

Possible discrepancies in GUT spectra

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Grand Unified Theories
importance of systematic control

Gauge invariance

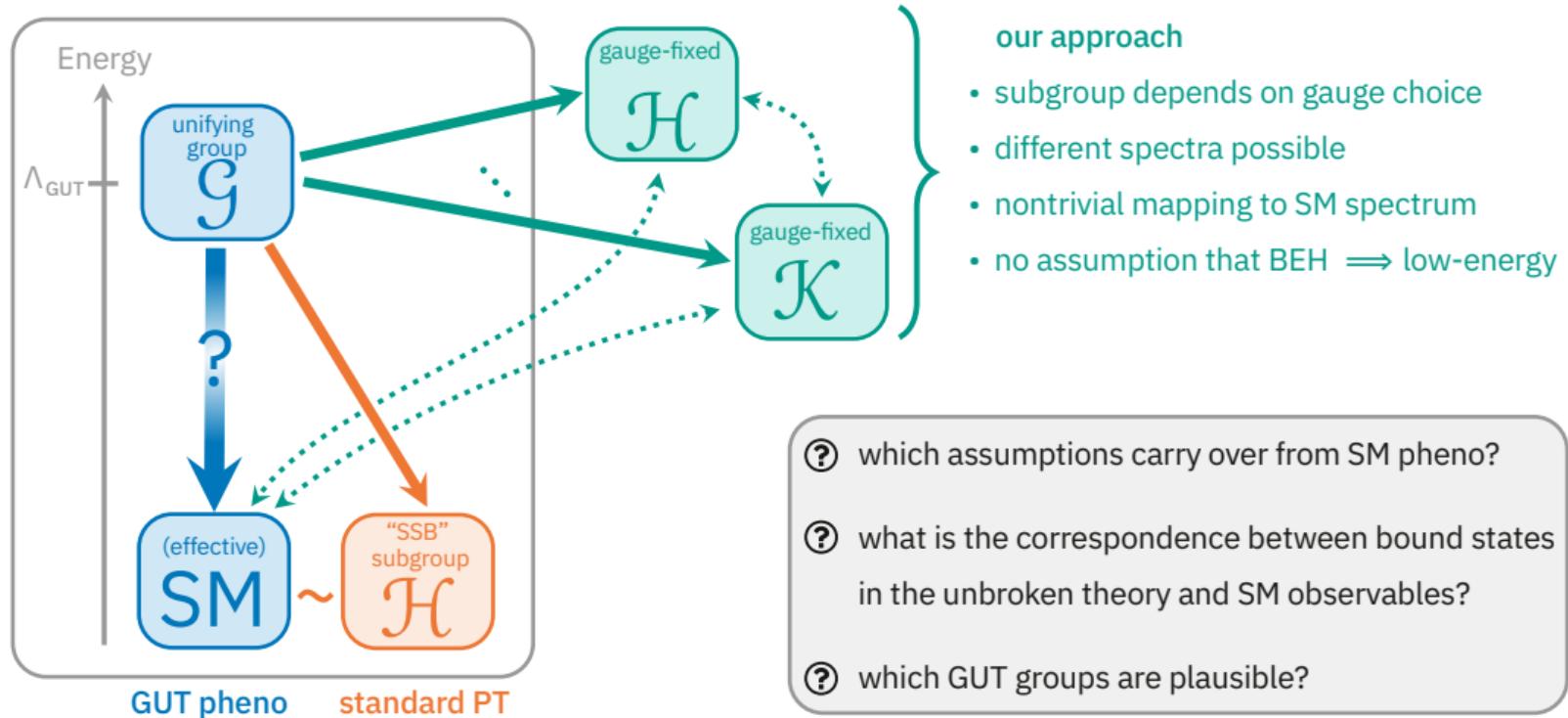
BRST breaks down for nonabelian theories
elementary fields are unphysical
the Fröhlich–Morchio–Strocchi mechanism

Lattice spectroscopy

toy SU(3) model to test FMS mechanism
discrepancies with naive perturbation theory

Main message:
naive perturbation theory can't be trusted
for predicting GUT spectra

