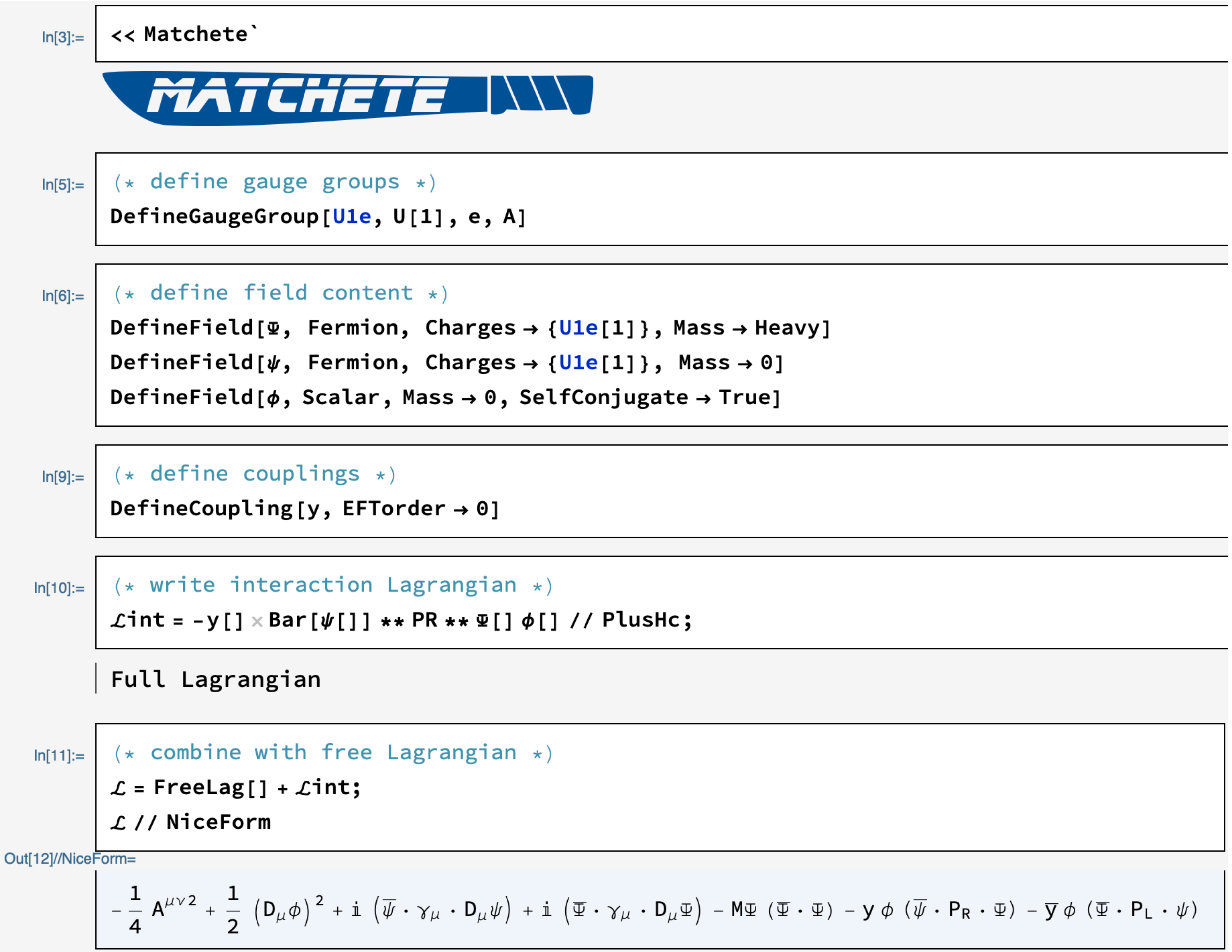

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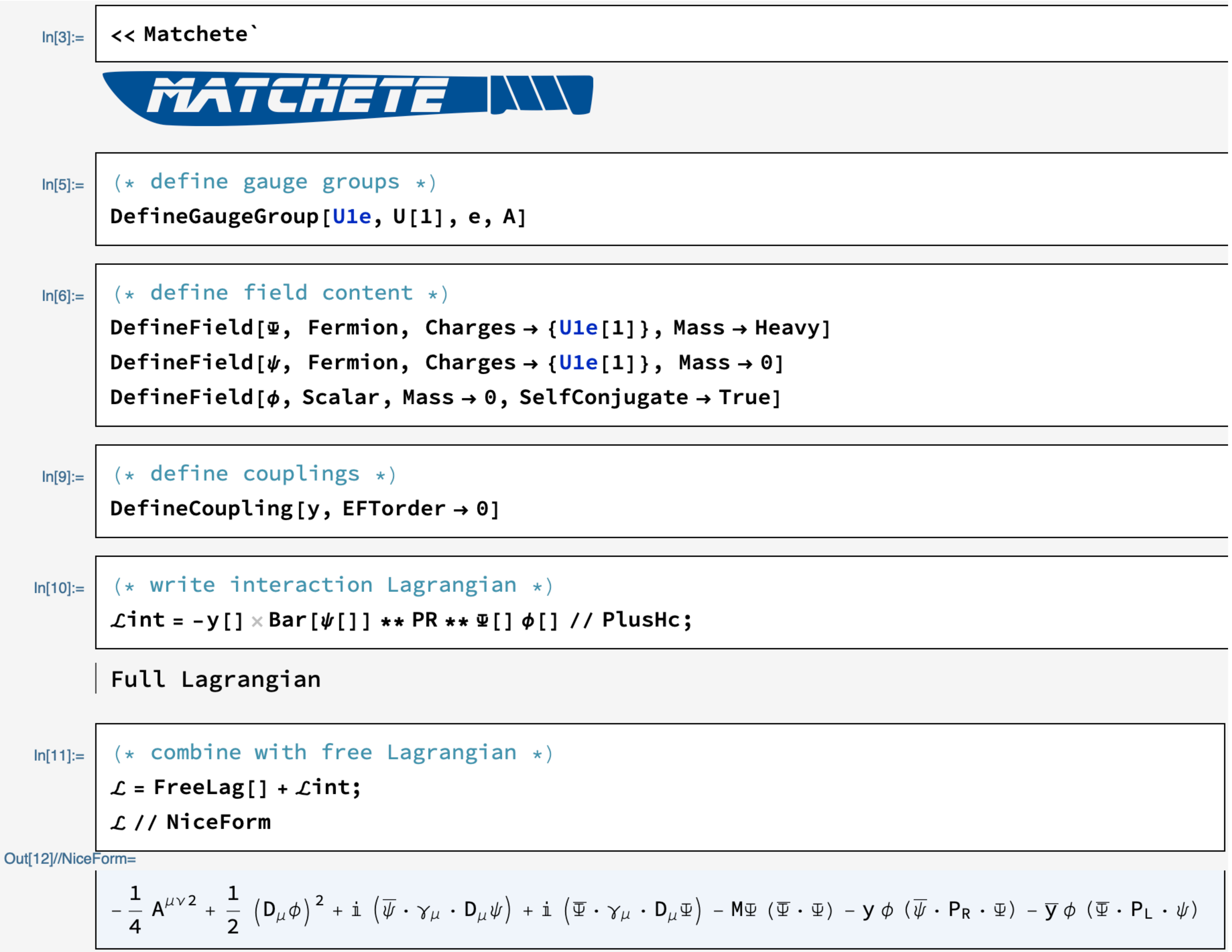

