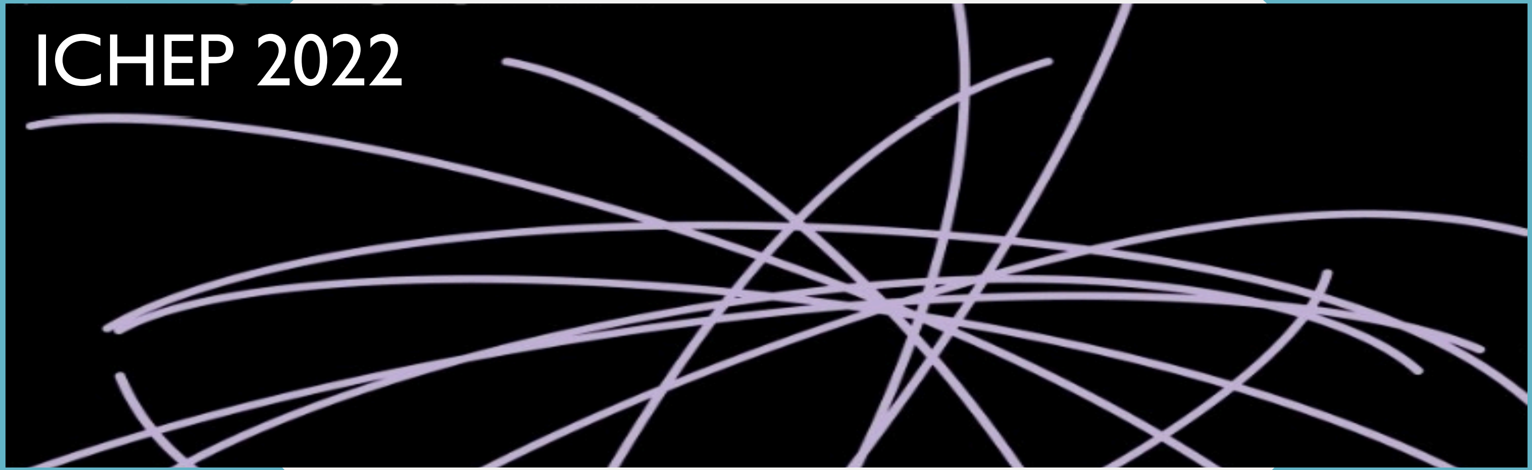
= QUANTUM ALGORITHM FOR QUERYING CAUSALITY OF MULTILOOP SCATTERING AMPLITUDES =



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Outline

- Loop-Tree Duality
  - Reformulation of LTD
  - Causal LTD representation
- Application of a quantum algorithm

LTD

- S. Catani, T. Gleisberg, F. Krauss, G. Rodrigo and J. C. Winter, “From loops to trees by-passing Feynman’s theorem,” JHEP 0809 (2008) 065.

- What does LTD do?

  Opens any loop diagram to a forest of connected trees.

- How does it do?

  Exploits the Cauchy residue theorem to reduce the dimension of the integration domain by one unit:

$$\int_{\boldsymbol{q}} \int dq_0 \prod_{j=1}^{N} G_F(q_j) = -2\pi i \int_{\vec{q}} \sum_{\boldsymbol{i}} Res_{\{Im\ q_{\{i,0\}}<0\}} \left[\prod_{j=1}^{N} G_F(q_j) \right]$$

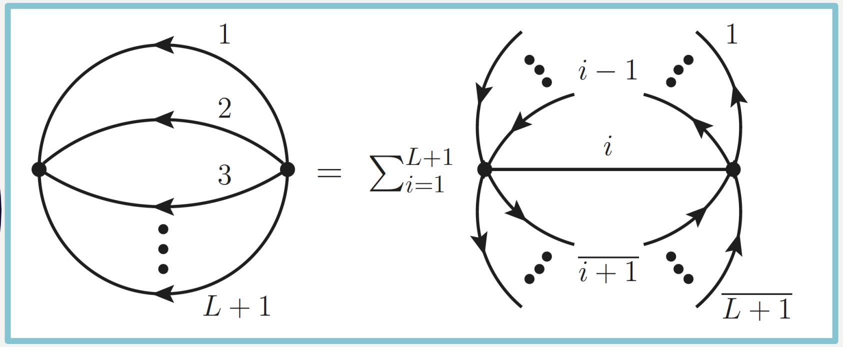
Minkowski space → Euclidean space

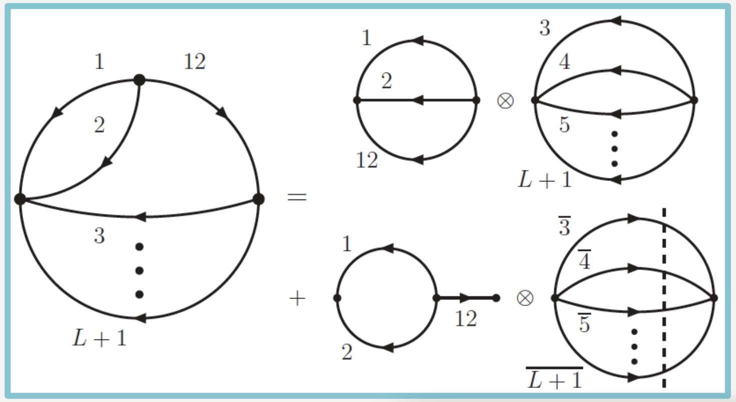
LTD - Opening

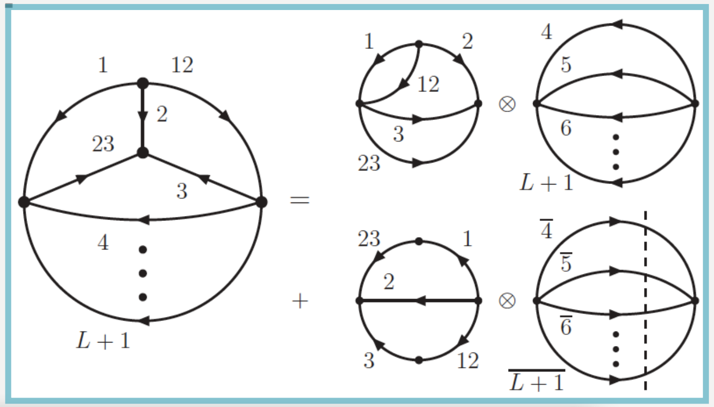
Nested residues

Reformulation of LTD to all orders

- Aguilera *et al.*, “Open loop amplitudes and causality to all orders and powers from the loop-tree duality”, Phys. Rev. Lett. 124 (2020) no.21, 211602.



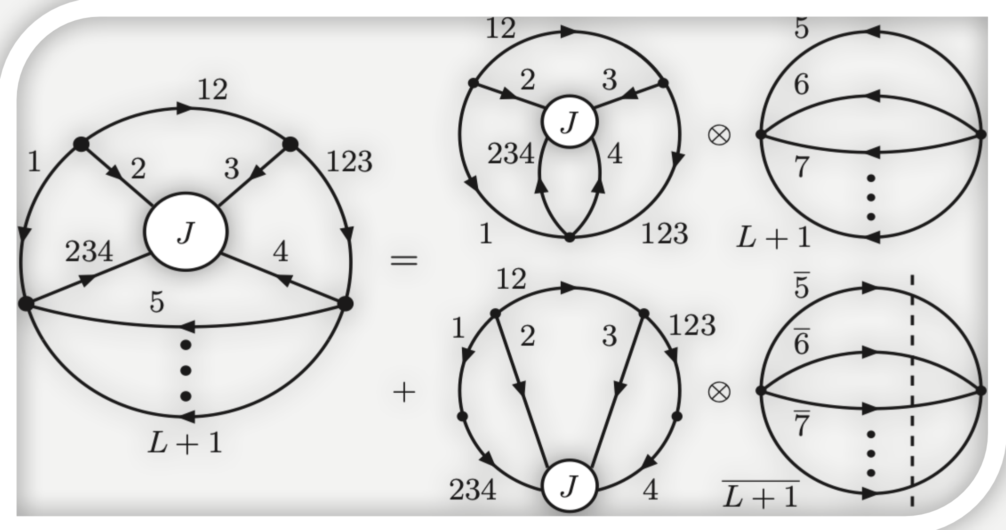




- S. Ramírez-Uribe *et al.*, “Universal opening of four-loop scattering amplitudes to trees”, JHEP 04, 129 (2021).

