

Dark Phase Transition and Gravitational Wave of Strongly Coupled Hidden Sectors

Zhi-Wei Wang

Lund University, Sweden

July 7th, 2022

ICHEP 2022, July 6th - 13th, Bologna

Motivations and what we do

- (Dark) composite dynamics: non perturbative physics, dynamical symmetry breaking, UV completion, naturalness
- (Dark) composite dynamics face challenges to be explored both theoretically and via experiments and thus any extra test is important
- We unify first principle lattice simulations and gravitational wave astronomy to constrain the dark sector

What composes the strongly coupled sector?

- Dark Yang-Mills theories
- Pure gluons \Rightarrow confinement-deconfinement phase transition
- Gluons + Fermions
 - Fermions in fundamental representation \Rightarrow chiral phase transition
 - Fermions in adjoint rep. \Rightarrow confinement & chiral phase transition
 - Fermions in 2-index symmetric rep. \Rightarrow confinement & chiral phase transition
- Gluons + Fermions + Scalars (not explored yet)

How to describe the strongly coupled sector?

- Pure gluons

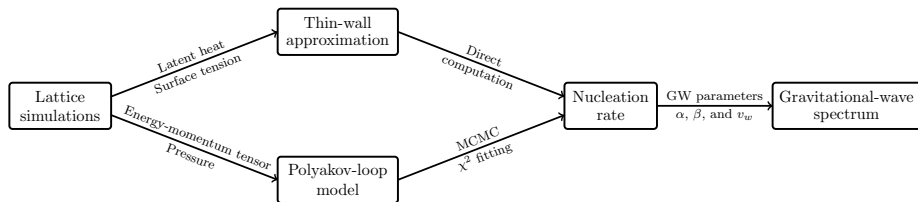
- Polyakov loop model (Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005; Kang, Zhu, Matsuzaki, JHEP 09 (2021) 060)
- Matrix Model (Halverson, Long, Maiti, Nelson, Salinas, JHEP **05** (2021) 154)
- Holographic QCD model (Ares, Henriksson, Hindmarsh, Hoyos, Jokela, PRD **105** (2022) 066020)

- Gluons + Fermions

- Polyakov loop improved Nambu-Jona-Lasinio model (Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003; Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
- linear sigma model (Helmboldt, Kubo, Woude, PRD **100** (2019) 055025)
- Polyakov Quark Meson model (Schaefer, Pawłowski, Wambach, PRD **76** (2007) 074023)

Procedure of pure gluon case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)



Polyakov Loop Model for Pure Gluons: I

- Pisarski first proposed the Polyakov-loop Model as an effective field theory to describe the confinement-deconfinement phase transition of $SU(N)$ gauge theory (Pisarski, PRD **62** (2000) 111501).
- In a local $SU(N)$ gauge theory, a **global center symmetry $Z(N)$** is used to distinguish confinement phase (unbroken phase) and deconfinement phase (broken phase)
- An order parameter for the $Z(N)$ symmetry is constructed using the Polyakov Loop (thermal Wilson line) (Polyakov, PLB **72** (1978) 477)

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) d\tau \right]$$

The symbol \mathcal{P} denotes path ordering and A_4 is the Euclidean temporal component of the gauge field

- The Polyakov Loop transforms like an adjoint field under local $SU(N)$ gauge transformations

Polyakov Loop Model for Pure Gluons: II

- Convenient to define the trace of the **Polyakov loop as an order parameter** for the $Z(N)$ symmetry

$$\ell(\vec{x}) = \frac{1}{N} \text{Tr}_c[\mathbf{L}],$$

where Tr_c denotes the trace in the colour space.

- Under a global $Z(N)$ transformation, the Polyakov loop ℓ transforms as a field with charge one

$$\ell \rightarrow e^{i\phi} \ell, \quad \phi = \frac{2\pi j}{N}, \quad j = 0, 1, \dots, (N-1)$$

- The expectation value of ℓ i.e. $\langle \ell \rangle$ has the **important property**:

$$\langle \ell \rangle = 0 \quad (T < T_c, \text{ Confined}); \quad \langle \ell \rangle > 0 \quad (T > T_c, \text{ Deconfined})$$

- At very high temperature, the vacua exhibit a N -fold degeneracy:

$$\langle \ell \rangle = \exp\left(i \frac{2\pi j}{N}\right) \ell_0, \quad j = 0, 1, \dots, (N-1)$$

where ℓ_0 is defined to be real and $\ell_0 \rightarrow 1$ as $T \rightarrow \infty$

