Electroweak flavour unification:

a new solution to the flavour puzzle

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Why a unification-based theory of flavour?

The Standard Model is a complicated QFT:

- 3 gauge couplings
- 5 fermions in “weird” representations (for one generation)
- 3 generations
- Peculiar Yukawa structure
- Higgs mechanism in electroweak sector gives weak, short-range forces + long-range EM
- Chiral symmetry breaking and confinement of QCD in the IR

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Unification of **forces and/or matter** attempts to explain all (or part of) this structure as a consequence of **something simpler** at high energies.

(Traditionally hinted at by the near-unification of gauge couplings at GUT scale)

There are two GUTs (one gauge coupling) that don't require extra fermions:

- SU(5) $\Psi \sim \mathbf{5} \oplus \overline{\mathbf{10}} \oplus \mathbf{1}$ Georgi, Glashow, 1974
- SO(10) $\Psi \sim \mathbf{16}$ Georgi, 1975, and Fritzsch, Minkowski, 1975

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Q: can we **unify** either SU(5) or SO(10) **with flavour**, thereby explaining the origin of three generations?

A: **No!** (at least not without extra fermions)

A provocative claim:

“If we want to unify the three generations of matter, we must forgo the complete unification of forces.”

Gauge Flavour Unification

To unify gauge and flavour symmetries, it turns out all roads go through **Pati-Salam** gauge group:

$$PS = SU(4) \times SU(2)_L \times SU(2)_R$$

Pati, Salam, 1974

$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

Two options:

1. Unify colour and flavour
2. Unify electroweak and flavour

Gauge Flavour Unification

1. Colour flavour unification

$SU(12) \times SU(2)_L \times SU(2)_R$
$\Psi_L \sim (\mathbf{12}, \mathbf{2}, \mathbf{1}),\ \Psi_R \sim (\mathbf{12}, \mathbf{1}, \mathbf{2})$

$G_F = SU(3)$

$SU(4) \times SU(2)_L \times SU(2)_R \times G_F$

$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$

Gauge Flavour Unification

Reminder:
The Lie group $Sp(6)$ is a subgroup of $SU(6)$:
$Sp(6) = \{U \in SU(6) | U^T \Omega U = \Omega\}$, where $\Omega = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$

1. Colour flavour unification

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2. Electroweak flavour unification

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$$G_F = SO(3)_L \times SO(3)_R$$

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$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

See also Kuo and Nakagawa in 1984

These are the *only* gauge-flavour unified groups with just 2 Ψs, assuming no BSM Weyls

[Allanach, Gripaios, Tooby-Smith, 2104.14555]

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This talk $\boxed{SU(4) \times Sp(6)_L \times Sp(6)_R}$
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$$G = SU(4) \times Sp(6)_L \times Sp(6)_R$$

Summarize our motivations:

1. Unification of quarks & leptons
2. Unification of three generations

This is a lot of matter unification!

3. The challenge: can we also explain the **flavour puzzle**?

$$\boxed{G = SU(4) \times Sp(6)_L \times Sp(6)_R}$$

Let’s build a model of flavour

$G = SU(4) \times Sp(6)_L \times Sp(6)_R$

Embedding the SM fields

Embed all SM chiral fermions in **2 fundamental fields** (nothing extra):

$$\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}) \sim \begin{pmatrix} u_1^r & u_2^r & u_3^r & d_1^r & d_2^r & d_3^r \\ u_1^g & u_2^g & u_3^g & d_1^g & d_2^g & d_3^g \\ u_1^b & u_2^b & u_3^b & d_1^b & d_2^b & d_3^b \\ \upsilon_1 & \upsilon_2 & \upsilon_3 & e_1 & e_2 & e_3 \end{pmatrix}, \qquad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6}) \sim \text{ similar}$$

Recap: in Pati-Salam, $H_1 \sim (\mathbf{1}, \mathbf{2}, \mathbf{2})$ and $H_{15} \sim (\mathbf{15}, \mathbf{2}, \mathbf{2})$

$\longrightarrow$ Embed SM Higgs in $H_1 \sim (\mathbf{1}, \mathbf{6}, \mathbf{6})$ and $H_{15} \sim (\mathbf{15}, \mathbf{6}, \mathbf{6})$, with Yukawa couplings:

$$\mathcal{L} = y_1 \mathrm{Tr}\left[\overline{\Psi_L} \Omega H_1 \Omega \Psi_R\right] + y_{15} \mathrm{Tr}\left[\overline{\Psi_L} \Omega H_{15} \Omega \Psi_R\right] + \bar{y}_1 \mathrm{Tr}\left[\overline{\Psi_L} \Omega H_1^* \Omega \Psi_R\right] + \bar{y}_{15} \mathrm{Tr}\left[\overline{\Psi_L} \Omega H_{15}^* \Omega \Psi_R\right]$$