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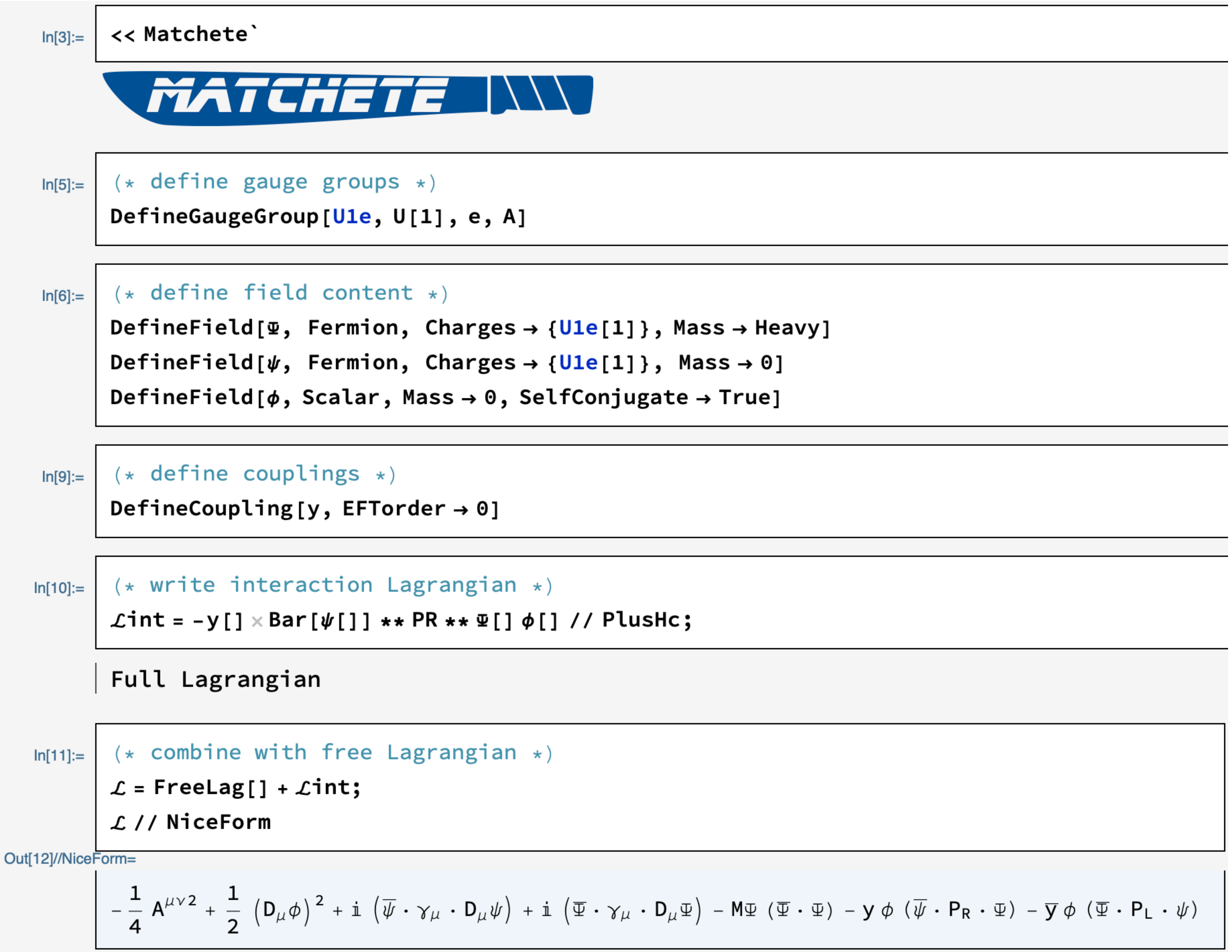