Gauge-invariant approach to grand unified theories



Elementary fields form an unphysical state space

nonabelian gauge group + local gauge-fixing condition:

no unique solutions beyond PT

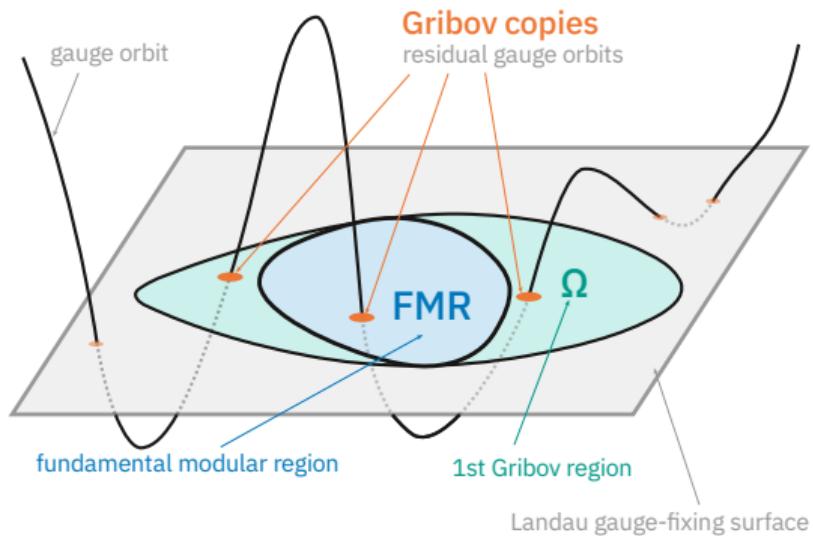
BRST insufficient to fix gauge

ξ -invariance $\not\Rightarrow$ gauge invariance

perturbative state space is gauge-dependent

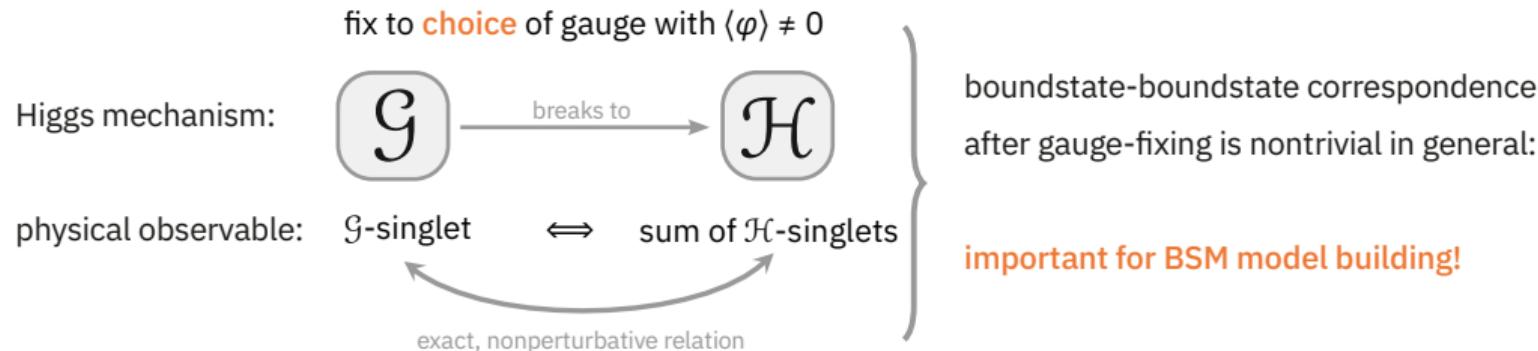
elementary fields (and e.g. Higgs vev)

are not reliable order parameters



Fröhlich–Morchio–Strocchi approach: composite states

elementary:	ψ	$W_\mu^{(a)}$	φ
composite:	$\varphi^\dagger \psi$	$i\varphi^\dagger D_\mu \varphi$	$\varphi^\dagger \varphi$
	fermion	vector boson	“Higgs”