Recall $\Omega = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$

The Pati-Salam Higgs fields have become flavoured

$$G = SU(4) \times Sp(6)_L \times Sp(6)_R$$

We must break $G \longrightarrow \cdots \longrightarrow SM$

We do so using an **(almost) minimal** set of scalars, via a **multi-scale** symmetry breaking pattern

Nothing else will be needed to generate realistic fermion masses and quark mixings

Type	Field	G_{EWF} irrep
SM fermions	Ψ_L	$(\mathbf{4},\mathbf{6},\mathbf{1})$
	Ψ_R	$(\mathbf{4},\mathbf{1},\mathbf{6})$
Higgs	H_1	$(\mathbf{1},\mathbf{6},\mathbf{6})$
	H_{15}	$(\mathbf{15},\mathbf{6},\mathbf{6})$
SSB scalars	S_L	$(\mathbf{1},\mathbf{14},\mathbf{1})$
	S_R	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{6})$
	Φ_L	$(\mathbf{1},\mathbf{14},\mathbf{1})$
	Φ_R	$(\mathbf{1},\mathbf{1},\mathbf{14})$

$G = SU(4) \times Sp(6)_L \times Sp(6)_R$

Step 1. Deconstruction of electroweak symmetry

At a high scale, break $Sp(6)_L \to SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3}$ via a scalar $S_L \sim (\mathbf{1}, \mathbf{14}, \mathbf{1})$

$$\begin{pmatrix} 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \\ 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \end{pmatrix}$$

See also Kuo, Nakagawa, 1984

We do something similar for right-sector, with $S_R \sim (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6})$ breaking

$$SU(4) \times Sp(6)_R \to SU(3) \times Sp(4)_{R,12} \times U(1)_R$$

(also breaking quark-lepton unification at high scale)

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon \Lambda_H$

v

$$SU(3) \times \prod_{i=1}^{3} SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$$

Step 2. Flavoured Higgses

Under the deconstruction, the Higgs fields split into **flavoured** components:

$$\begin{aligned} H_{1,15} \mapsto & [\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{1}]_{-3} \oplus [\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{1}]_{-3} \oplus \underline{[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{1}]_{-3}} \\ & \oplus [\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{1}]_{3} \oplus [\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{1}]_{3} \oplus \underline{[\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{1}]_{3}} \\ & \oplus [\mathbf{1},(\mathbf{2},\mathbf{1},\mathbf{1}),\mathbf{4}]_{0} \oplus [\mathbf{1},(\mathbf{1},\mathbf{2},\mathbf{1}),\mathbf{4}]_{0} \oplus [\mathbf{1},(\mathbf{1},\mathbf{1},\mathbf{2}),\mathbf{4}]_{0} \\ & \oplus \{SU(3) \text{ triplets and octets for } H_{15}\}\,. \end{aligned}$$

SM Higgs

- Higgs vev falls into small number of these family-aligned components
- This **picks out one family to be heavy, defining the third family**
- Other fermions **massless** at renormalizable level

We assume the **other Higgs components are heavy**, and **integrated out at a scale Λ_H**

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon\Lambda_H$

v

Step 3. Breaking to the SM

$$SU(3) \times \prod_{i=1}^{3} SU(2)_{L,i} \times Sp(4)_{R,12} \times U(1)_R$$

Type	Field	G_{EWF} irrep
SM fermions	Ψ_L	$(\mathbf{4}, \mathbf{6}, \mathbf{1})$
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	S_R	$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6})$
	Φ_L	$(\mathbf{1}, \mathbf{14}, \mathbf{1})$
	Φ_R	$(\mathbf{1}, \mathbf{1}, \mathbf{14})$

Λ_L, Λ_R

Λ_H

$\epsilon\Lambda_H$

v

Last two scalars Φ_L and Φ_R break to SM. The 2-index **14** reps provide link fields:

$$\Phi_L \to \mathbf{1}^{\oplus 2} \oplus \underbrace{(\mathbf{2},\mathbf{2},\mathbf{1})}_{\Phi_L^{12}} \oplus (\mathbf{2},\mathbf{1},\mathbf{2}) \oplus \underbrace{(\mathbf{1},\mathbf{2},\mathbf{2})}_{\Phi_L^{23}}$$

$$\langle \Phi_L^{12} \rangle = \epsilon_L^{12}\Lambda_H,\ \langle \Phi_L^{23} \rangle = \epsilon_L^{23}\Lambda_H : SU(2)_{L,1} \times SU(2)_{L,2} \times SU(2)_{L,3} \longrightarrow SU(2)_L$$