Summary of Pure Gluon Facts

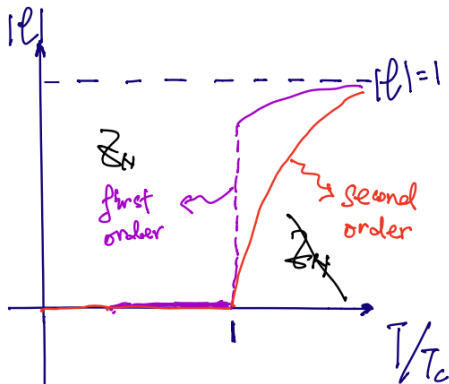
Temperature

Free Gluon
 Z_4 is broken

At $T_{\text{confinement}}$

Confinement
glue ball

Z_4 is restored



Second Order for $SU(2)$
First order $SU(N)$ ($N \geq 3$)

Effective Potential of the Polyakov Loop Model: I

- The simplest effective potential preserving the Z_N symmetry in the polynomial form is given by (Pisarski, PRD **62** (2000) 111501)

$$V_{\text{PLM}}^{(\text{poly})} = T^4 \left(-\frac{b_2(T)}{2} |\ell|^2 + b_4 |\ell|^4 + \dots - b_3 (\ell^N + \ell^{*N}) \right)$$

$$\text{where } b_2(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3 + a_4 \left(\frac{T_0}{T} \right)^4$$

"..." represent any required lower dimension operator than ℓ^N

- For the $SU(3)$ case, there is also an alternative logarithmic form

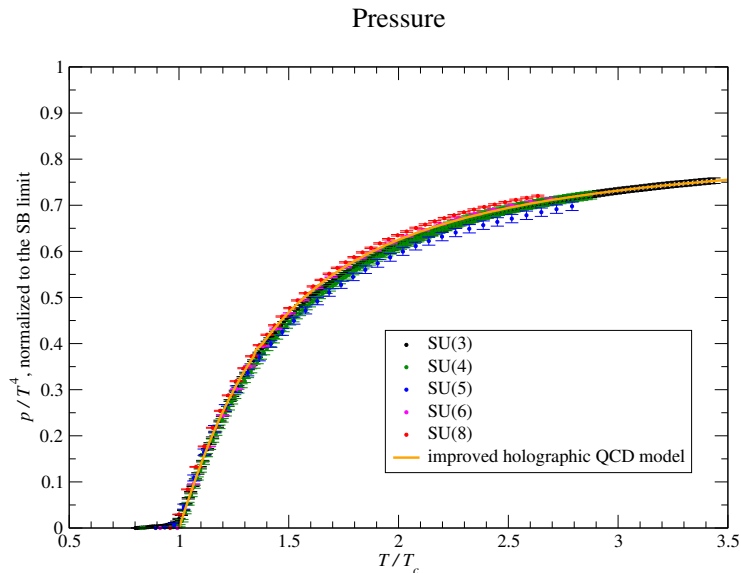
$$V_{\text{PLM}}^{(3\log)} = T^4 \left(-\frac{a(T)}{2} |\ell|^2 + b(T) \ln(1 - 6|\ell|^2 + 4(\ell^{*3} + \ell^3) - 3|\ell|^4) \right)$$

$$a(T) = a_0 + a_1 \left(\frac{T_0}{T} \right) + a_2 \left(\frac{T_0}{T} \right)^2 + a_3 \left(\frac{T_0}{T} \right)^3, \quad b(T) = b_3 \left(\frac{T_0}{T} \right)^3$$

- The a_i, b_i coefficients in $V_{\text{PLM}}^{(\text{poly})}$ and $V_{\text{PLM}}^{(3\log)}$ are determined by fitting the lattice results

Fitting the Coefficients Using the Lattice Results: I

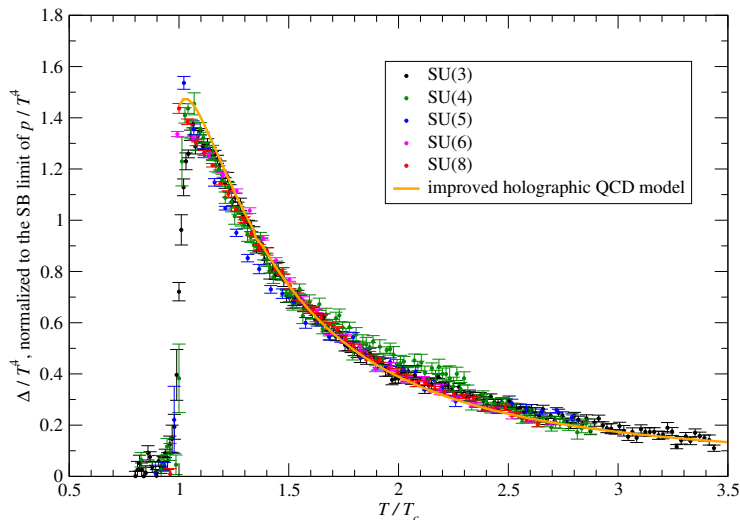
Marco Panero, Phys.Rev.Lett. 103 (2009) 232001