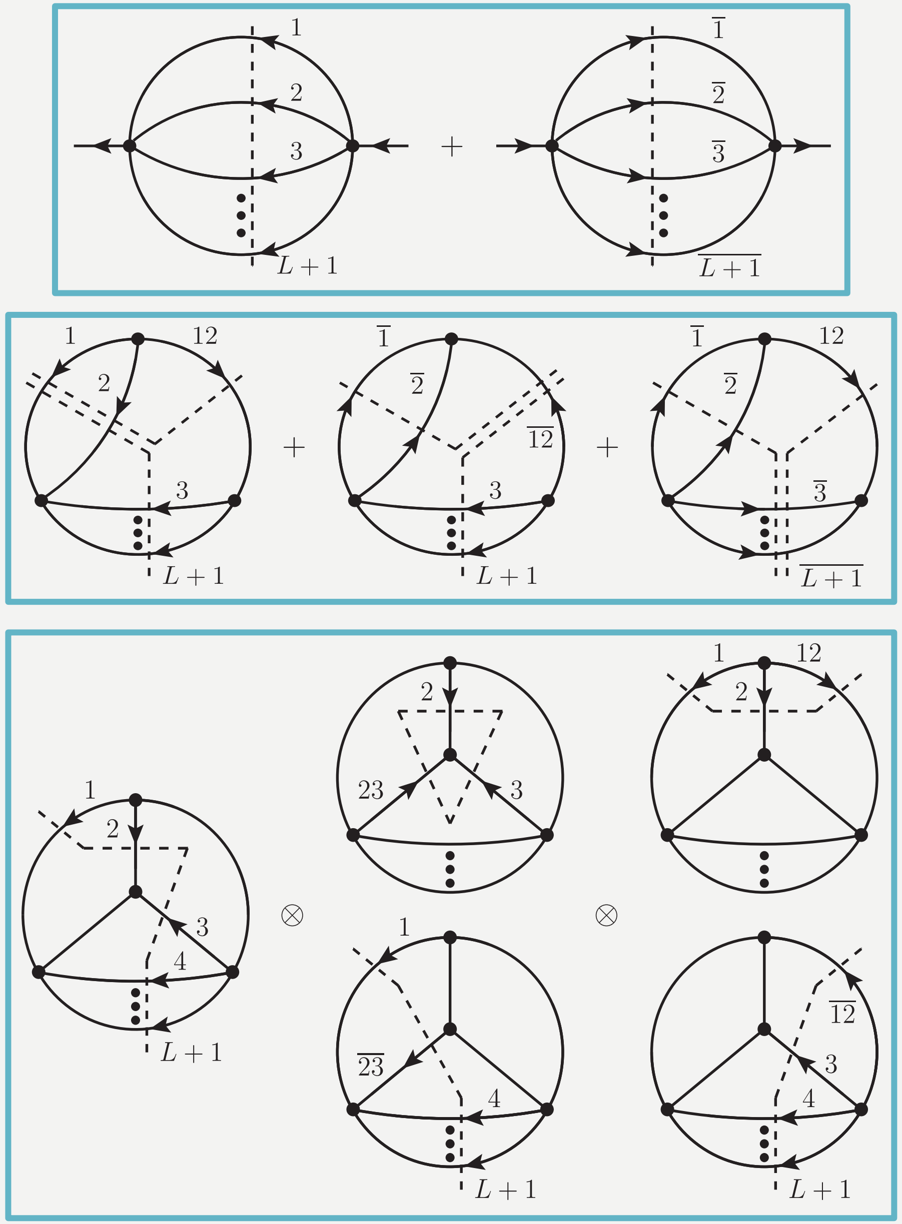
Unified description

$J \equiv 23 \cup 34 \cup 24$

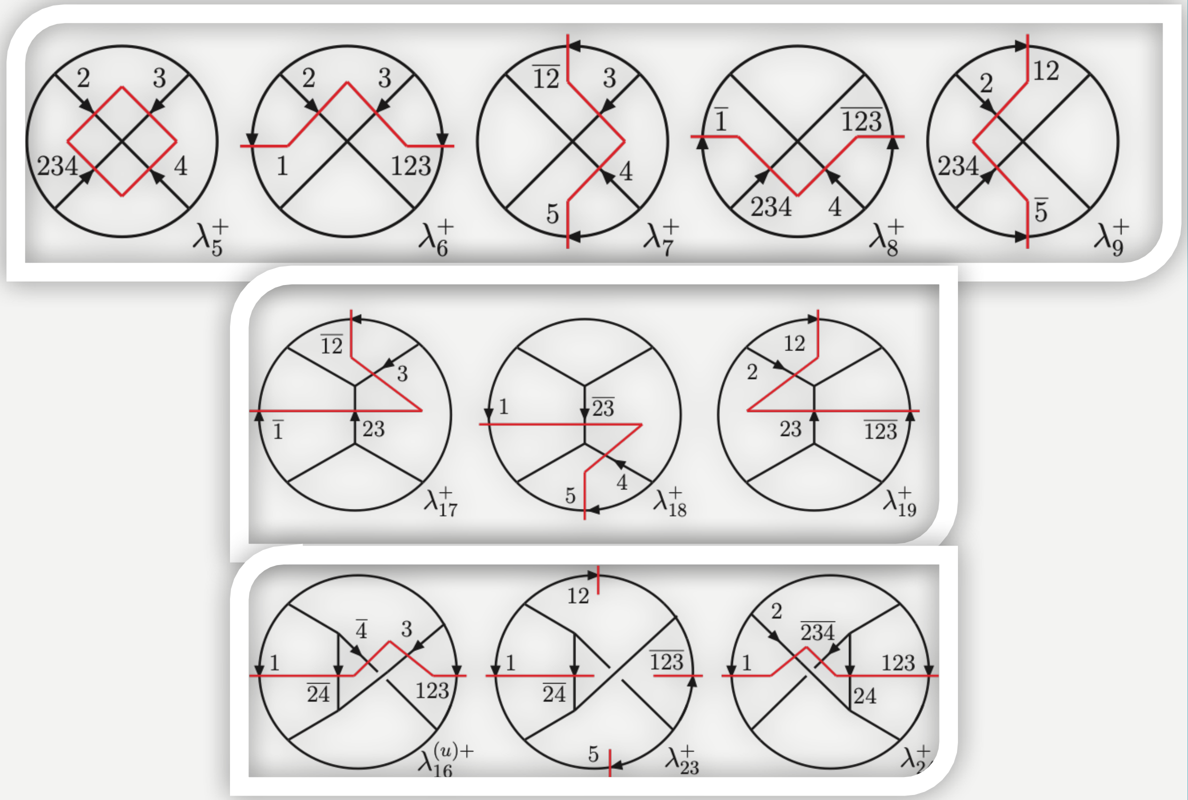


LTD - Causal representation

○ Aguilera, Hernández, Rodrigo, Sborlini, Torres, "Causal representation of multi-loop Feynman integrands within the loop-tree duality", JHEP 01, 069 (2021).



Causal representation is given in terms of products of causal propagators, interpreted as entangled thresholds.