$\langle \Phi_R \rangle$ more complicated… also decomposes as “link fields”, break $Sp(4)_{R,12} \times U(1)_R \to U(1)_Y$

These steps **generate effective Yukawa** couplings for **all light fermions**:

Integrate out heavy Higgs components

Λ_H

↓

High-dimension operators

$$\mathcal{O}^{4+n} \sim \left(\frac{\Phi_{L/R}}{\Lambda_H}\right)^n \overline{\psi_L} H \psi_R$$

↓

$\epsilon\Lambda_H$

Match onto SM Yukawas

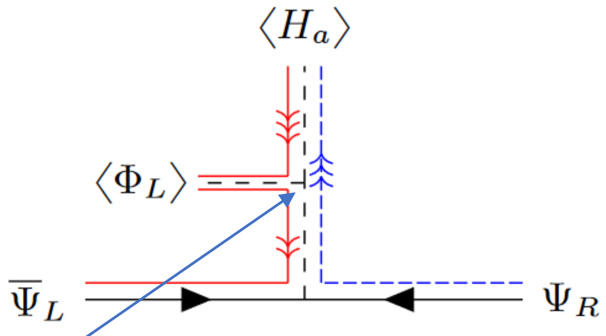
$$\epsilon_{L/R}^n \, \overline{\psi_L} H \psi_R$$

Hierarchies!

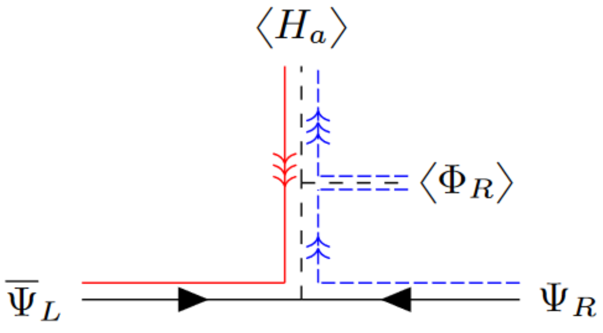
EFT of light fermion Yukawas: a sketch

Dimension 5: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi$

$$\begin{pmatrix} & & \times \\ & \times & \end{pmatrix}$$



$Y_{23}^{u,d,e}$



$Y_{32}^{u,d,e}$

Scalar potential $\supset \Lambda_H \mathrm{Tr}\left(\Omega^T H_1^\dagger \Omega \Phi_L \Omega H_1\right) + \cdots$

$\Longrightarrow$

$$\frac{y_1 \beta_L^1}{2\Lambda_H} \phi_L^{23} \left(\overline{Q}_2 \mathcal{H}_1 D_3 + \overline{Q}_2 \overline{\mathcal{H}}_1 U_3 + \overline{L}_2 \mathcal{H}_1 E_3\right)$$

EFT of light fermion Yukawas: a sketch

Other light Yukawas are generated at higher operator dimension in the EFT

<u>The upshot</u>: all Yukawa matrices have the hierarchical structure

$$\frac{M^f}{v} \sim \begin{pmatrix} \epsilon_L^{12}\epsilon_L^{23}\epsilon_R^{12}\epsilon_R^{23} & \epsilon_L^{12}\epsilon_L^{23}\epsilon_R^{23} & \epsilon_L^{12}\epsilon_L^{23} \\ \epsilon_L^{23}\epsilon_R^{12}\epsilon_R^{23} & \epsilon_L^{23}\epsilon_R^{23} & \epsilon_L^{23} \\ \epsilon_R^{12}\epsilon_R^{23} & \epsilon_R^{23} & 1 \end{pmatrix}$$

for $f \in u, d, e$.

Quark masses and mixings

Extract observables using matrix perturbation theory:

Mass eigenvalues:

$$y_{u,d,e} \approx \left|\frac{\det(\mathbf{h}^{u,d,e})}{\mathbf{k}^{u,d,e}_{11}}\right| \epsilon_L^{12}\epsilon_R^{12}\epsilon_L^{23}\epsilon_R^{23},$$

$$y_{c,s,\mu} \approx \left|\frac{\mathbf{k}^{u,d,e}_{11}}{\mathbf{h}^{u,d,e}_{33}}\right| \epsilon_L^{23}\epsilon_R^{23},$$

$$y_{t,b,\tau} \approx \left|\mathbf{h}^{u,d,e}_{33}\right|,$$

The $\mathbf{h}^{u,d,e}_{ij}$ and $\mathbf{k}^{u,d,e}_{ij}$ are combinations of our EFT coefficients.