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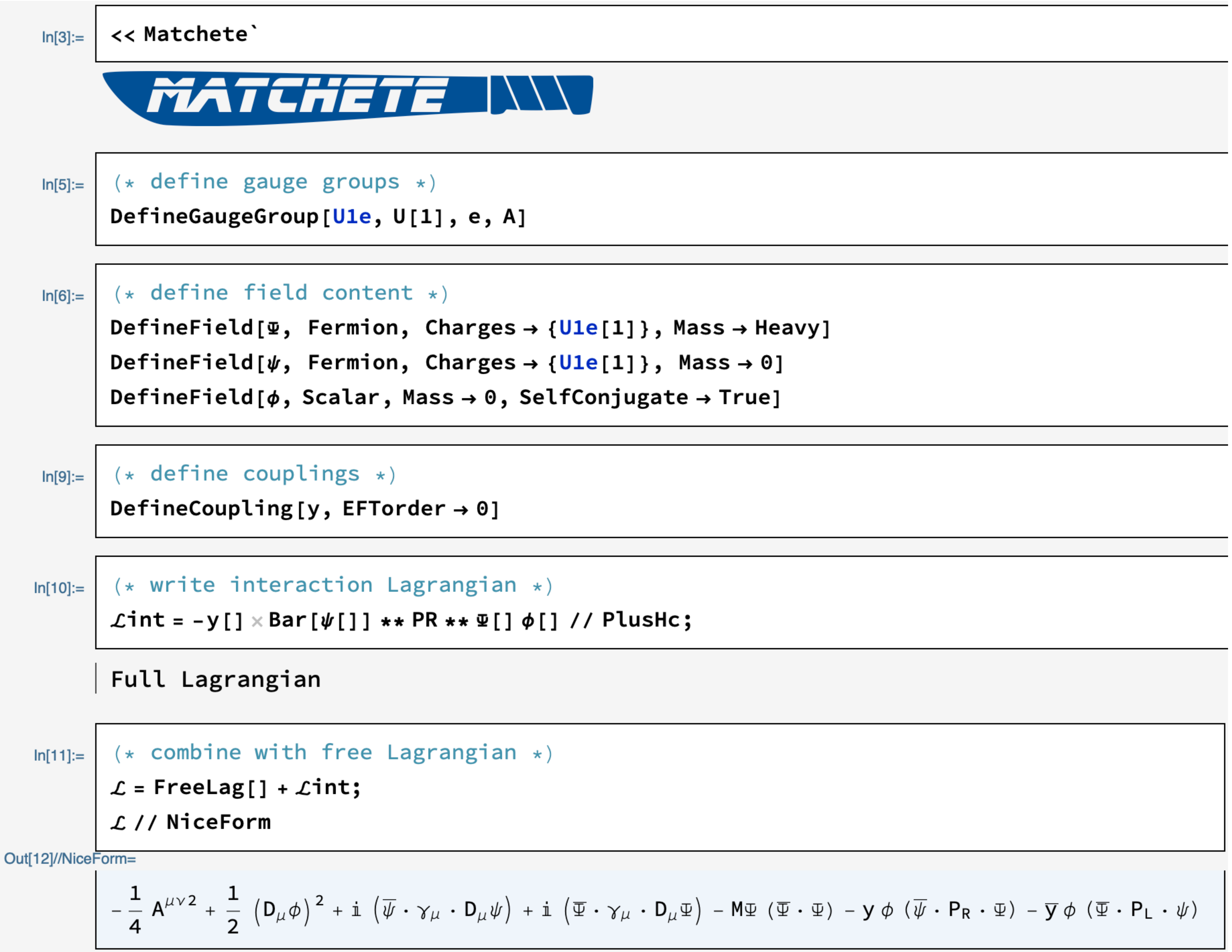

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Predefined models: SM, SMEFT…

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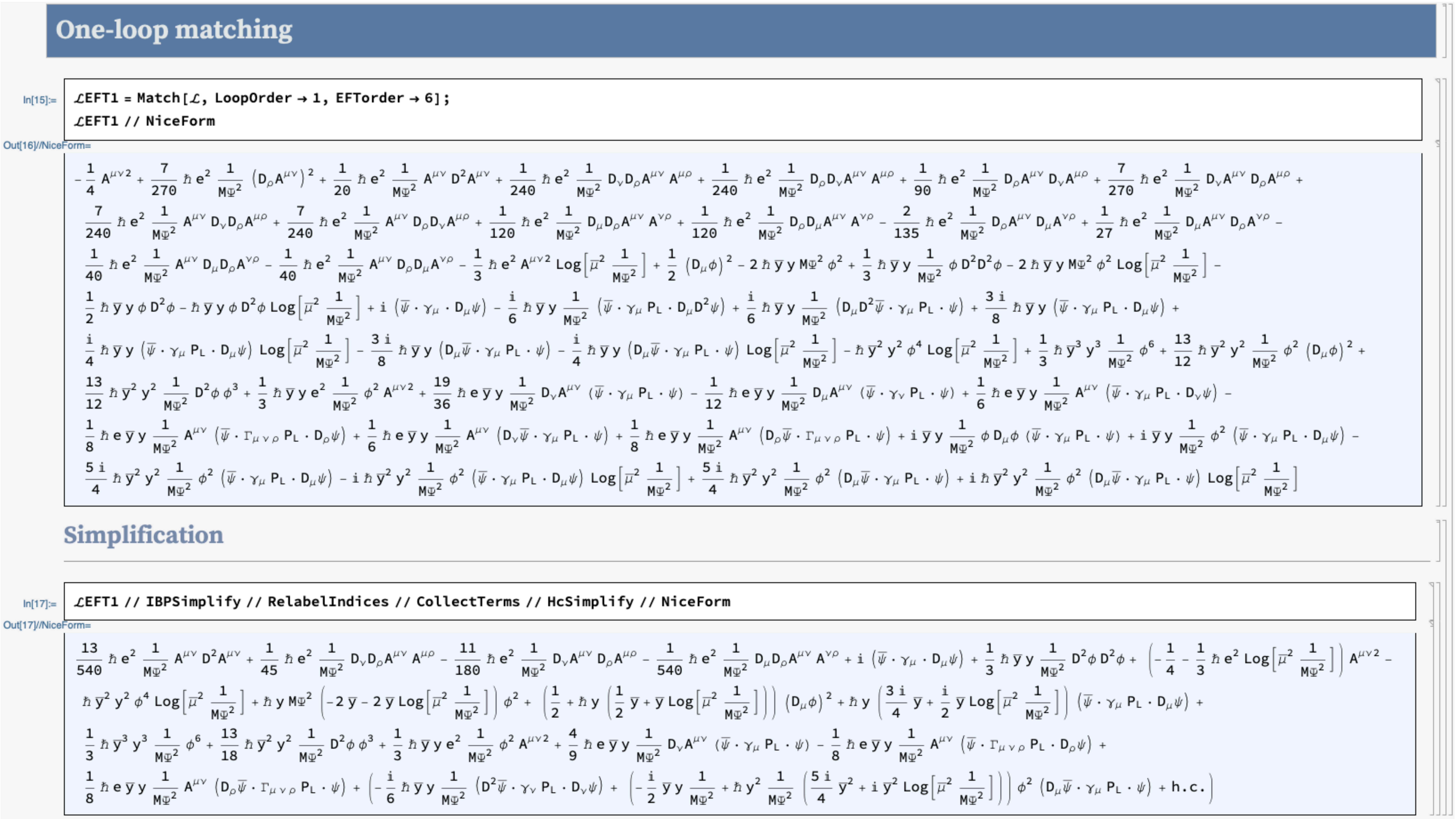


One-loop matching

In[15]:=

```
ℒEFT1 = Match[ℒ, LoopOrder → 1, EFTorder → 6];
ℒEFT1 // NiceForm
```