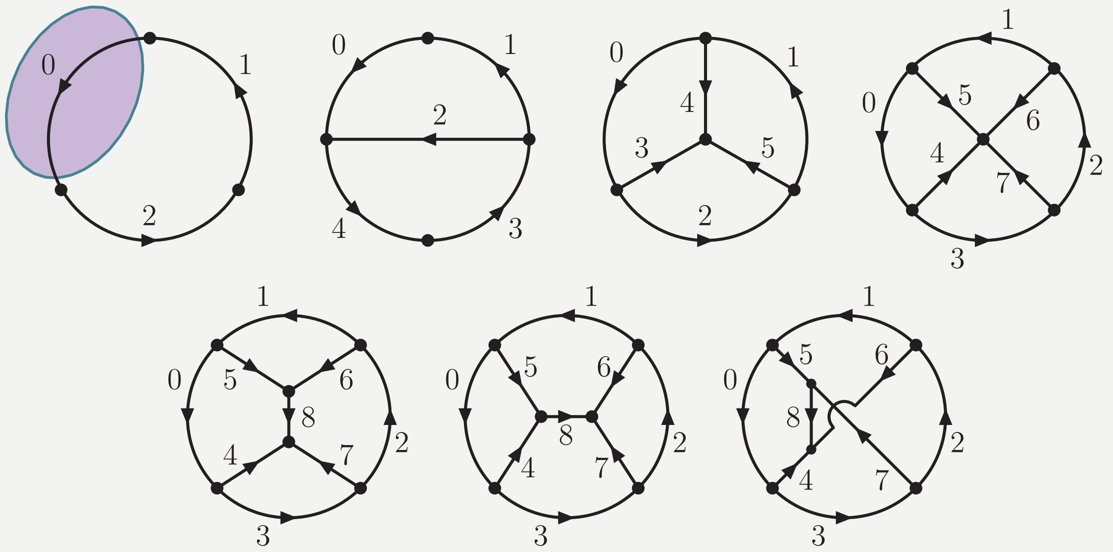
○ S. Ramírez-Uribe *et al.*, "Universal opening of four-loop scattering amplitudes to trees", JHEP 04, 129 (2021).

Quantum algorithm

Bootstrap the LTD causal representation of representative multiloop topologies.

- S. Ramírez-Uribe, A. E. Rentería-Olivo, G. Rodrigo, G. F. R. Sborlini, and L. Vale Silva, "Quantum algorithm for Feynman loop integrals" , JHEP 05, 100 (2022), arXiv:2105.08703 [hep-ph].

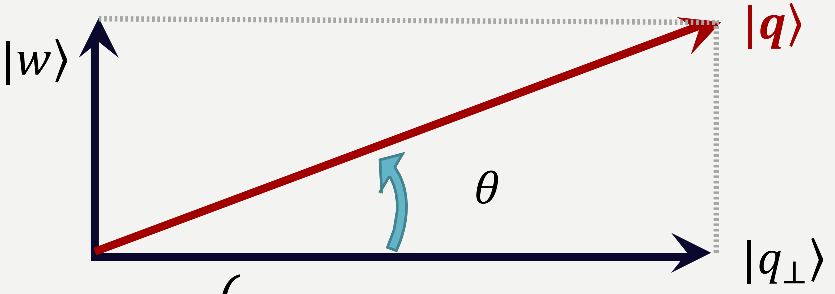
Two possible states:
$|1\rangle$ *or* $|0\rangle$



Grover's quantum algorithm

Quantum algorithm

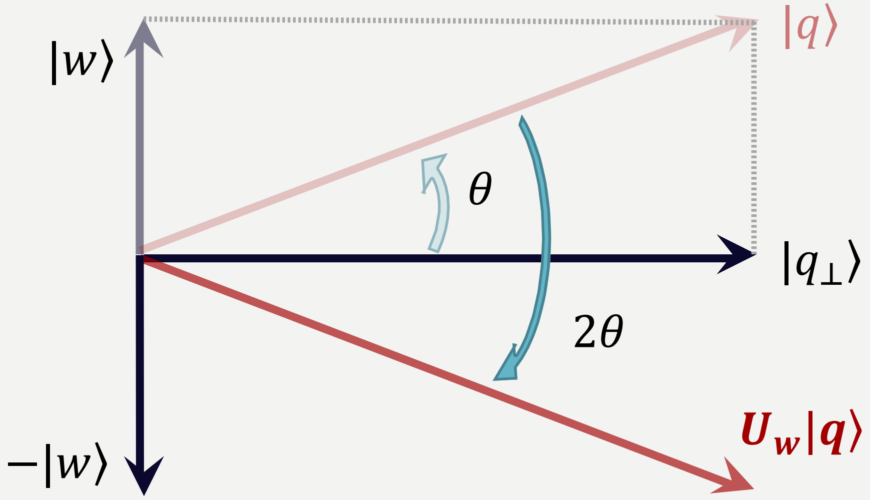
- Superposition



$$|q\rangle = \begin{cases} \frac{1}{\sqrt{N}} \sum |x\rangle \\ \cos\theta\, |q_\perp\rangle + \sin\theta\, |w\rangle \end{cases}$$

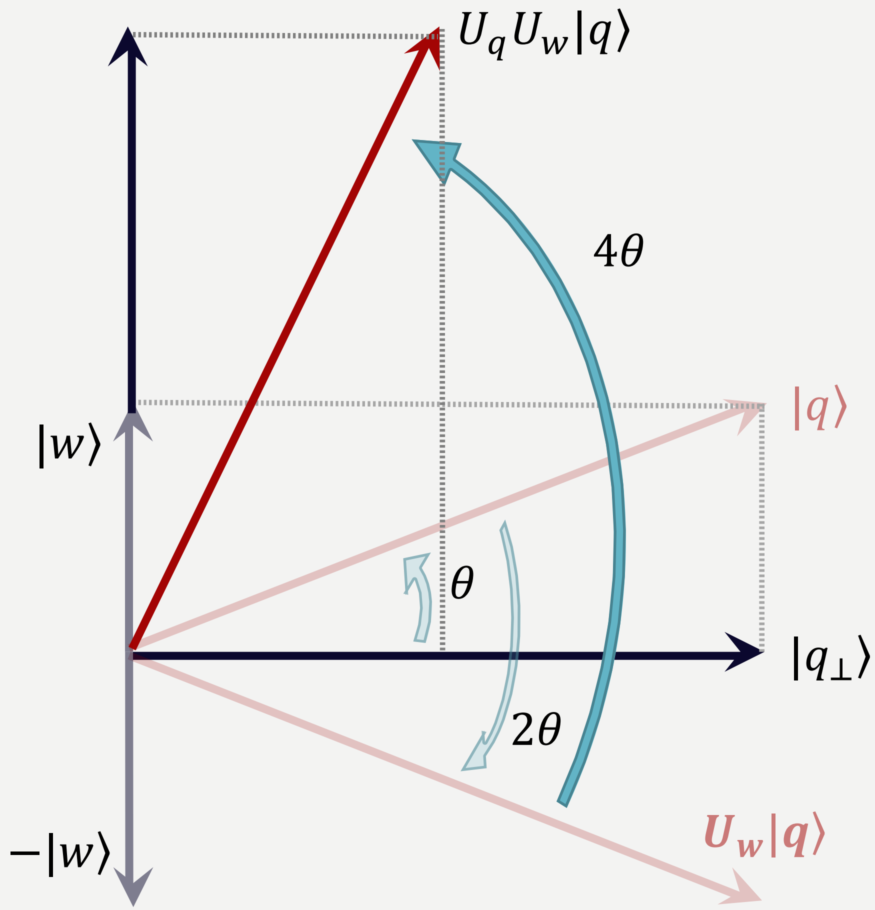
$|q_\perp\rangle$: Orthogonal state
$|w\rangle$: Winning state
θ: Mixing angle