Fitting the Coefficients Using the Lattice Results: II

Marco Panero, Phys.Rev.Lett. 103 (2009) 232001

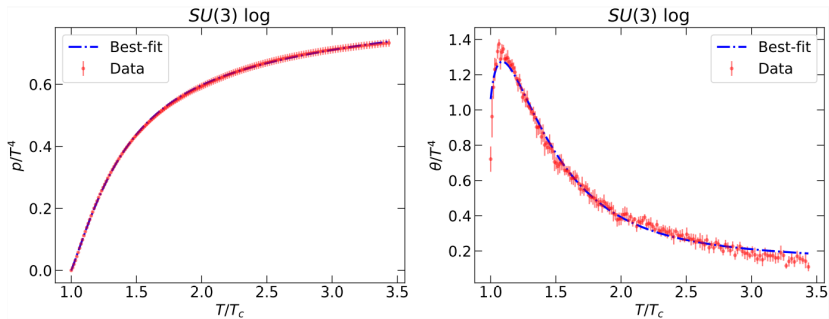
Trace of the energy-momentum tensor



Fitting the Coefficients Using the Lattice Results: III

(Huang, Reichert, Sannino and Z-WW, PRD **104** (2021) 035005)

Fitted to lattice data of pressure and the trace of energy momentum tensor.



Fitting the Coefficients Using the Lattice Results: IV

(Huang, Reichert, Sannino and Z-WW, PRD **104** (2021) 035005)

Table: The parameters for the best-fit points.

N	3	$3 \log$	4	5	6	8
a_0	3.72	4.26	9.51	14.3	16.6	28.7
a_1	-5.73	-6.53	-8.79	-14.2	-47.4	-69.8
a_2	8.49	22.8	10.1	6.40	108	134
a_3	-9.29	-4.10	-12.2	1.74	-147	-180
a_4	0.27		0.489	-10.1	51.9	56.1
b_3	2.40	-1.77		-5.61		
b_4	4.53		-2.46	-10.5	-54.8	-90.5
b_6			3.23		97.3	157
b_8					-43.5	-68.9

Include Fermions

(K. Fukushima, PLB **591** (2004) 277; Ratti, Thaler Weise, PRD **73** (2006) 014019)

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.

- The Polyakov-loop-Nambu-Jona-Lasinio (PNJL) model is used to describe phase-transition dynamics in dark gauge-fermion sectors
- The **finite-temperature grand potential** of the PNJL models can be generically written as

$$V_{\text{PNJL}} = V_{\text{PLM}}[\ell, \ell^*] + V_{\text{cond}}[\langle\bar{\psi}\psi\rangle] + V_{\text{zero}}[\langle\bar{\psi}\psi\rangle] + V_{\text{medium}}[\langle\bar{\psi}\psi\rangle, \ell, \ell^*]$$

- $V_{\text{PLM}}[\ell, \ell^*]$ is the Polyakov loop model potential (discussed above)
- $V_{\text{cond}}[\langle\bar{\psi}\psi\rangle]$ represents the condensate energy
- $V_{\text{zero}}[\langle\bar{\psi}\psi\rangle]$ denotes the fermion zero-point energy
- The medium potential $V_{\text{medium}}[\langle\bar{\psi}\psi\rangle, \ell, \ell^*]$ encodes the interactions between the chiral and gauge sector which arises from an integration over the quark fields coupled to a background gauge field

- The PNJL Lagrangian can be generically written as:

$$\mathcal{L}_{\text{PNJL}} = \mathcal{L}_{\text{pure-gauge}} + \mathcal{L}_{4\text{F}} + \mathcal{L}_{6\text{F}} + \mathcal{L}_k$$

- Without losing generality, we consider below massless 3-flavour case in fundamental representation of $SU(3)$ gauge symmetry
- Here, $\mathcal{L}_{4\text{F}}$ is the four-quark interaction which reads:

$$\mathcal{L}_{4\text{F}} = G_S \sum_{a=0}^8 [(\bar{\psi} \lambda^a \psi)^2 + (\bar{\psi} i \gamma^5 \lambda^a \psi)^2], \quad \psi = (u, d, s)^T$$