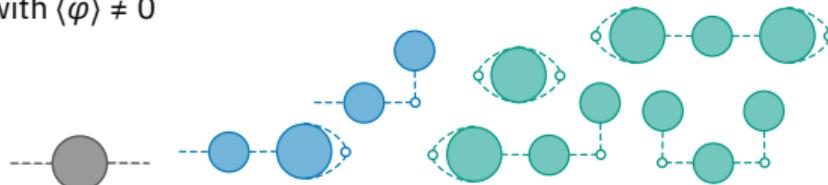
Compare: composite “Higgs” and 1^- vector singlet

[here: $SU(N)$ Yang–Mills with single fundamental scalar]

expand in **choice of gauge** with $\langle \varphi \rangle \neq 0$

$$\text{e.g. } \varphi(x) = v\hat{n} + \eta(x)$$

$$h(x) = 2 \operatorname{Re}[\hat{n}^\dagger \eta(x)]$$



$$\underbrace{\langle (\varphi^\dagger \varphi)(x)(\varphi^\dagger \varphi)(y) \rangle_c}_{\text{bound-state mass}} = v^2 \langle h(x)h(y) \rangle_c + \underbrace{2v \langle h(x)(\eta^\dagger \eta)(y) \rangle_c + \langle (\eta^\dagger \eta)(x)(\eta^\dagger \eta)(y) \rangle_c}_{\text{extra terms neglected in standard picture}}$$

coincides with standard PT

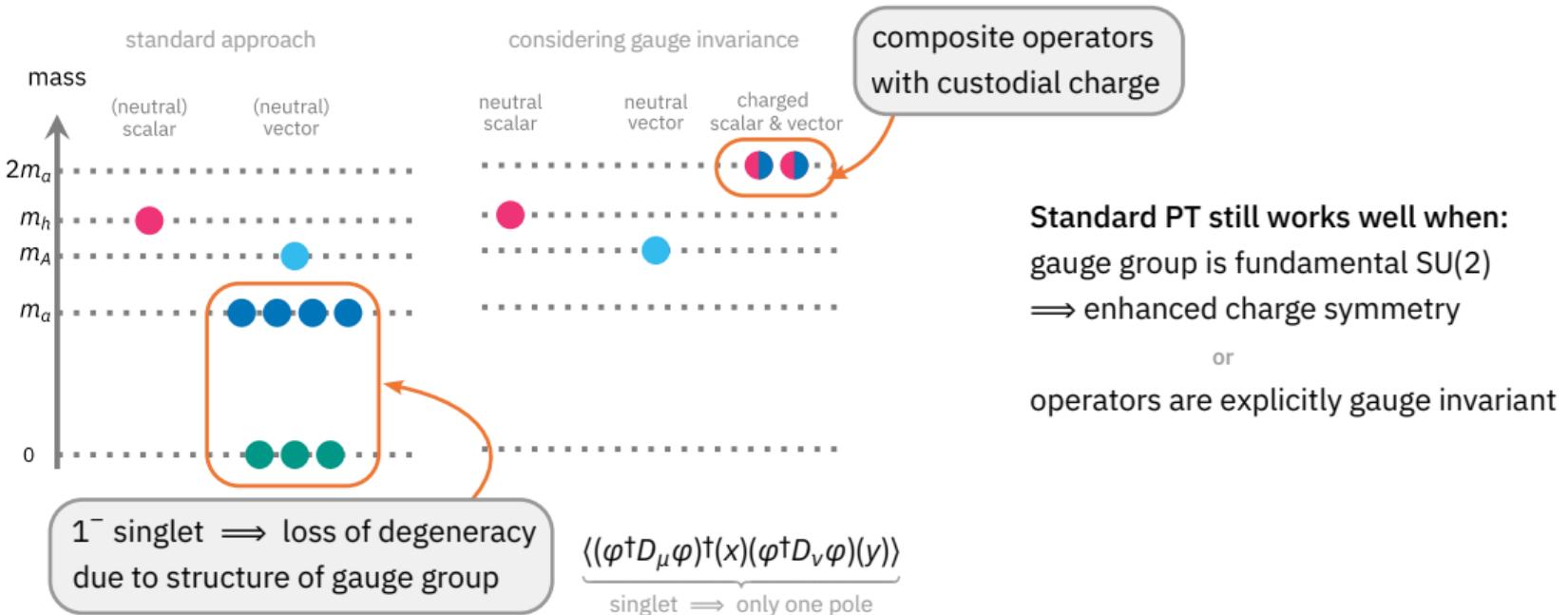
$$\underbrace{\langle (\varphi^\dagger D_\mu \varphi)^\dagger(x)(\varphi^\dagger D_\nu \varphi)(y) \rangle_c}_{\text{singlet} \implies \text{only one pole}} = v^2 c^{ab} \langle W_\mu^{(a)}(x) W_\nu^{(b)}(y) \rangle_c + \underbrace{O(\eta/v) + \dots}_{\text{don't affect pole structure}}$$

conflicts with standard PT
for $SU(N > 2)$

poles coincide to all orders in perturbation theory!