Hierarchies in mixing angles:
Choose $\epsilon_L^{12} \sim \lambda$ (Cabibbo), $\epsilon_L^{23} \sim |V_{cb}| \sim \lambda^2$

Hierarchies in mass ratios:
Choose $\epsilon_R^{12} \sim \lambda^2$, $\epsilon_R^{23} \sim \lambda$

CKM matrix $V_{\text{CKM}} = V_L^u V_L^{d*} \approx$

$$\begin{pmatrix} 1-\left(\left|\frac{\mathbf{k}^d_{21}}{\mathbf{k}^d_{11}}\right|^2+\left|\frac{\mathbf{k}^u_{21}}{\mathbf{k}^u_{11}}\right|^2-2\frac{\mathbf{k}^{u*}_{21}\mathbf{k}^d_{21}}{\mathbf{k}^{u*}_{11}\mathbf{k}^d_{11}}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{k}^d_{21}}{\mathbf{k}^d_{11}}-\frac{\mathbf{k}^{u*}_{21}}{\mathbf{k}^{u*}_{11}}\right)\epsilon_L^{12} & \left(\frac{\mathbf{k}^{u*}_{31}}{\mathbf{k}^{u*}_{11}}+\frac{\mathbf{h}^d_{13}}{\mathbf{h}^d_{33}}-\frac{\mathbf{h}^d_{23}}{\mathbf{h}^d_{33}}\frac{\mathbf{k}^{u*}_{21}}{\mathbf{k}^{u*}_{11}}\right)\epsilon_L^{12}\epsilon_L^{23} \\ \left(\frac{\mathbf{k}^u_{21}}{\mathbf{k}^u_{11}}-\frac{\mathbf{k}^{d*}_{21}}{\mathbf{k}^{d*}_{11}}\right)\epsilon_L^{12} & 1-\left(\left|\frac{\mathbf{k}^d_{21}}{\mathbf{k}^d_{11}}\right|^2+\left|\frac{\mathbf{k}^u_{21}}{\mathbf{k}^u_{11}}\right|^2-2\frac{\mathbf{k}^u_{21}\mathbf{k}^{d*}_{21}}{\mathbf{k}^u_{11}\mathbf{k}^{d*}_{11}}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{h}^d_{23}}{\mathbf{h}^d_{33}}-\frac{\mathbf{h}^{u*}_{23}}{\mathbf{h}^{u*}_{33}}\right)\epsilon_L^{23} \\ \left(\frac{\mathbf{k}^{d*}_{31}}{\mathbf{k}^{d*}_{11}}+\frac{\mathbf{h}^u_{13}}{\mathbf{h}^u_{33}}-\frac{\mathbf{h}^u_{23}}{\mathbf{h}^u_{33}}\frac{\mathbf{k}^{d*}_{21}}{\mathbf{k}^{d*}_{11}}\right)\epsilon_L^{12}\epsilon_L^{23} & \left(\frac{\mathbf{h}^u_{23}}{\mathbf{h}^u_{33}}-\frac{\mathbf{h}^{d*}_{23}}{\mathbf{h}^{d*}_{33}}\right)\epsilon_L^{23} & 1 \end{pmatrix}$$

Our CKM matches onto the Wolfenstein parametrization

Wolfenstein, 1983

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Hierarchies in **mass ratios**:
Choose $\epsilon_R^{12} \sim \lambda^2$, $\epsilon_R^{23} \sim \lambda$

... And there is **enough freedom** in the EFT coefficients to fit all the data ☑