- Six-fermion interaction $\mathcal{L}_{6\text{F}}$ denotes the Kobayashi-Maskawa-'t Hooft (KMT) term breaking $U(1)_A$ down to Z_3 (generically Z_{N_f} for N_f flavours)

$$\mathcal{L}_{6\text{F}} = G_D [\det(\bar{\psi}_{Li} \psi_{Rj}) + \det(\bar{\psi}_{Ri} \psi_{Lj})]$$

The Condensate Energy and Zero Point Energy

(Fukushima, Skokov, PPNP 96 (2017) 154)

- The total condensate energy is extracted from \mathcal{L}_{4F} and \mathcal{L}_{6F} using mean field approximation:

$$V_{\text{cond}} = 6G_S\sigma^2 + \frac{1}{2}G_D\sigma^3, \quad \sigma \equiv \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle = \frac{1}{3}\langle \bar{\psi}\psi \rangle$$

- The $2\langle \bar{u}u \rangle \bar{u}u$ and $\langle \bar{u}u \rangle^2 \bar{u}u$ terms from \mathcal{L}_{4F} and \mathcal{L}_{6F} respectively contribute to the constituent quark mass of u
- The total constituent quark mass from \mathcal{L}_{4F} and \mathcal{L}_{6F} is:

$$M = -4G_S\sigma - \frac{1}{4}G_D\sigma^2$$

- The expression for the zero-point energy is given by:

$$V_{\text{zero}}[\langle \bar{\psi}\psi \rangle] = -\dim(R) 2N_f \int \frac{d^3p}{(2\pi)^3} E_p, \quad E_p = \sqrt{\vec{p}^2 + M^2}$$

E_p is the energy of a free quark with constituent mass M and three-momentum \vec{p} and a sharp three-momentum cutoff Λ is chosen

Medium Potential: Finite Temperature Contribution

- In the standard NJL model, the medium effect (finite temperature contribution) is implemented by the grand canonical partition function
- In the PNJL model, we can simply do the following replacement to include the contribution from Polyakov loop

$$V_{\text{medium}} = -2N_c T \sum_{u,d,s} \int \frac{d^3 p}{(2\pi)^3} \left(\ln \left[1 + e^{-\beta(E-\mu)} \right] + \ln \left[1 + e^{-\beta(E+\mu)} \right] \right) \\ \rightarrow -2T \sum_{u,d,s} \int \frac{d^3 p}{(2\pi)^3} \text{Tr}_c \left\{ \left(\ln \left[1 + \mathbf{L} e^{-\beta(E-\mu)} \right] + \ln \left[1 + \mathbf{L}^\dagger e^{-\beta(E+\mu)} \right] \right) \right\}$$

- \mathbf{L} is the Polyakov loop:

$$\mathbf{L}(\vec{x}) = \mathcal{P} \exp \left[i \int_0^{1/T} A_4(\vec{x}, \tau) d\tau \right]$$

- Summary of parameters: G_S, G_D, Λ ; Observables: M, f_π, m_σ

Second Part: Bubble Nucleation and Gravitational Wave

Bubble Nucleation: Generic Discussion

- In a first-order phase transition, the transition occurs via bubble nucleation and it is essential to compute the nucleation rate
- The tunnelling rate due to thermal fluctuations from the metastable vacuum to the stable one is suppressed by the three-dimensional Euclidean action $S_3(T)$

$$\Gamma(T) = T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2} e^{-S_3(T)/T}$$

- The generic three-dimensional Euclidean action reads

$$S_3(T) = 4\pi \int_0^\infty dr r^2 \left[\frac{1}{2} \left(\frac{d\rho}{dr} \right)^2 + V_{\text{eff}}(\rho, T) \right],$$

where ρ denotes a generic scalar field with mass dimension one, $[\rho] = 1$

Bubble Nucleation: Confinement Phase Transition

(Huang, Reichert, Sannino and Z-WW, PRD **104** (2021) 035005)

- Confinement phase transition occurs for pure gluon and adjoint fermions
- $[\ell] = 0$ dimensionless while $[\rho] = 1$, we rewrite ρ as $\rho = \ell T$ and convert the radius into a dimensionless quantity $r' = r T$:

$$S_3(T) = 4\pi T \int_0^\infty dr' r'^2 \left[\frac{1}{2} \left(\frac{d\ell}{dr'} \right)^2 + V'_{\text{eff}}(\ell, T) \right],$$

which has the same form as the above generic equation.