Out[16]//NiceForm=

$$
\begin{aligned}
&-\frac{1}{4} A^{\mu\nu\,2} + \frac{7}{270}\hbar e^2 \frac{1}{M\Psi^2}\left(D_\rho A^{\mu\nu}\right)^2 + \frac{1}{20}\hbar e^2 \frac{1}{M\Psi^2} A^{\mu\nu} D^2 A^{\mu\nu} + \frac{1}{240}\hbar e^2 \frac{1}{M\Psi^2} D_\nu D_\rho A^{\mu\nu} A^{\mu\rho} + \frac{1}{240}\hbar e^2 \frac{1}{M\Psi^2} D_\rho D_\nu A^{\mu\nu} A^{\mu\rho} + \frac{1}{90}\hbar e^2 \frac{1}{M\Psi^2} D_\rho A^{\mu\nu} D_\nu A^{\mu\rho} + \frac{7}{270}\hbar e^2 \frac{1}{M\Psi^2} D_\nu A^{\mu\nu} D_\rho A^{\mu\rho} + \\
&\frac{7}{240}\hbar e^2 \frac{1}{M\Psi^2} A^{\mu\nu} D_\nu D_\rho A^{\mu\rho} + \frac{7}{240}\hbar e^2 \frac{1}{M\Psi^2} A^{\mu\nu} D_\rho D_\nu A^{\mu\rho} + \frac{1}{120}\hbar e^2 \frac{1}{M\Psi^2} D_\mu D_\rho A^{\mu\nu} A^{\nu\rho} + \frac{1}{120}\hbar e^2 \frac{1}{M\Psi^2} D_\rho D_\mu A^{\mu\nu} A^{\nu\rho} - \frac{2}{135}\hbar e^2 \frac{1}{M\Psi^2} D_\rho A^{\mu\nu} D_\mu A^{\nu\rho} + \frac{1}{27}\hbar e^2 \frac{1}{M\Psi^2} D_\mu A^{\mu\nu} D_\rho A^{\nu\rho} - \\
&\frac{1}{40}\hbar e^2 \frac{1}{M\Psi^2} A^{\mu\nu} D_\mu D_\rho A^{\nu\rho} - \frac{1}{40}\hbar e^2 \frac{1}{M\Psi^2} A^{\mu\nu} D_\rho D_\mu A^{\nu\rho} - \frac{1}{3}\hbar e^2 A^{\mu\nu\,2}\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] + \frac{1}{2}\left(D_\mu\phi\right)^2 - 2\hbar\,\bar{y}\,y\,M\Psi^2\,\phi^2 + \frac{1}{3}\hbar\,\bar{y}\,y\,\frac{1}{M\Psi^2}\,\phi\,D^2D^2\phi - 2\hbar\,\bar{y}\,y\,M\Psi^2\,\phi^2\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] - \\
&\frac{1}{2}\hbar\,\bar{y}\,y\,\phi\,D^2\phi - \hbar\,\bar{y}\,y\,\phi\,D^2\phi\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] + i\left(\bar{\psi}\cdot\gamma_\mu\cdot D_\mu\psi\right) - \frac{i}{6}\hbar\,\bar{y}\,y\,\frac{1}{M\Psi^2}\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\mu D^2\psi\right) + \frac{i}{6}\hbar\,\bar{y}\,y\,\frac{1}{M\Psi^2}\left(D_\mu D^2\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) + \frac{3\,i}{8}\hbar\,\bar{y}\,y\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) + \\
&\frac{i}{4}\hbar\,\bar{y}\,y\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\mu\psi\right)\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] - \frac{3\,i}{8}\hbar\,\bar{y}\,y\left(D_\mu\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) - \frac{i}{4}\hbar\,\bar{y}\,y\left(D_\mu\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right)\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] - \hbar\,\bar{y}^2 y^2\phi^4\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] + \frac{1}{3}\hbar\,\bar{y}^3 y^3\frac{1}{M\Psi^2}\phi^6 + \frac{13}{12}\hbar\,\bar{y}^2 y^2\frac{1}{M\Psi^2}\phi^2\left(D_\mu\phi\right)^2 + \\
&\frac{13}{12}\hbar\,\bar{y}^2 y^2\frac{1}{M\Psi^2}D^2\phi\,\phi^3 + \frac{1}{3}\hbar\,\bar{y}\,y\,e^2\frac{1}{M\Psi^2}\phi^2 A^{\mu\nu\,2} + \frac{19}{36}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}D_\nu A^{\mu\nu}\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) - \frac{1}{12}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}D_\mu A^{\mu\nu}\left(\bar{\psi}\cdot\gamma_\nu P_L\cdot\psi\right) + \frac{1}{6}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}A^{\mu\nu}\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\nu\psi\right) - \\
&\frac{1}{8}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}A^{\mu\nu}\left(\bar{\psi}\cdot\Gamma_{\mu\nu\rho}P_L\cdot D_\rho\psi\right) + \frac{1}{6}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}A^{\mu\nu}\left(D_\nu\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) + \frac{1}{8}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}A^{\mu\nu}\left(D_\rho\bar{\psi}\cdot\Gamma_{\mu\nu\rho}P_L\cdot\psi\right) + i\,\bar{y}\,y\,\frac{1}{M\Psi^2}\phi\,D_\mu\phi\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) + i\,\bar{y}\,y\,\frac{1}{M\Psi^2}\phi^2\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) - \\
&\frac{5\,i}{4}\hbar\,\bar{y}^2 y^2\frac{1}{M\Psi^2}\phi^2\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) - i\,\hbar\,\bar{y}^2 y^2\frac{1}{M\Psi^2}\phi^2\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\mu\psi\right)\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] + \frac{5\,i}{4}\hbar\,\bar{y}^2 y^2\frac{1}{M\Psi^2}\phi^2\left(D_\mu\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) + i\,\hbar\,\bar{y}^2 y^2\frac{1}{M\Psi^2}\phi^2\left(D_\mu\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right)\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right]
\end{aligned}
$$

Simplification

In[17]:=

```
ℒEFT1 // IBPSimplify // RelabelIndices // CollectTerms // HcSimplify // NiceForm
```