Gauge invariance qualitatively changes the spectrum

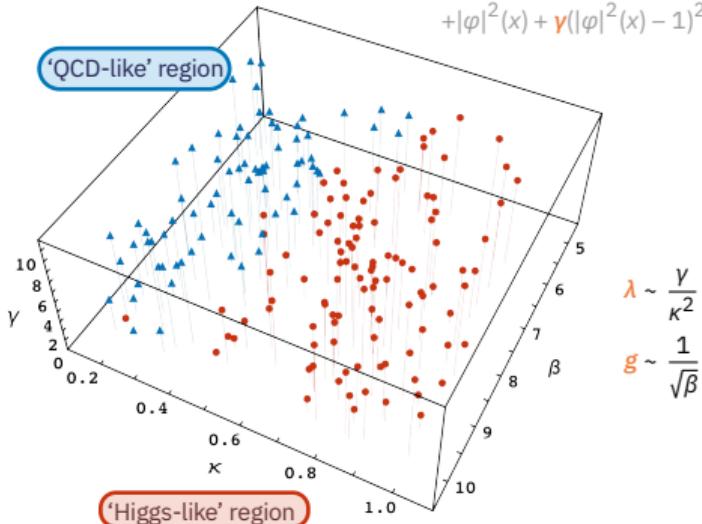
[here: SU(N) Yang–Mills with single fundamental scalar]



Testing FMS on the lattice: a toy SU(3) + YM model

$$\mathcal{L} = \frac{1}{2} \text{tr}(W_{\mu\nu}W^{\mu\nu}) + |D\varphi|^2 - \lambda(|\varphi|^2 - f^2)^2$$

$$\beta \text{Re} \sum_{\mu < \nu} [\mathbb{I} - U_{\mu\nu}(x)] - \kappa \sum_{\pm\mu} \varphi^\dagger(x) U_\mu^R(x) \varphi(x + \hat{\mu}) \\ + |\varphi|^2(x) + y(|\varphi|^2(x) - 1)^2$$