- The bubble profile (instanton solution) is obtained by solving the E.O.M. of the $S_3(T)$

$$\frac{d^2 \ell(r')}{dr'^2} + \frac{2}{r'} \frac{d\ell(r')}{dr'} - \frac{\partial V'_{\text{eff}}(\ell, T)}{\partial \ell} = 0$$

- The boundary conditions (deconfinement \rightarrow confinement) are

$$\frac{d\ell(r' = 0, T)}{dr'} = 0, \quad \lim_{r' \rightarrow 0} \ell(r', T) = 0$$

- We used the method of overshooting/undershooting (Python package)

Bubble Profile of Confinement Phase Transition

(Huang, Reichert, Sannino and Z-WW, PRD **104** (2021) 035005)

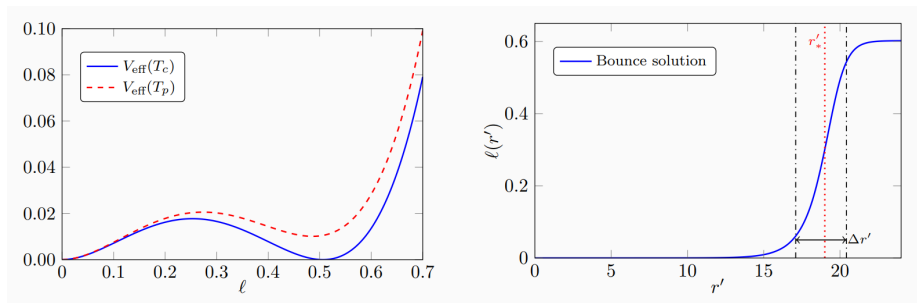


Figure: The bubble radius is indicated by r'_* and the wall width by $\Delta r'$. Inside of the bubble ($r' \ll r'_*$) lying the **confinement phase**, the Z_N symmetry is unbroken and $\langle \ell \rangle = 0$, while outside of the bubble ($r \gg r'_*$) lying the **deconfinement phase**, the Z_N symmetry is broken and $\langle \ell \rangle > 0$.

Bubble Nucleation: Chiral Phase Transition

(Reichert, Sannino, Z-W W and Zhang, JHEP 01 (2022) 003, arXiv:2109.11552)

- Chiral phase transition occurs when including fermions
- $\bar{\sigma}$ is classically nonpropagating in PNJL and it's kinetic term is induced only via quantum fluctuations
- We thus include its wave-function renormalization Z_σ with

$$Z_\sigma^{-1} = - \frac{d\Gamma_{\sigma\sigma}(q^0, \mathbf{q}, \bar{\sigma})}{d\mathbf{q}^2} \Big|_{q^0=0, \mathbf{q}^2=0}$$

- The three-dimensional Euclidean action and E.O.M. are modified to:

$$S_3(T) = 4\pi \int_0^\infty dr r^2 \left[\frac{Z_\sigma^{-1}}{2} \left(\frac{d\bar{\sigma}}{dr} \right)^2 + V_{\text{eff}}(\bar{\sigma}, T) \right]$$

$$\frac{d^2\bar{\sigma}}{dr^2} + \frac{2}{r} \frac{d\bar{\sigma}}{dr} - \frac{1}{2} \frac{\partial \log Z_\sigma}{\partial \bar{\sigma}} \left(\frac{d\bar{\sigma}}{dr} \right)^2 = Z_\sigma \frac{\partial V_{\text{eff}}}{\partial \bar{\sigma}}$$

- The associated boundary conditions:

$$\frac{d\bar{\sigma}(r=0, T)}{dr} = 0,$$

$$\lim_{r \rightarrow \infty} \bar{\sigma}(r, T) = 0$$

Gravitational Wave Parameters: Inverse Duration Time

- The phase-transition temperature T_* is often identified with the nucleation temperature T_n defined as the temperature where the rate of bubble nucleation per Hubble volume and time is order one: $\Gamma/H^4 \sim \mathcal{O}(1)$
- More accurately, we can use **percolation temperature** T_p : the temperature at which 34% of false vacuum is converted
- For sufficiently fast phase transitions, the decay rate is approximated by:

$$\Gamma(T) \approx \Gamma(t_*)e^{\beta(t-t_*)}$$

- The inverse duration time then follows as

$$\beta = -\left. \frac{d}{dt} \frac{S_3(T)}{T} \right|_{t=t_*}$$

- The dimensionless version $\tilde{\beta}$ is defined relative to the Hubble parameter H_* at the characteristic time t_*

$$\tilde{\beta} = \frac{\beta}{H_*} = T \left. \frac{d}{dT} \frac{S_3(T)}{T} \right|_{T=T_*},$$

where we used that $dT/dt = -H(T)T$.

Gravitational Wave Parameters: Strength Parameter I

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022)003, arXiv:2109.11552.)