Out[17]//NiceForm=

$$
\begin{aligned}
&\frac{13}{540}\hbar e^2\frac{1}{M\Psi^2}A^{\mu\nu}D^2A^{\mu\nu} + \frac{1}{45}\hbar e^2\frac{1}{M\Psi^2}D_\nu D_\rho A^{\mu\nu}A^{\mu\rho} - \frac{11}{180}\hbar e^2\frac{1}{M\Psi^2}D_\nu A^{\mu\nu}D_\rho A^{\mu\rho} - \frac{1}{540}\hbar e^2\frac{1}{M\Psi^2}D_\mu D_\rho A^{\mu\nu}A^{\nu\rho} + i\left(\bar{\psi}\cdot\gamma_\mu\cdot D_\mu\psi\right) + \frac{1}{3}\hbar\,\bar{y}\,y\,\frac{1}{M\Psi^2}D^2\phi\,D^2\phi + \left(-\frac{1}{4} - \frac{1}{3}\hbar e^2\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right]\right)A^{\mu\nu\,2} - \\
&\hbar\,\bar{y}^2 y^2\phi^4\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right] + \hbar\,y\,M\Psi^2\left(-2\,\bar{y} - 2\,\bar{y}\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right]\right)\phi^2 + \left(\frac{1}{2} + \hbar\,y\left(\frac{1}{2}\bar{y} + \bar{y}\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right]\right)\right)\left(D_\mu\phi\right)^2 + \hbar\,y\left(\frac{3\,i}{4}\bar{y} + \frac{i}{2}\bar{y}\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right]\right)\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot D_\mu\psi\right) + \\
&\frac{1}{3}\hbar\,\bar{y}^3 y^3\frac{1}{M\Psi^2}\phi^6 + \frac{13}{18}\hbar\,\bar{y}^2 y^2\frac{1}{M\Psi^2}D^2\phi\,\phi^3 + \frac{1}{3}\hbar\,\bar{y}\,y\,e^2\frac{1}{M\Psi^2}\phi^2 A^{\mu\nu\,2} + \frac{4}{9}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}D_\nu A^{\mu\nu}\left(\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) - \frac{1}{8}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}A^{\mu\nu}\left(\bar{\psi}\cdot\Gamma_{\mu\nu\rho}P_L\cdot D_\rho\psi\right) + \\
&\frac{1}{8}\hbar\,e\,\bar{y}\,y\,\frac{1}{M\Psi^2}A^{\mu\nu}\left(D_\rho\bar{\psi}\cdot\Gamma_{\mu\nu\rho}P_L\cdot\psi\right) + \Bigg(-\frac{i}{6}\hbar\,\bar{y}\,y\,\frac{1}{M\Psi^2}\left(D^2\bar{\psi}\cdot\gamma_\nu P_L\cdot D_\nu\psi\right) + \left(-\frac{i}{2}\bar{y}\,y\,\frac{1}{M\Psi^2} + \hbar\,y^2\frac{1}{M\Psi^2}\left(\frac{5\,i}{4}\bar{y}^2 + i\,\bar{y}^2\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right]\right)\right)\phi^2\left(D_\mu\bar{\psi}\cdot\gamma_\mu P_L\cdot\psi\right) + \text{h.c.}\Bigg)
\end{aligned}
$$

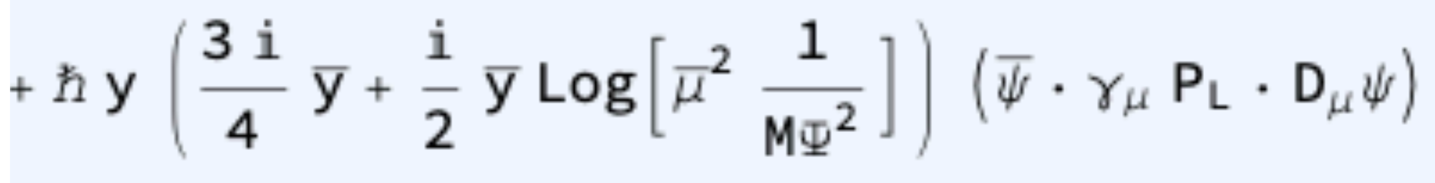
Specify loop order:
0 | 1

Specify EFT order:
4 | 5 | 6 | 7 | 8 | …

Perform simplifications

Comparison between code and by hand

Matchete output:



$$+ \hbar\, y \left(\frac{3\, i}{4}\, \bar{y} + \frac{i}{2}\, \bar{y}\, \mathrm{Log}\left[\bar{\mu}^2\, \frac{1}{M\Psi^2}\right]\right) \left(\bar{\psi} \cdot \gamma_\mu\, P_L \cdot D_\mu \psi\right)$$

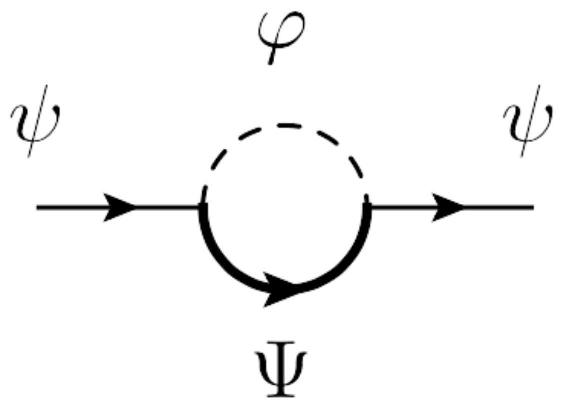
Diagrammatic computation by hand:

- Define EFT basis

$$\mathcal{L}_{\text{eff}}^{(M^0)} = \frac{c_\varphi}{2}(\partial_\mu\varphi)(\partial^\mu\varphi) - \frac{c_m}{2}\varphi^2 - \frac{c_\lambda}{4!}\varphi^4 - \frac{c_A}{4}F_{\mu\nu}F^{\mu\nu} + c_\psi \bar{\psi}_L(i\not{D})\psi_L$$

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{(M^{-2})} =& \frac{c_1}{2M^2}\varphi\Box^2\varphi + \frac{c_2}{4M^2}F_{\mu\nu}\Box F^{\mu\nu} - \frac{ic_3}{2M^2}\left[\bar{\psi}D^2\not{D}P_L\psi + \text{h.c.}\right] \\ &+ \frac{c_4 e}{M^2}\left[F_{\nu\rho}\bar{\psi}\Gamma^{\mu\nu\rho}P_L D_\mu\psi + \text{h.c.}\right] - \frac{c_5 e}{M^2}(\partial_\nu F^{\mu\nu})\bar{\psi}\gamma_\mu P_L\psi \\ &+ \frac{c_6}{2M^2}\bar{\psi}\varphi(i\not{D})\varphi P_L\psi + \frac{c_7}{4!M^2}\varphi^2\Box\varphi^2 - \frac{c_8}{8M^2}F_{\mu\nu}F^{\mu\nu}\varphi^2 + \frac{c_9}{6!M^2}\varphi^6\end{aligned}$$