- Oracle



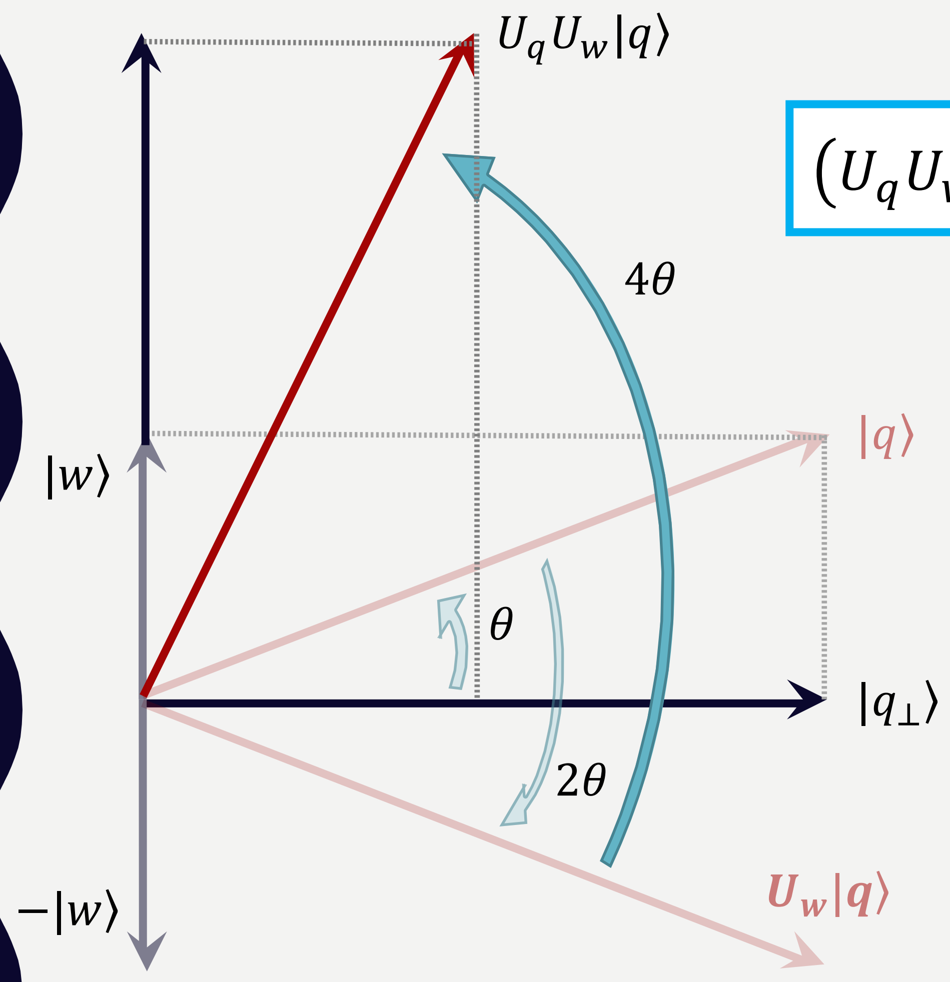
Flips the state $|x\rangle$ if $x \in |w\rangle$ and leaves it unchanged otherwise.

- Diffusion



Performs a reflection around the initial state $|q\rangle$

Quantum algorithm



$$(U_q U_w)^t |q\rangle = \cos\theta_t\, |q_\perp\rangle + \sin\theta_t\, |w\rangle \qquad \theta_t = (2t+1)\theta$$

To consider → $\theta \leq \pi/6$ ($r \leq N/4$)

What if $r > N/4$?

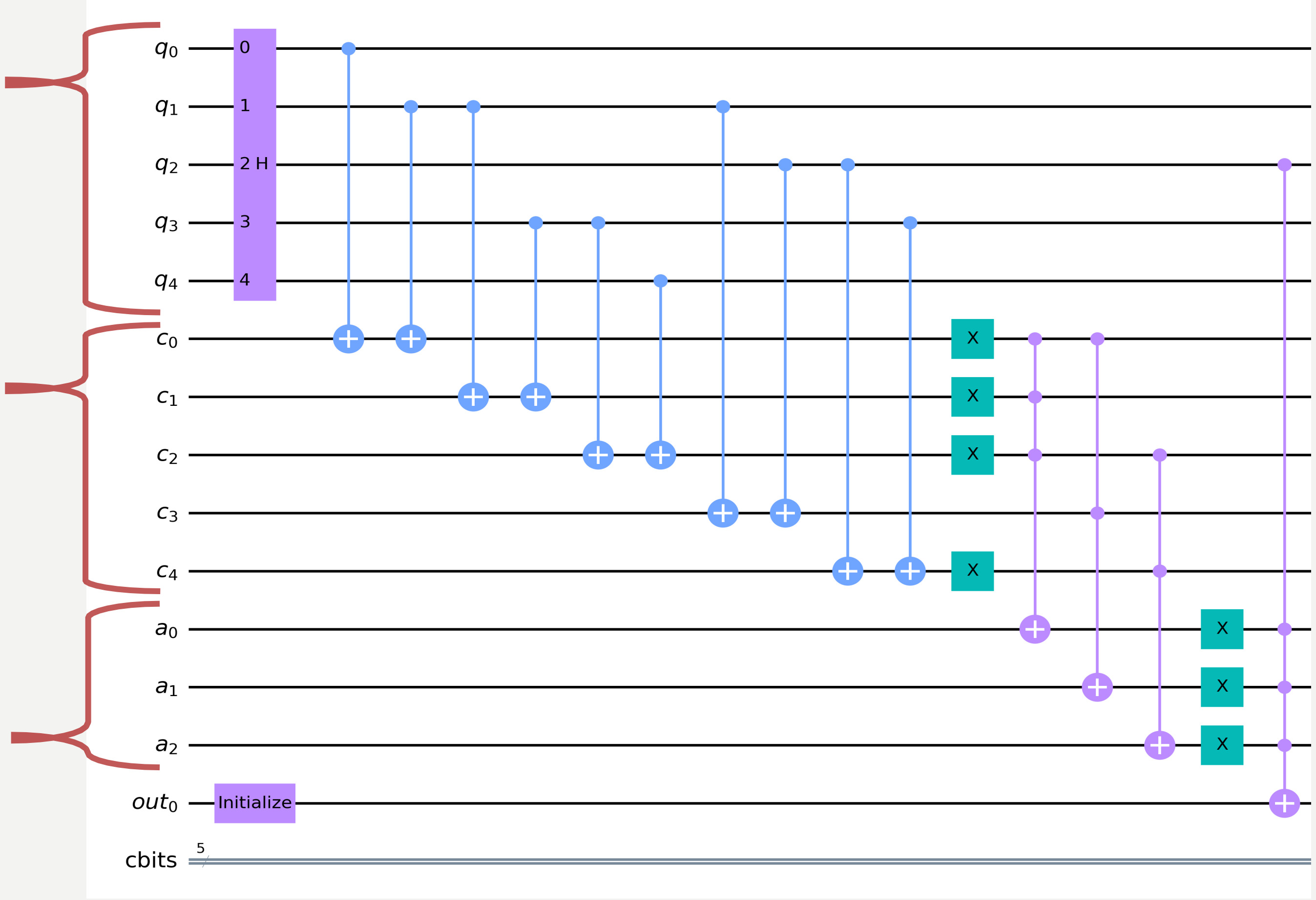
Quantum algorithm

$|q\rangle$ Encodes the states of the internal propagators.