- We define the strength parameter α from the **trace of the energy-momentum tensor** θ weighted by the enthalpy

$$\alpha = \frac{1}{3} \frac{\Delta\theta}{w_+} = \frac{1}{3} \frac{\Delta e - 3\Delta p}{w_+}, \quad \Delta X = X^{(+)} - X^{(-)}, \text{ for } X = (\theta, e, p)$$

(+) denotes the meta-stable phase (outside of the bubble) while (−) denotes the stable phase (inside of the bubble).

- The relations between enthalpy w , pressure p , and energy e are given by

$$w = \frac{\partial p}{\partial \ln T}, \quad e = \frac{\partial p}{\partial \ln T} - p,$$

which are extracted from the effective potential with

$$p^{(\pm)} = -V_{\text{eff}}^{(\pm)}$$

Gravitational Wave Parameters: Strength Parameter II

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005

Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

- α is thus given by

$$\alpha = \frac{1}{3} \frac{4\Delta V_{\text{eff}} - T \frac{\partial \Delta V_{\text{eff}}}{\partial T}}{-T \frac{\partial V_{\text{eff}}^{(+)}}{\partial T}},$$

- For confinement phase transition: $\alpha \approx 1/3$ (ΔV_{eff} is negligible since $e_+ \gg p_+$ and $e_- \sim p_- \sim 0$ in PLM potential)
- For chiral phase transition: we find smaller values, $\alpha \sim \mathcal{O}(10^{-2})$, due to the fact that more relativistic d.o.f.s participate in the phase transition
- Relativistic SM d.o.f.s do not contribute to our definition of α since they are fully decoupled from the phase transition but these d.o.f.s will play a role to dilute the GW signals

Gravitational-wave spectrum

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

- Contributions from bubble collision and turbulence are subleading
- The GW spectrum from sound waves is given by

$$h^2 \Omega_{\text{GW}}(f) = h^2 \Omega_{\text{GW}}^{\text{peak}} \left(\frac{f}{f_{\text{peak}}} \right)^3 \left[\frac{4}{7} + \frac{3}{7} \left(\frac{f}{f_{\text{peak}}} \right)^2 \right]^{-\frac{7}{2}}$$

- The peak frequency

$$f_{\text{peak}} \simeq 1.9 \cdot 10^{-5} \text{ Hz} \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \left(\frac{T}{100 \text{ GeV}} \right) \left(\frac{\tilde{\beta}}{v_w} \right)$$

- The peak amplitude

$$h^2 \Omega_{\text{GW}}^{\text{peak}} \simeq 2.65 \cdot 10^{-6} \left(\frac{v_w}{\tilde{\beta}} \right) \left(\frac{\kappa_{sw} \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{\frac{1}{3}} \Omega_{\text{dark}}^2$$

- The factor Ω_{dark}^2 accounts for the dilution of the GWs by the non-participating SM d.o.f.

$$\Omega_{\text{dark}} = \frac{\rho_{\text{rad,dark}}}{\rho_{\text{rad,tot}}} = \frac{g_{*,\text{dark}}}{g_{*,\text{dark}} + g_{*,\text{SM}}}$$

GW Signatures for Arbitrary N in the Pure Gluon Case

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

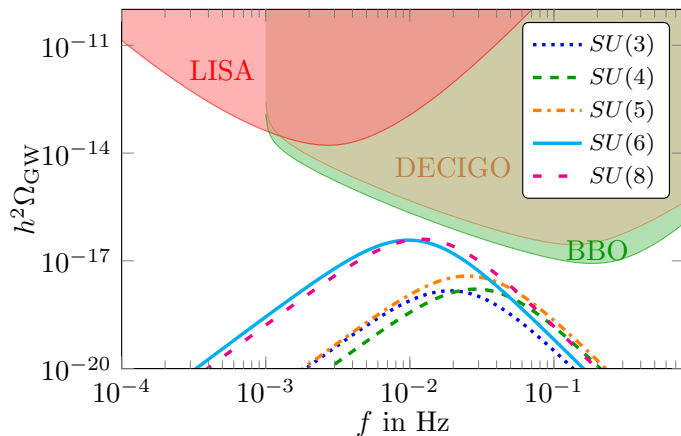


Figure: The dependence of the GW spectrum on the number of dark colours is shown for the values $N = 3, 4, 5, 6, 8$. All spectra are plotted with the bubble wall velocity set to the Chapman-Jouguet detonation velocity and with $T_c = 1$ GeV.