$$\lambda \sim \frac{\gamma}{\kappa^2}$$

$$g \sim \frac{1}{\sqrt{\beta}}$$

Generalisation of SM gauge-weak sector
single scalar $\varphi \in \text{SU}(3)$

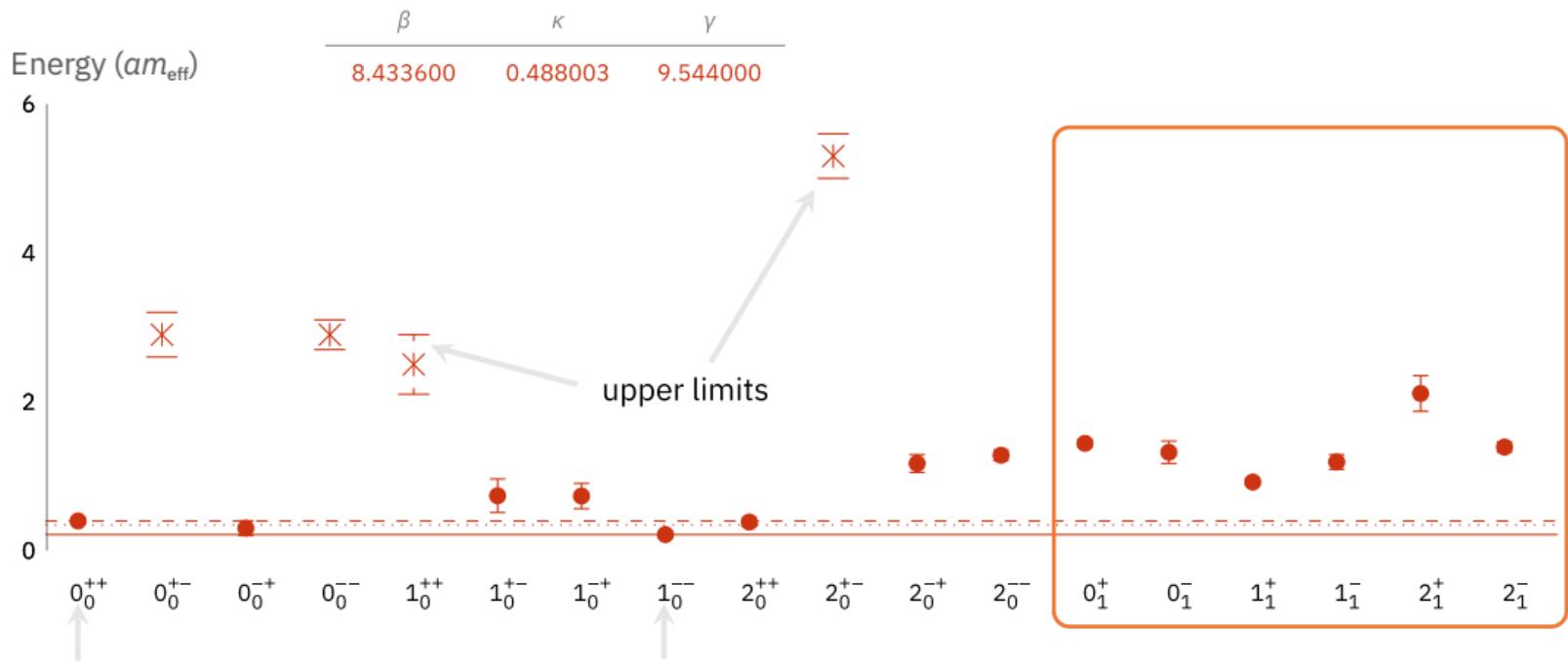
Breaks to nontrivial gauge group
 $\text{SU}(3) \rightarrow \text{SU}(2)$

Nontrivial custodial group
global $\text{U}(1)$

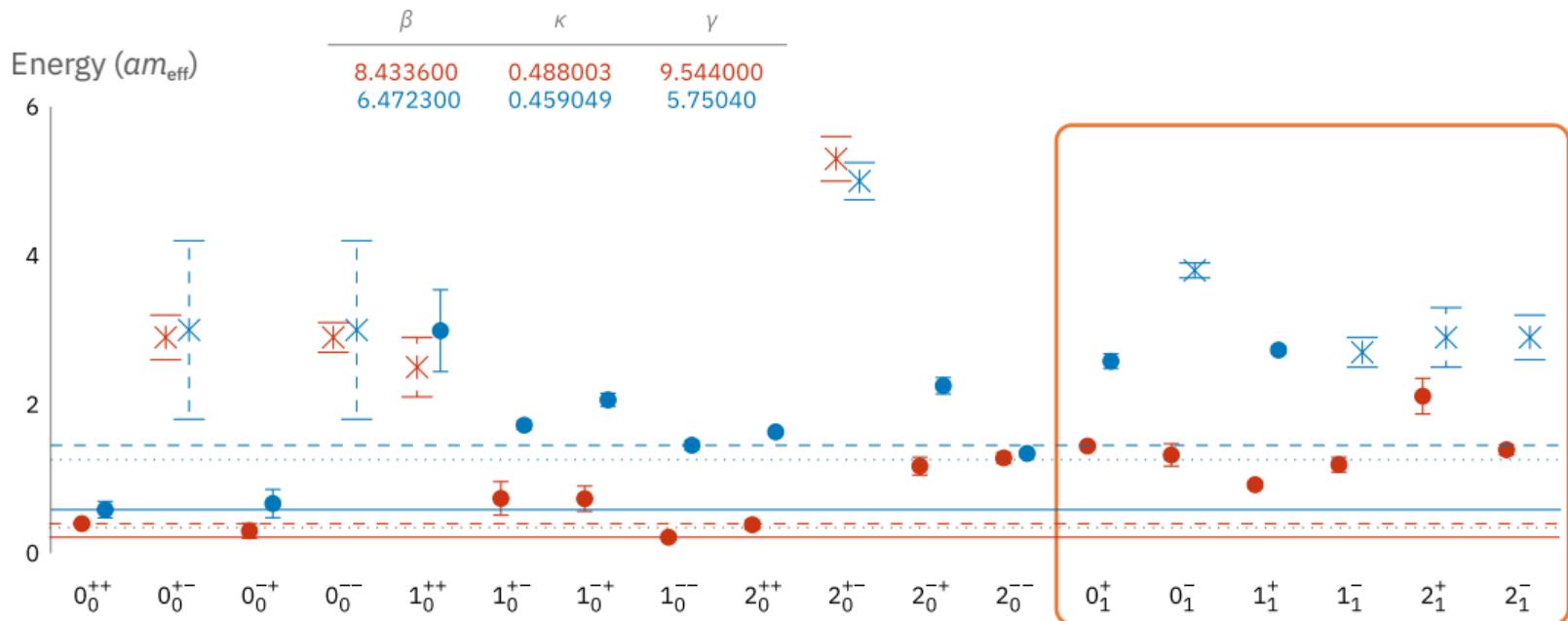
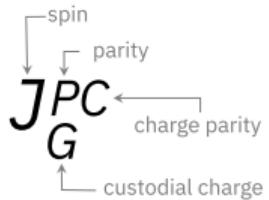
- ② what is the stable spectrum?
- ② are the lighter states charged?
- ② do lattice results support FMS?