- Compute amplitude in the full theory



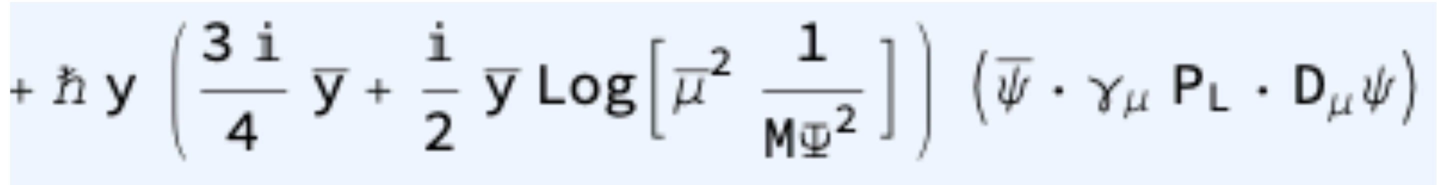
$$= \frac{i\alpha_y}{8\pi}\not{p}P_L\left\{\left(\Delta_\mu + \frac{3}{2}\right) + \frac{2p^2}{3M^2}\right\}$$

- Compute amplitude in the EFT and match

$$c_\psi = \frac{\alpha_y}{8\pi}\left(\log\frac{\mu^2}{M^2} + \frac{3}{2}\right)$$

Comparison between code and by hand

Matchete output:



$$+\hbar\, y \left(\frac{3\,i}{4}\,\bar{y} + \frac{i}{2}\,\bar{y}\,\mathrm{Log}\left[\bar{\mu}^2\,\frac{1}{M\Psi^2}\right]\right)\left(\bar{\psi}\cdot\gamma_\mu\,P_L\cdot D_\mu\psi\right)$$

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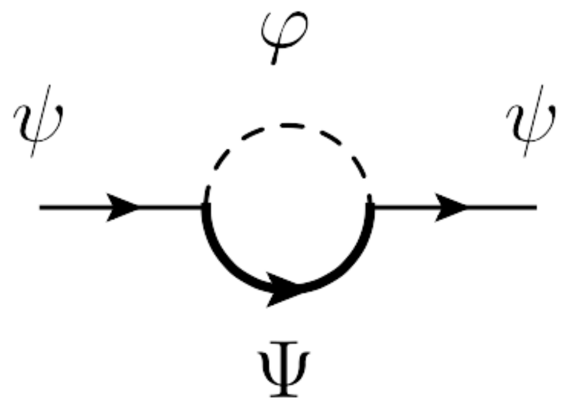


$$= \frac{i\alpha_y}{8\pi}\not{p}P_L\left\{\left(\Delta_\mu + \frac{3}{2}\right) + \frac{2p^2}{3M^2}\right\}$$

- Compute amplitude in the EFT and match

$$c_\psi = \frac{\alpha_y}{8\pi}\left(\log\frac{\mu^2}{M^2} + \frac{3}{2}\right)$$

Full diagrammatic calculation by hand

Corrections to:

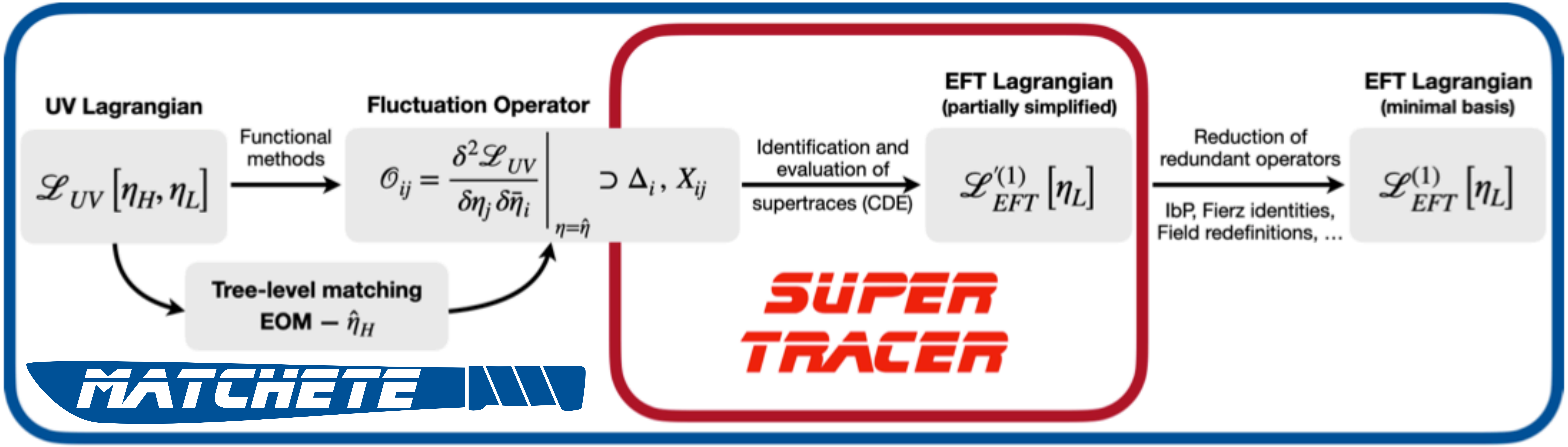
- wave-function: 3 graphs
- 3-point vertices: 11 graphs
- 4-point vertices: 23 graphs
- 5-point vertices: 30 graphs
- 6-point vertex: 5! graphs