A Landscape of GW Signatures with Pure Gluon

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

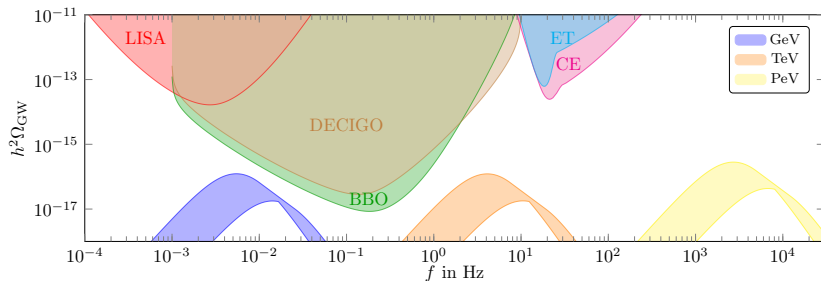


Figure: We display the GW spectrum of the $SU(6)$ phase transition for different confinement scales including $T_c = 1$ GeV, 1 TeV, and 1 PeV. We compare it to the power-law integrated sensitivity curves of LISA, BBO, DECIGO, CE, and ET.

Signal to Noise Ratio

(Huang, Reichert, Sannino and Z-W W, PRD **104** (2021) 035005)

$$\text{SNR} = \sqrt{\frac{3\text{year}}{s} \int_{f_{\min}}^{f_{\max}} df \left(\frac{h^2 \Omega_{\text{GW}}}{h^2 \Omega_{\text{det}}} \right)^2}$$

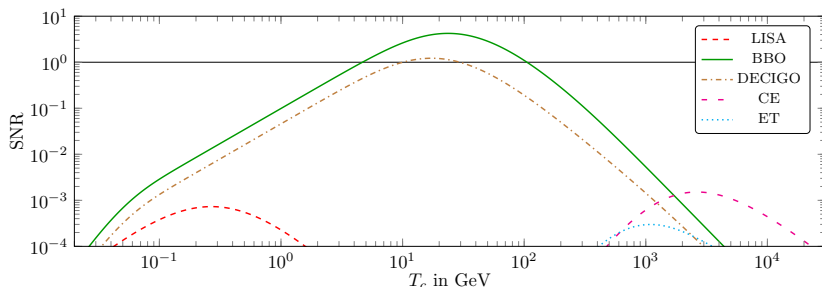


Figure: We display the SNR for the phase transition in a dark $SU(6)$ sector as a function of the confinement temperature T_c from experiments of LISA, BBO, DECIGO, CE, and ET. We assume an observation time of three years.

Landscape of GW spectrum with three Dirac fermions

(Reichert, Sannino, Z-W W and Zhang, JHEP 01 (2022) 003, arXiv:2109.11552.)

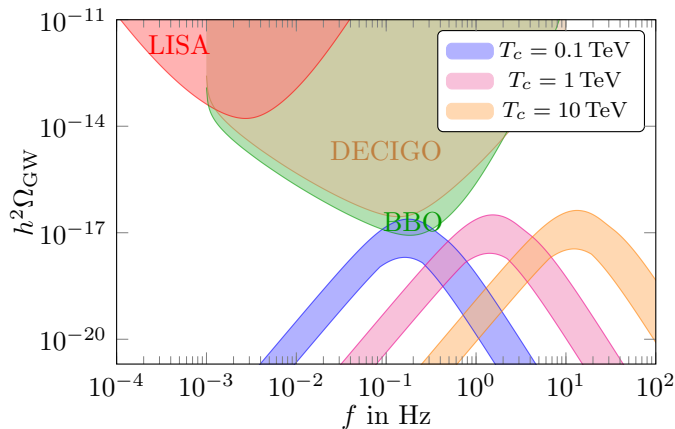


Figure: Gravitational-wave spectrum with three Dirac fermions in the fundamental representation for different critical temperatures.

Representation Matters

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

Rep.	flavour	chiral PT	conf.-deconf.
Fund.	3	1st	X
adjoint	1	2nd	1st
2-index Sym.	1	2nd	1st

Table: Representations versus different phase transitions.

- Need small N_f to remain below the conformal Banks-Zaks window.

Signal to Noise Ratio for Different Representations

(Reichert, Sannino, Z-W W and Zhang, JHEP 01 (2022) 003, arXiv:2109.11552.)