SU(3) fundamental spectrum: additional U(1)-charged states

JPC
 G
 spin
 parity
 charge parity
 custodial charge



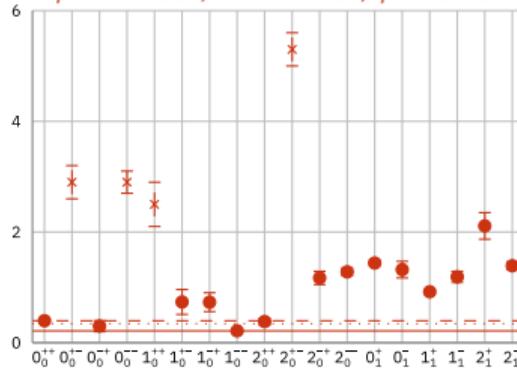
SU(3) fundamental spectrum: additional U(1)-charged states



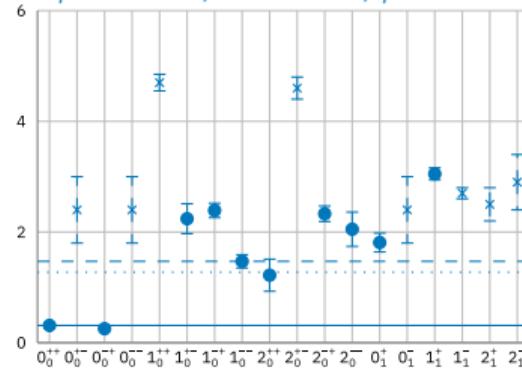
Phase transition: Higgs-like \rightarrow “QCD-like”

[energy scale = am_{eff} , $V \rightarrow \infty$]