$|c\rangle$ Stores binary clauses that probe if two adjacent edges are oriented in the same direction.

$$c_{ij} \equiv (q_i = q_j) \qquad \overline{c_{ij}} \equiv (q_i \neq q_j)$$

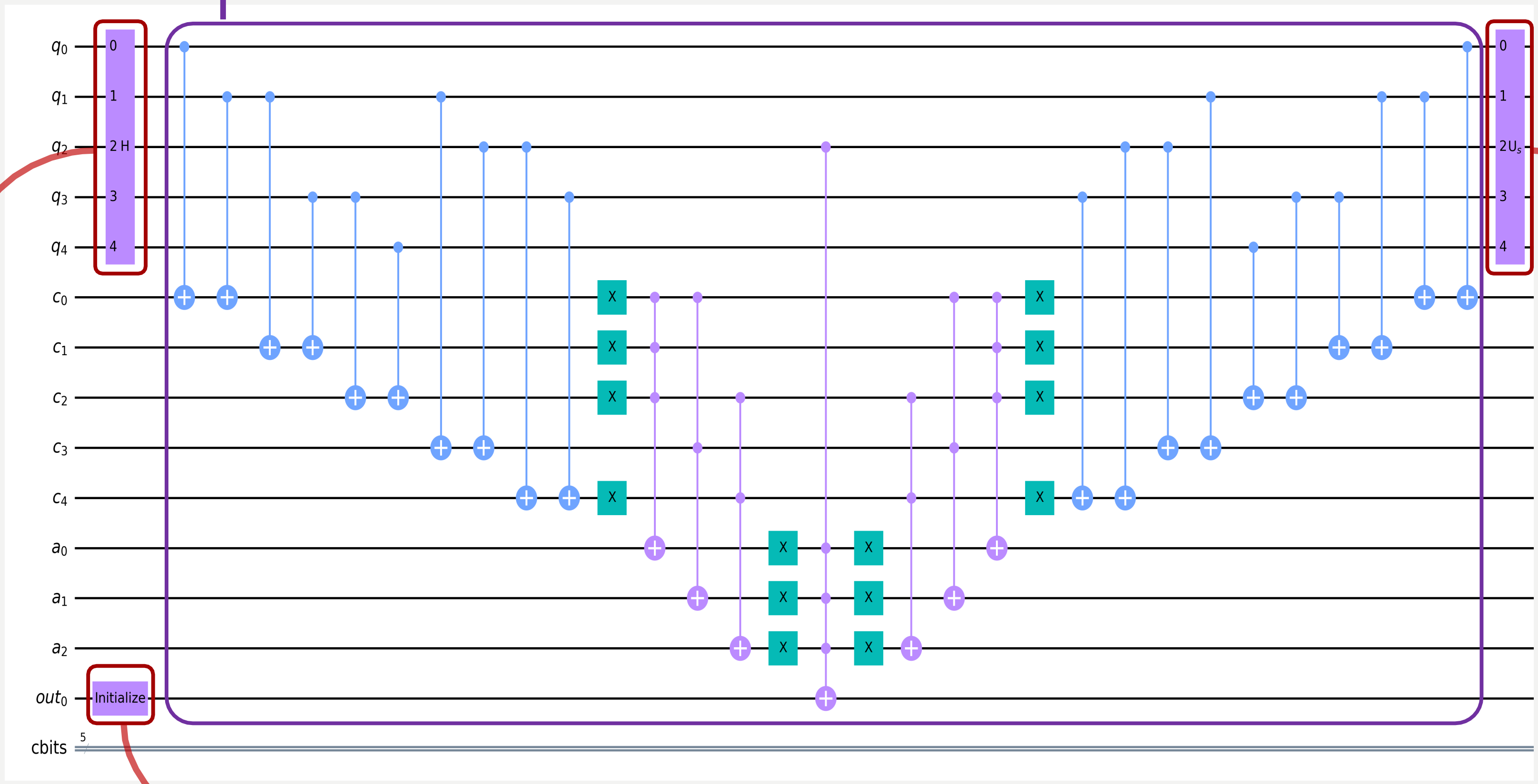
$|a\rangle$ Stores the loop clauses probing if all the qubits in each subloop form a cyclic circuit.



Quantum algorithm

The oracle $\Longrightarrow$ $U_w|q\rangle|c\rangle|a\rangle|out_0\rangle = \begin{cases} |out_0\rangle \\ -|out_0\rangle \end{cases}$

Superposition $\Rightarrow H|q\rangle$

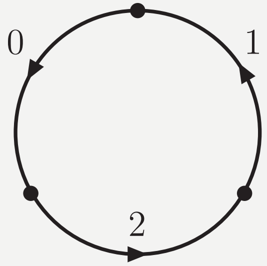


Diffuser U_q

The Grover's marker initialized to the Bell state $|out_0\rangle = |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$