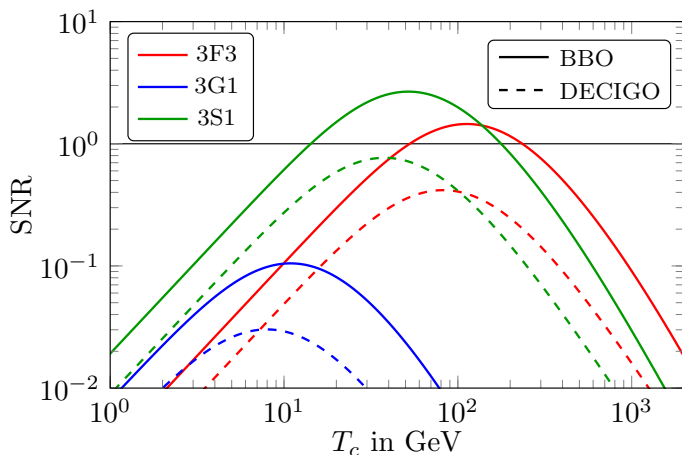


Figure: Signal-to-noise ratio as a function of the critical temperature for the best-case scenarios of each model at BBO and DECIGO with an observation time of 3 years.

Thank you for your attention!

The Constituent Quark Mass and Zero Point Energy: II

(Fukushima, Skokov, PPNP **96** (2017) 154)

- The integration can be carried analytically and the result is:

$$V_{\text{zero}}[\langle\bar{\psi}\psi\rangle] = -\frac{\dim(\mathbf{R})N_f\Lambda^4}{8\pi^2}\left[(2+\xi^2)\sqrt{1+\xi^2} + \frac{\xi^4}{2}\ln\frac{\sqrt{1+\xi^2}-1}{\sqrt{1+\xi^2}+1}\right],$$

in which $\xi \equiv \frac{M}{\Lambda}$.

GW parameters α , β and PNJL observables

(Reichert, Sannino, Z-W W and Zhang, JHEP **01** (2022) 003, arXiv:2109.11552.)

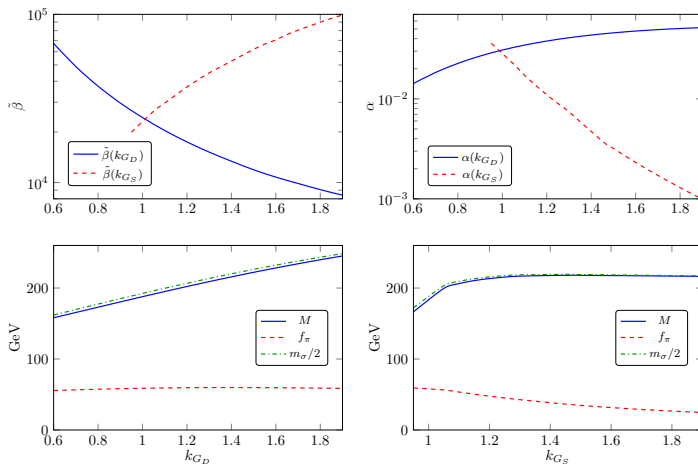


Figure: The GW parameters $\tilde{\beta}$, α with the observables M , f_π , and m_σ as a function of $G_S = k_{GS} \cdot 4.6 \text{ GeV}^{-2}$ and $G_D = k_{GD} \cdot (-743 \text{ GeV}^{-5})$. We use $T_c = 100 \text{ GeV}$, the ratio $\Lambda/T_0 = 3.54$. Below $k_{G_S, \text{crit}} = 0.882$, no chiral symmetry breaking occurs.

The Efficiency Factor κ

- The efficiency factor for the sound waves κ_{SW} consist of the factor κ_v as well as an additional suppression due to the length of the sound-wave period τ_{SW}

$$\kappa_{\text{SW}} = \sqrt{\tau_{\text{SW}}} \kappa_v$$

- τ_{SW} is dimensionless and measured in units of the Hubble time

$$\tau_{\text{SW}} = 1 - 1/\sqrt{1 + 2 \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta} \bar{U}_f}} \Rightarrow \tau_{\text{SW}} \sim \frac{(8\pi)^{\frac{1}{3}} v_w}{\tilde{\beta} \bar{U}_f} \text{ for } \beta \gg 1$$

where \bar{U}_f is the root-mean-square fluid velocity

$$\bar{U}_f^2 = \frac{3}{v_w(1+\alpha)} \int_{c_s}^{v_w} d\xi \xi^2 \frac{v(\xi)^2}{1-v(\xi)^2} \simeq \frac{3}{4} \frac{\alpha}{1+\alpha} \kappa_v$$

- τ_{SW} is suppressed for large β occurring often in strongly coupled sectors
- κ_v was numerically fitted to simulation results depends α and v_w . At the Chapman-Jouguet detonation velocity it reads

$$\kappa_v(v_w = v_J) = \frac{\sqrt{\alpha}}{0.135 + \sqrt{0.98 + \alpha}}$$