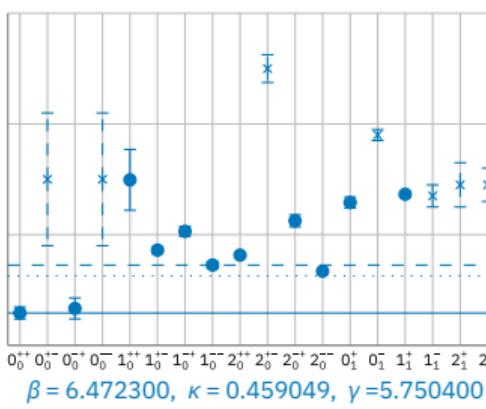
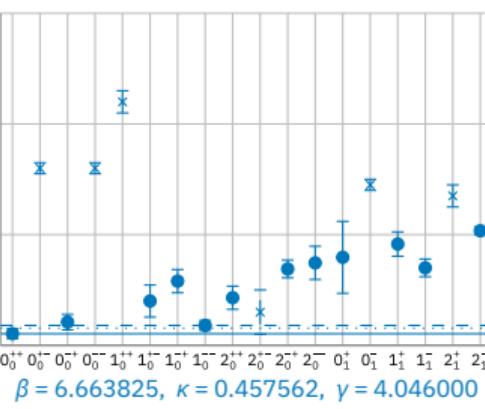
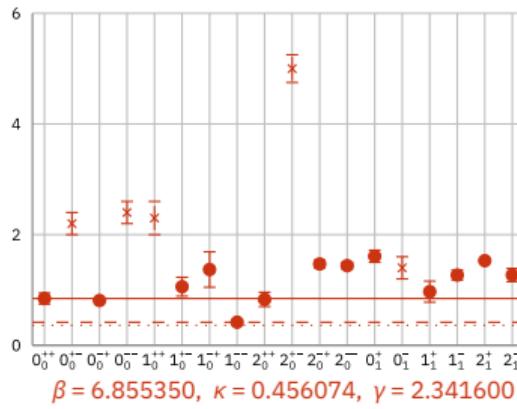
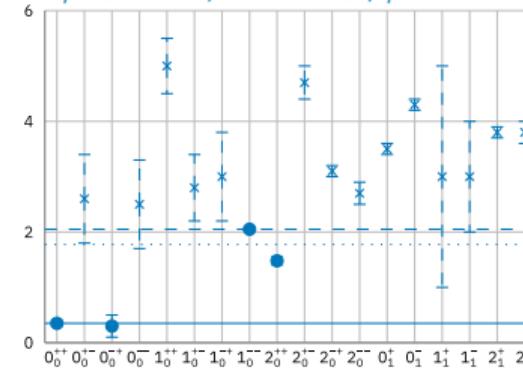
$$\beta = 8.436000, \kappa = 0.488003, \gamma = 9.544000$$



$$\beta = 8.393775, \kappa = 0.447893, \gamma = 8.646150$$



$$\beta = 8.353950, \kappa = 0.407782, \gamma = 7.748300$$



Outlook and implications for BSM phenomenology

Systematic control matters

gauge invariance has a qualitative effect on nonperturbative spectra
qualitative differences, even at small coupling

Results

qualitative differences from pure Yang–Mills, and from SU(2)
FMS: nontrivial field theory effects can still be treated perturbatively

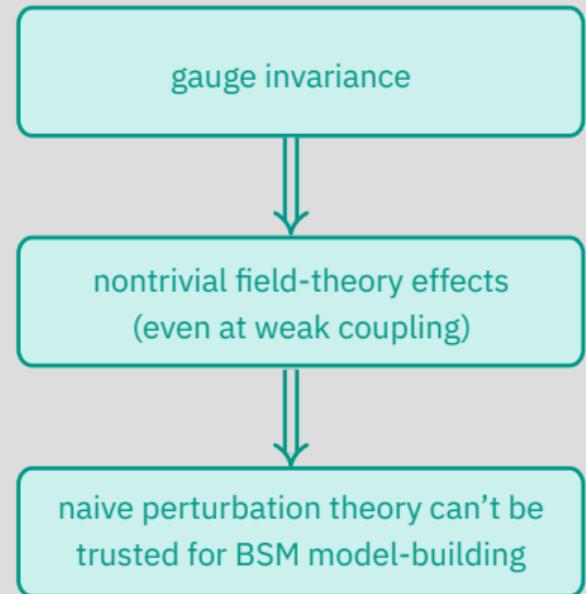
Applications and next steps

constraining GUTs (where lattice tests unfeasible)
meson decay, LFUV...
adjoint case: multiple symmetry breaking patterns