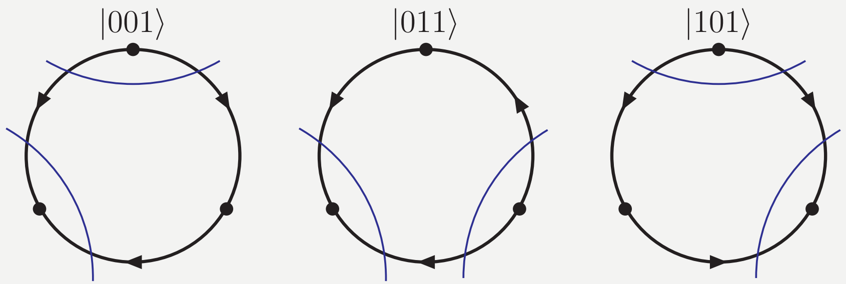
Quantum algorithm

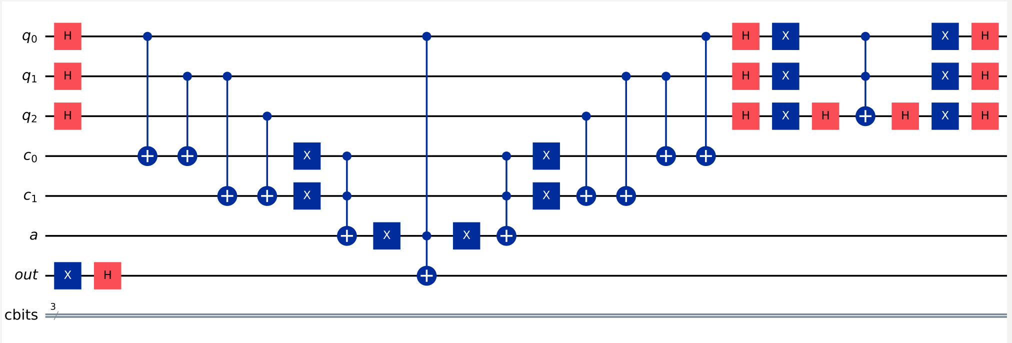
Qiskit

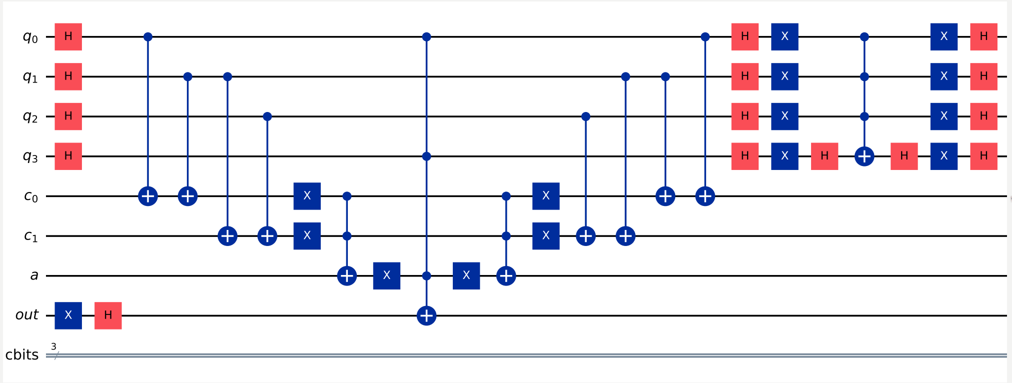


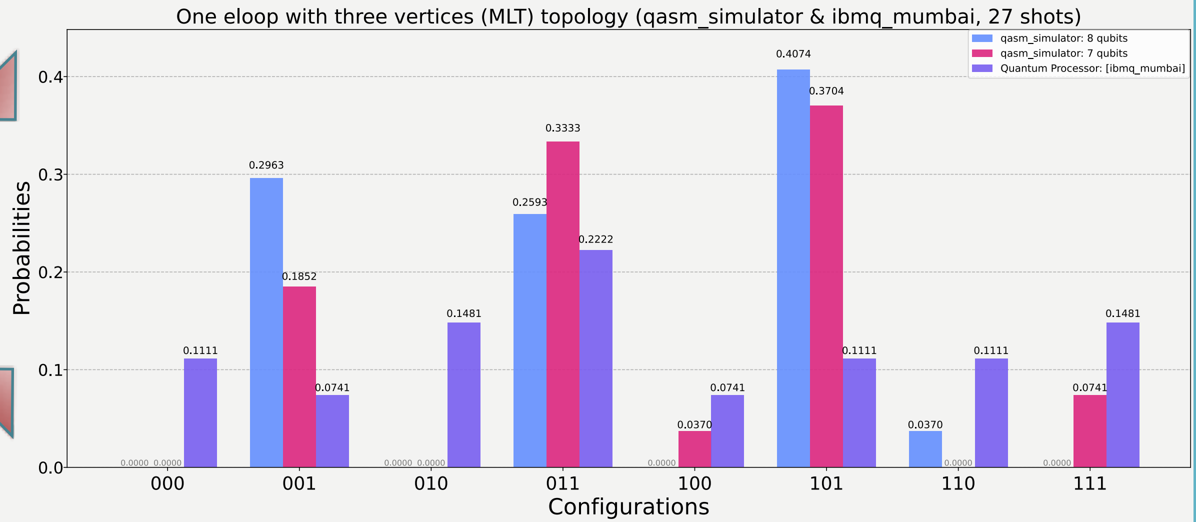
$$a_0 = \neg\,(c_{01} \wedge c_{12})$$
$$f^{(1)}(a, q) = a_0 \wedge q_0 \wedge q_3$$

ancillary qubit

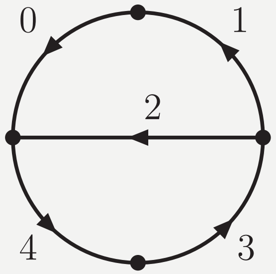






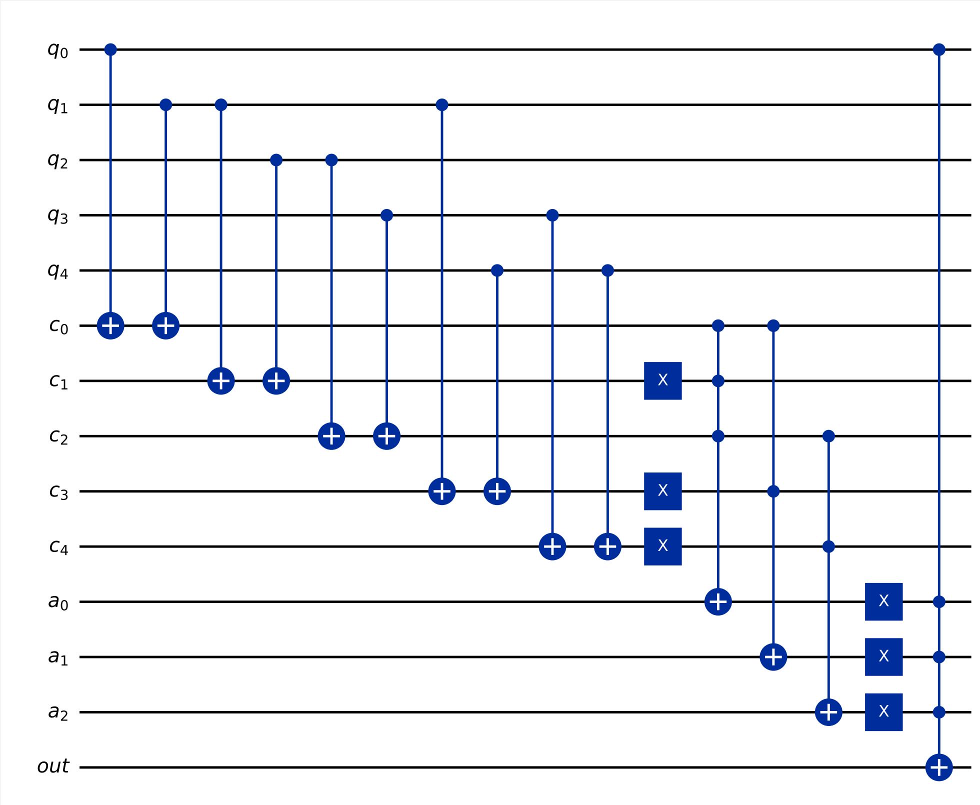


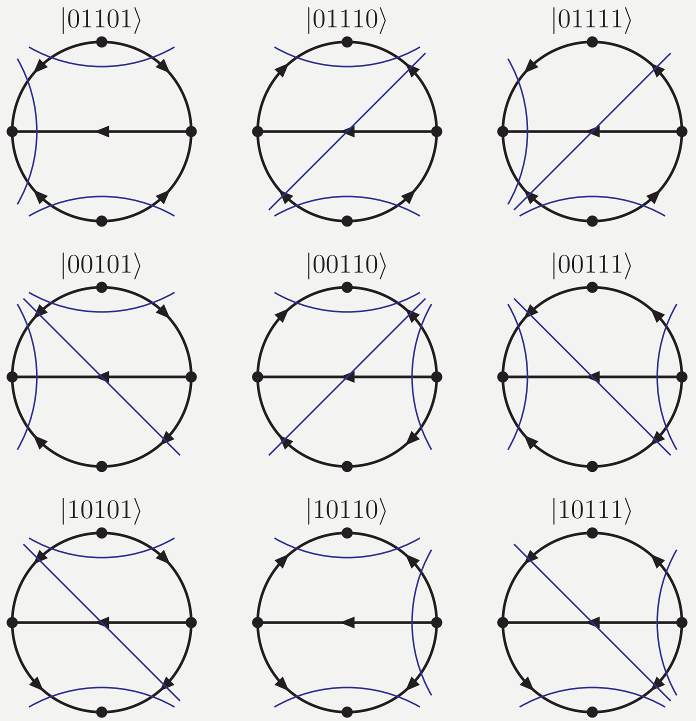
Quantum algorithm

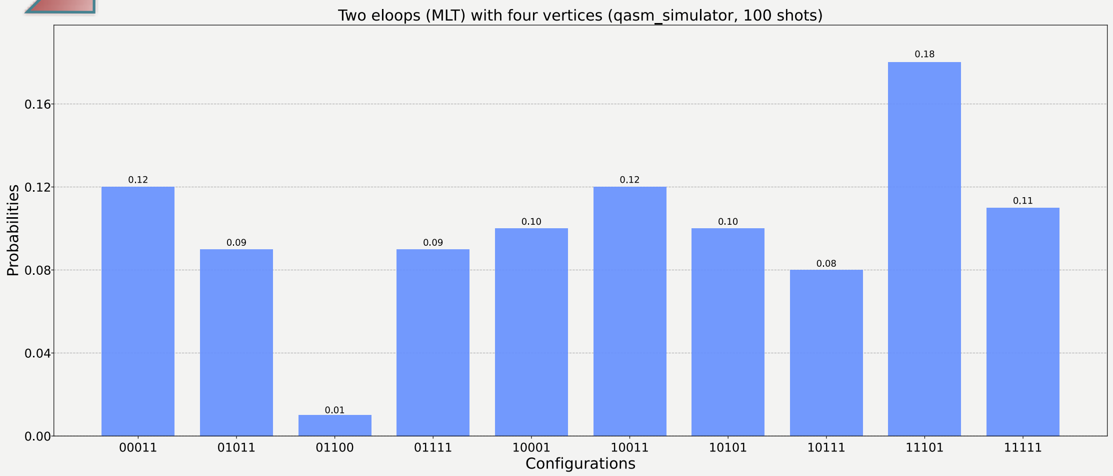


$$a_0 = \neg\,(c_{01} \wedge c_{13} \wedge c_{34})$$
$$a_1 = \neg\,(c_{01} \wedge \bar{c}_{12})$$
$$a_2 = \neg\,(c_{23} \wedge c_{34})$$