after 3 weeks of computation agree with program output

✓

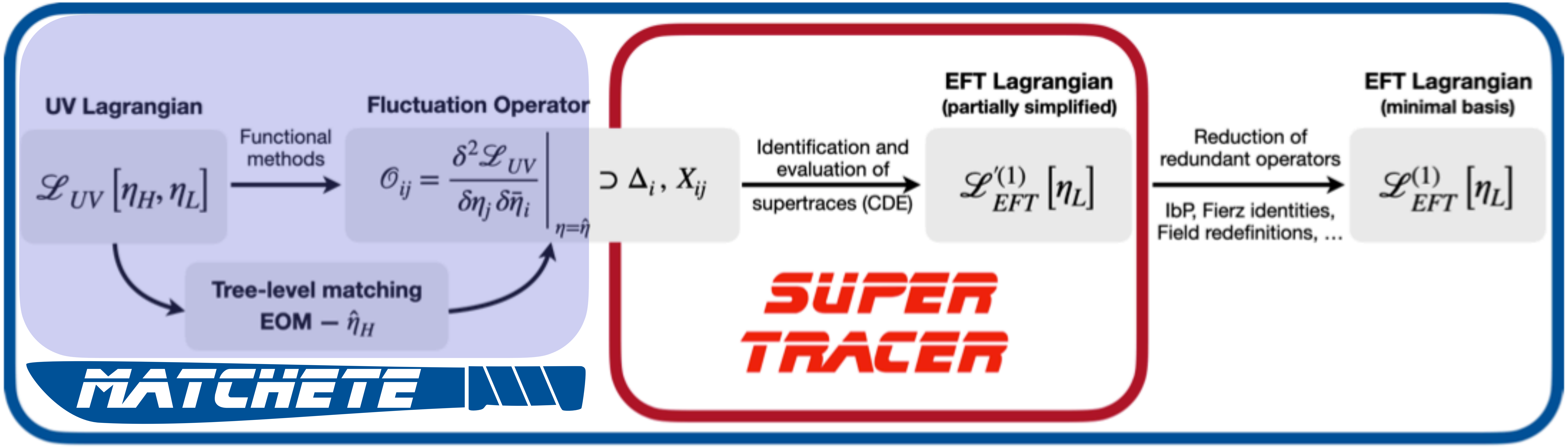
v.s. ∼ 30 s for the computer

Conclusion on the status



Conclusion on the status

- ✓ User-friendly interface to input Lagrangian
- ✓ Tree-level matching
- ✓ GroupMagic: subpackage for handling and contraction of Clebsch-Gordon coefficients
- \- Treatment of heavy vectors (gauge fixing and ghosts)



UV Lagrangian

$\mathscr{L}_{UV}[\eta_H, \eta_L]$

Functional methods

Fluctuation Operator

$$\mathcal{O}_{ij} = \frac{\delta^2 \mathscr{L}_{UV}}{\delta \eta_j \delta \bar{\eta}_i}\bigg|_{\eta=\hat{\eta}} \supset \Delta_i, X_{ij}$$

Tree-level matching

EOM – $\hat{\eta}_H$

Identification and evaluation of supertraces (CDE)

EFT Lagrangian
(partially simplified)

$\mathscr{L}'^{(1)}_{EFT}[\eta_L]$

Reduction of redundant operators

IbP, Fierz identities, Field redefinitions, ...

EFT Lagrangian
(minimal basis)

$\mathscr{L}^{(1)}_{EFT}[\eta_L]$

Conclusion on the status

- ✓ User-friendly interface to input Lagrangian
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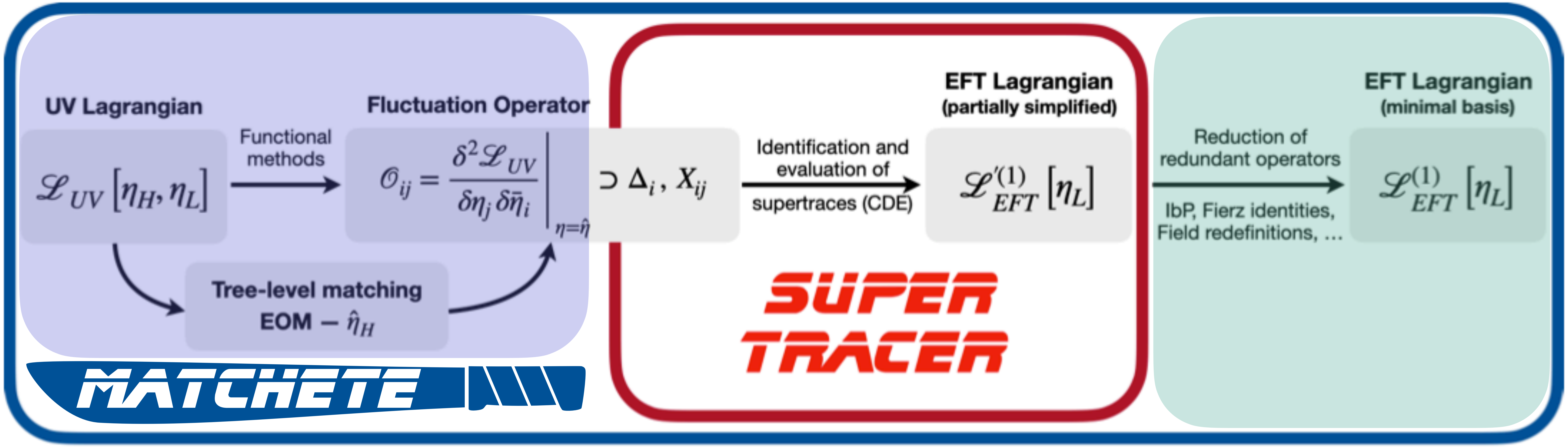

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$\mathcal{O}_{ij} = \left.\frac{\delta^2 \mathscr{L}_{UV}}{\delta \eta_j \, \delta \bar{\eta}_i}\right|_{\eta=\hat{\eta}} \supset \Delta_i, X_{ij}$

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Reduction of redundant operators

IbP, Fierz identities, Field redefinitions, ...

EFT Lagrangian (minimal basis)

$\mathscr{L}^{(1)}_{EFT}[\eta_L]$

MATCHETE

SUPER TRACER

- ✓ Integration by part
- ● Field redefinitions
- \- Kinetic diagonalization
- ✓ Fierz identities (spinorial) and evanescent operators

Future prospects

Future features:

- Computation of EFT β-functions (RGE and evanescent operators)
- Interface with other codes: smelli, Flavio (WCxf) … → phenomenology pipeline
- Generation of UFO files for Madgraph for UV theory or EFT

Long term features:

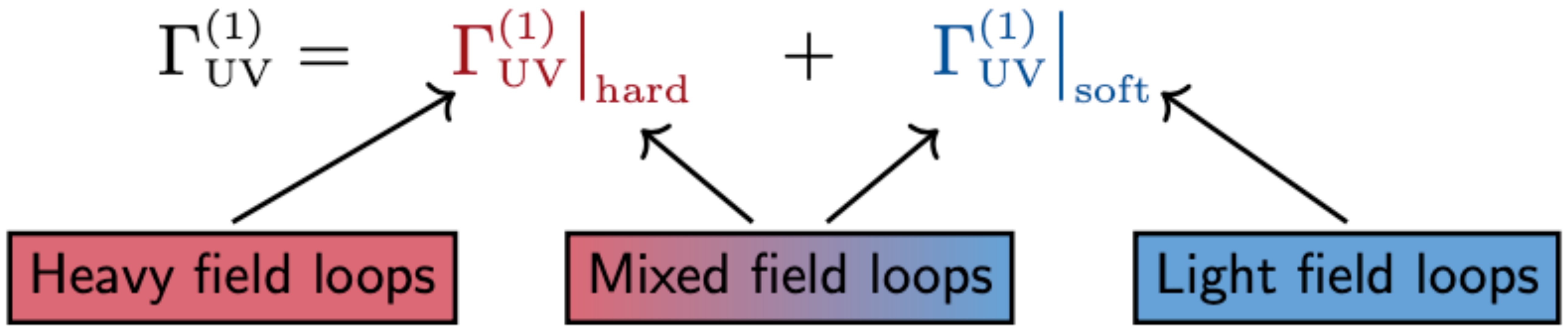
- Automatic spontaneous symmetry breaking
- Multi-scales matching and running

Thank you for listening!

Any questions?

Expansion by regions

Provides a method for scale separation in dimensional regularization: [Beneke, Smirnov, hep-ph/9711391; Jantzen, 1111.2589]



$$\Gamma^{(1)}_{\rm UV} = \Gamma^{(1)}_{\rm UV}\Big|_{\rm hard} + \Gamma^{(1)}_{\rm UV}\Big|_{\rm soft}$$

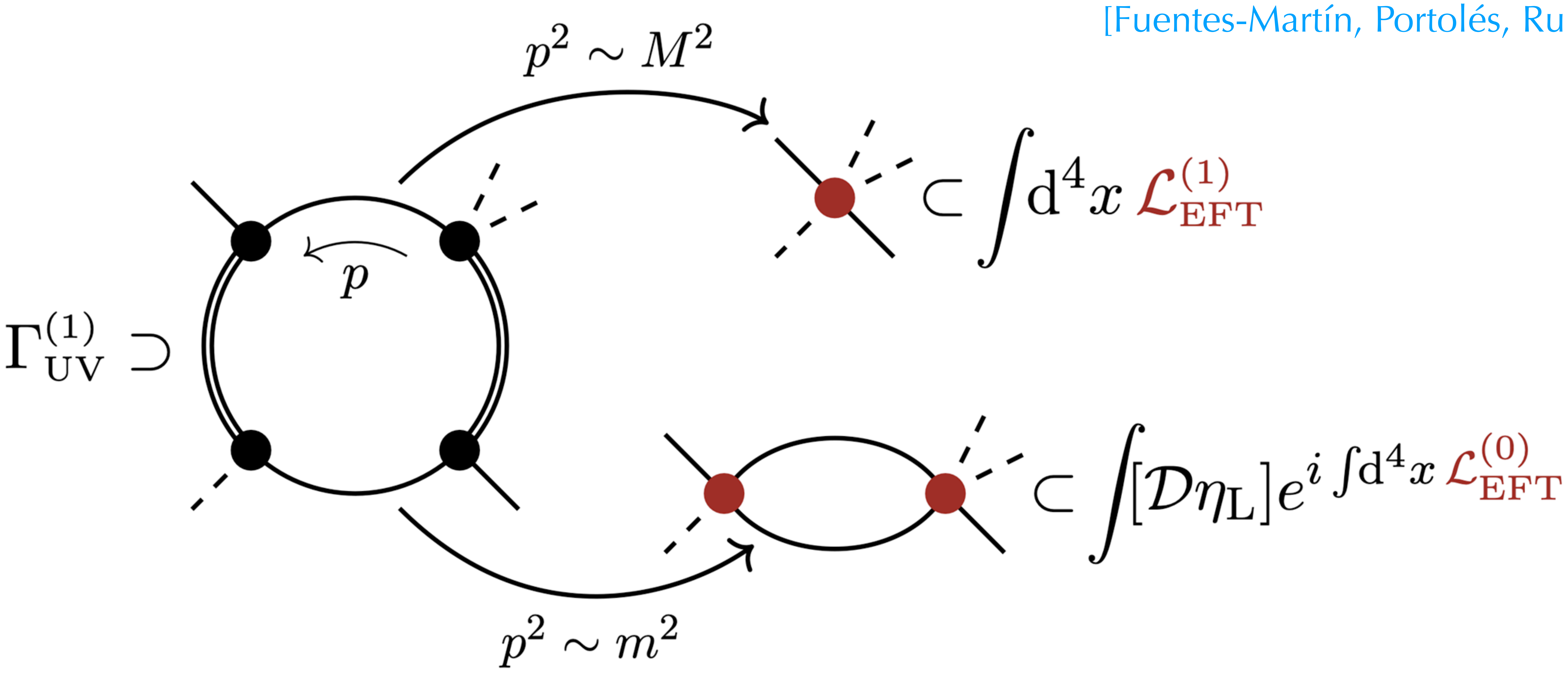
Heavy field loops

Mixed field loops

Light field loops

No need to subtract non-local EFT contributions in only the hard part of the loop is considered

[Fuentes-Martín, Portolés, Ruiz-Femenía, 1607.02142]



$$\Gamma^{(1)}_{\rm UV} \supset$$

$$\subset \int \mathrm{d}^4x\, \mathcal{L}^{(1)}_{\rm EFT}$$

$$\subset \int [\mathcal{D}\eta_{\rm L}]\, e^{i\int \mathrm{d}^4x\, \mathcal{L}^{(0)}_{\rm EFT}}$$

Automation pipeline and tools

Procedure:

1. Match UV theory to an EFT
2. Use RGE of the EFT to run to the scale of experiments
   - Possibly multiple matching steps
3. Compare EFT to data

➡ Variety of theories and complexity of computation require automation

partial automation

STrEAM
Cohen, Lu, Zhang [2012.07851]

MatchingTools
Criado [1710.06445]

CoDEx
Das Bakshi, Chakrabortty, Patra [1808.04403]

SUPER TRACER
Fuentes-Martin, König, Pages, Thomsen, FW [2012.08506]

full automation

Matchmakereft
Carmona, Lazopoulos, Olgoso, Santiago [2112.10787]
[diagrammatic technique]

MATCHETE
Fuentes-Martin, König, Pages, Thomsen, FW [in preparation]
[functional technique]