Possible discrepancies in GUT spectra

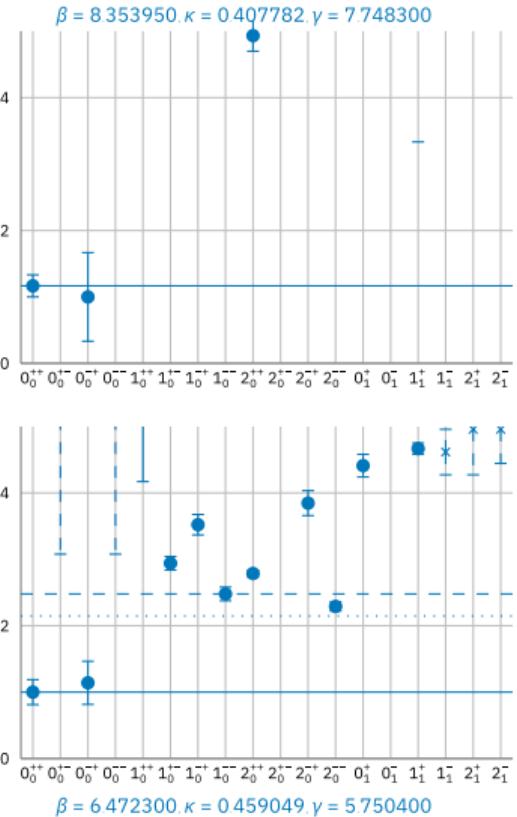
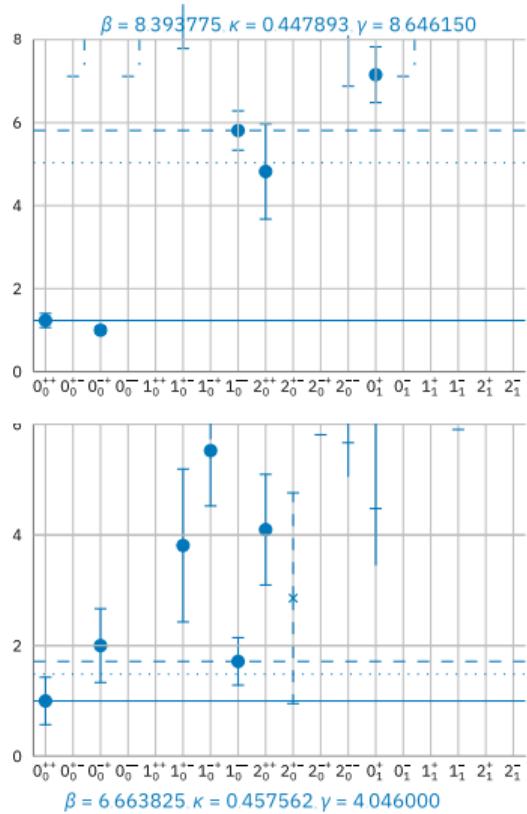
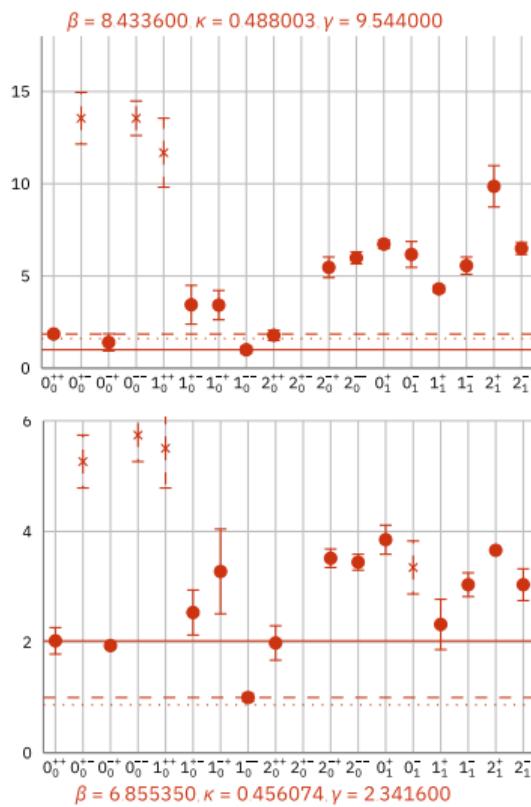
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Phase transition: Higgs-like \rightarrow “QCD-like”

[$V \rightarrow \infty$, normalised to lightest mass]



$\beta = 6.855350, \kappa = 0.456074, \gamma = 2.341600$

$\beta = 6.663825, \kappa = 0.457562, \gamma = 4.046000$

$\beta = 6.472300, \kappa = 0.459049, \gamma = 5.750400$