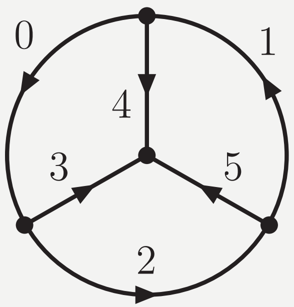
$$f^{(2)}(a,q) = (a_0 \wedge a_1 \wedge a_2) \wedge q_2$$





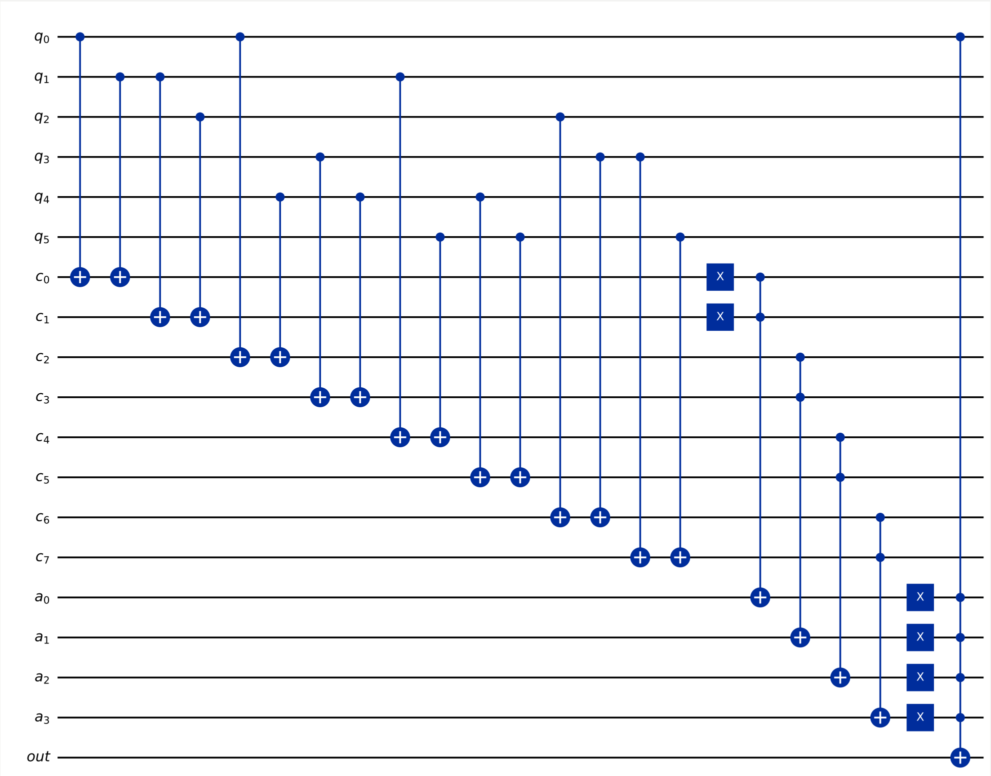


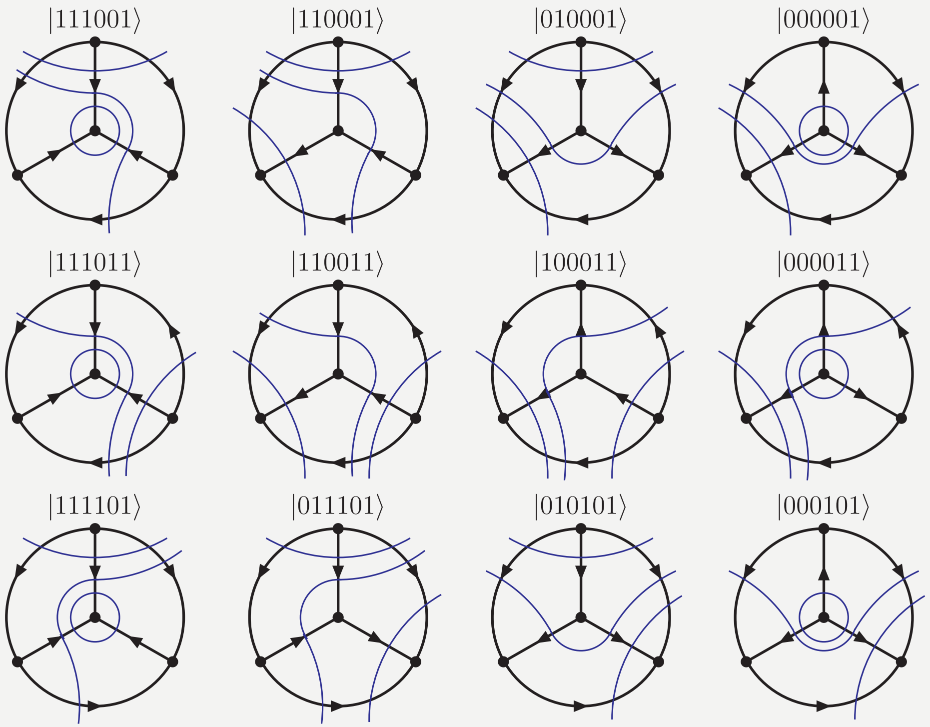
Quantum algorithm

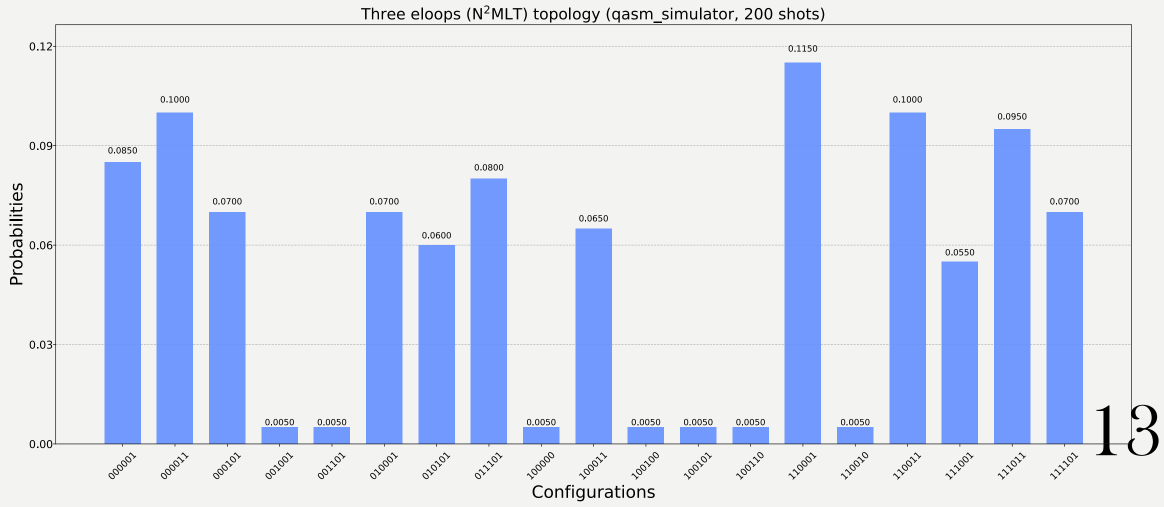


$$a_0 = \neg\,(c_{01} \wedge c_{12})$$
$$a_1 = \neg\,(\bar{c}_{04} \wedge \bar{c}_{34})$$
$$a_2 = \neg\,(\bar{c}_{15} \wedge \bar{c}_{45})$$
$$a_3 = \neg\,(\bar{c}_{23} \wedge \bar{c}_{35})$$

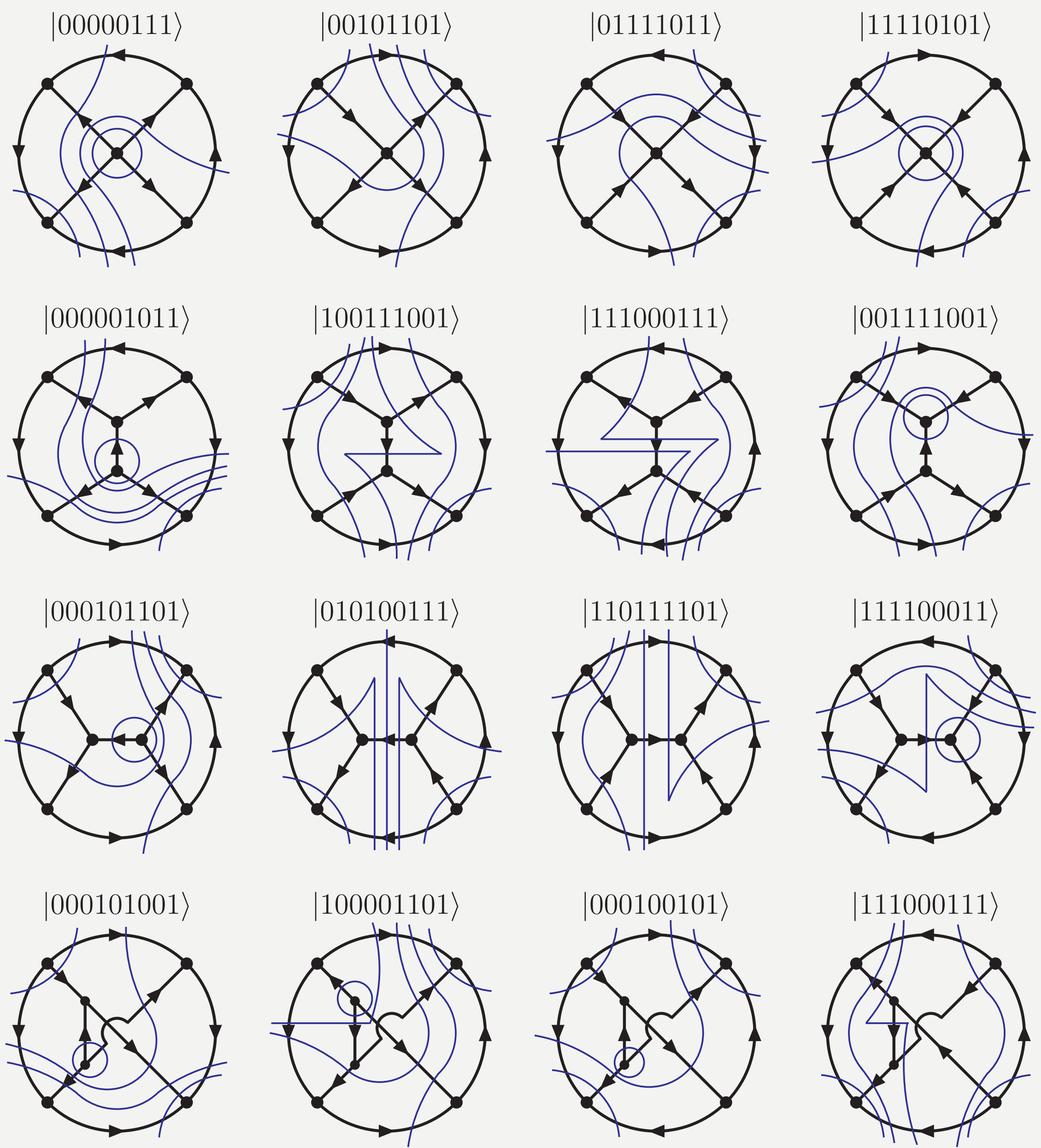
$$f^{(3)}(a,q) = (a_0 \wedge a_1 \wedge a_2 \wedge a_3) \wedge q_0$$







Quantum algorithm

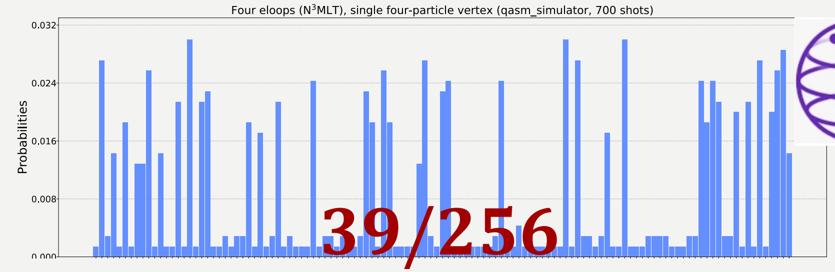


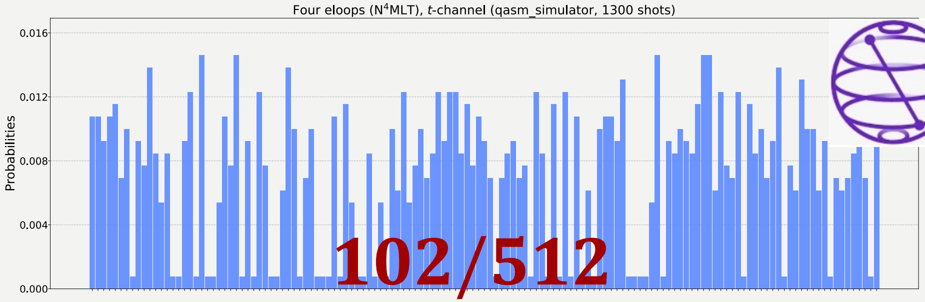
N^3MLT

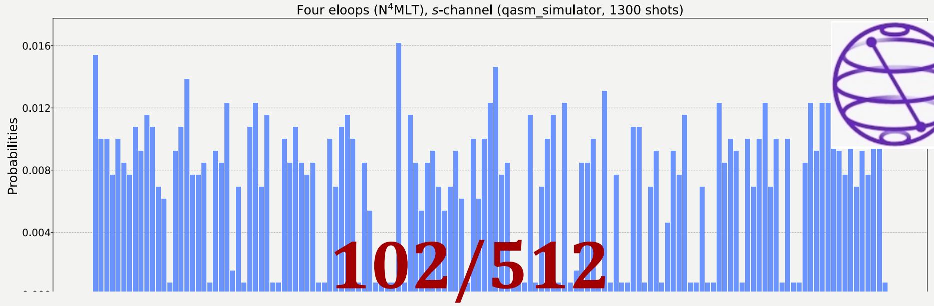
$t-channel$

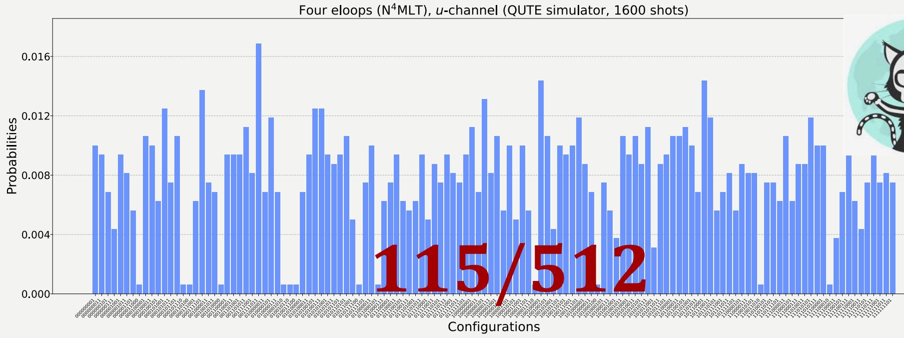
$s-channel$

$u-channel$









Conclusions

- We have applied a **quantum algorithm** to Feynman loop integrals.
- A modification of Grover's quantum algorithm was implemented in two quantum simulators, **IBM Qiskit** and **QUTE** by CTIC.
- Causal singular configurations of multiloop Feynman integrals have been efficiently identified.


Gracias!