CKM matrix $V_{\text{CKM}} = V_L^u V_L^{d*} \approx$

$$\begin{pmatrix} 1 - \left(\left|\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d}\right|^2 + \left|\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u}\right|^2 - 2\frac{\mathbf{k}_{21}^{u*}\mathbf{k}_{21}^d}{\mathbf{k}_{11}^{u*}\mathbf{k}_{11}^d}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d} - \frac{\mathbf{k}_{21}^{u*}}{\mathbf{k}_{11}^{u*}}\right)\epsilon_L^{12} & \left(\frac{\mathbf{k}_{31}^{u*}}{\mathbf{k}_{11}^{u*}} + \frac{\mathbf{h}_{13}^d}{\mathbf{h}_{33}^d} - \frac{\mathbf{h}_{23}^d}{\mathbf{h}_{33}^d}\frac{\mathbf{k}_{21}^{u*}}{\mathbf{k}_{11}^{u*}}\right)\epsilon_L^{12}\epsilon_L^{23} \\ \left(\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u} - \frac{\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{d*}}\right)\epsilon_L^{12} & 1 - \left(\left|\frac{\mathbf{k}_{21}^d}{\mathbf{k}_{11}^d}\right|^2 + \left|\frac{\mathbf{k}_{21}^u}{\mathbf{k}_{11}^u}\right|^2 - 2\frac{\mathbf{k}_{21}^{u}\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{u}\mathbf{k}_{11}^{d*}}\right)\frac{(\epsilon_L^{12})^2}{2} & \left(\frac{\mathbf{h}_{23}^d}{\mathbf{h}_{33}^d} - \frac{\mathbf{h}_{23}^{u*}}{\mathbf{h}_{33}^{u*}}\right)\epsilon_L^{23} \\ \left(\frac{\mathbf{k}_{31}^{d*}}{\mathbf{k}_{11}^{d*}} + \frac{\mathbf{h}_{13}^u}{\mathbf{h}_{33}^u} - \frac{\mathbf{h}_{23}^u}{\mathbf{h}_{33}^u}\frac{\mathbf{k}_{21}^{d*}}{\mathbf{k}_{11}^{d*}}\right)\epsilon_L^{12}\epsilon_L^{23} & \left(\frac{\mathbf{h}_{23}^u}{\mathbf{h}_{33}^u} - \frac{\mathbf{h}_{23}^{d*}}{\mathbf{h}_{33}^{d*}}\right)\epsilon_L^{23} & 1 \end{pmatrix}$$

Our CKM matches onto the Wolfenstein parametrization

Wolfenstein, 1983

$$\boxed{G = SU(4) \times Sp(6)_L \times Sp(6)_R}$$

Our EWFU model explains

- The **origin of 3 generations**
- The **hierarchical** structure of **fermion masses** and **quark mixing** angles

in terms of a flavour-enriched version of Pati—Salam **unification**

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in terms of a flavour-enriched version of Pati—Salam **unification**

Protons are stable in this UV model. So the symmetry breaking scales can be brought low…

How low can you go?

We have the following heavy gauge bosons in our model:

		Heavy scales ($\Lambda_{L,R}$)	Intermediate scale ($\epsilon\Lambda_H$)
Name	G_{SM} representation	Number (origin)	Number (origin)
Charged Z'	$(\mathbf{1},\mathbf{1})_6$	3 (S_R)	3 (Φ_R)
U_1 leptoquark	$(\bar{\mathbf{3}},\mathbf{1})_{-4}$	1 (S_R)	—
(W', Z') triplet	$(\mathbf{1},\mathbf{3})_0$ ($\mathbb{R}$)	3 (S_L)	2 (Φ_L)
Real Z'	$(\mathbf{1},\mathbf{1})_0$ ($\mathbb{R}$)	3 (S_L), 5 (S_R)	4 (Φ_R)

Heavy

The light states – all flavoured versions of the EW gauge bosons

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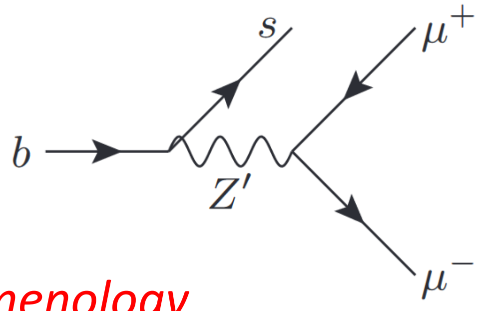
Heavy

The light states – all flavoured versions of the EW gauge bosons

Generic consequences:
New sources of **quark flavour violation** and **LF(U)V**

For example, consider the (W',Z') triplets from Φ_L.
The lightest Z' couples to $Q_{L,2}$, $Q_{L,3}$, $L_{L,2}$, $L_{L,3}$ …



<u>What's next?</u> thorough investigation of low-energy phenomenology

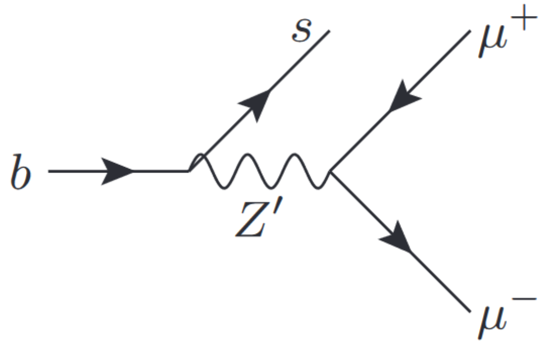
Thank you!

Buon appetito!

Backup slides

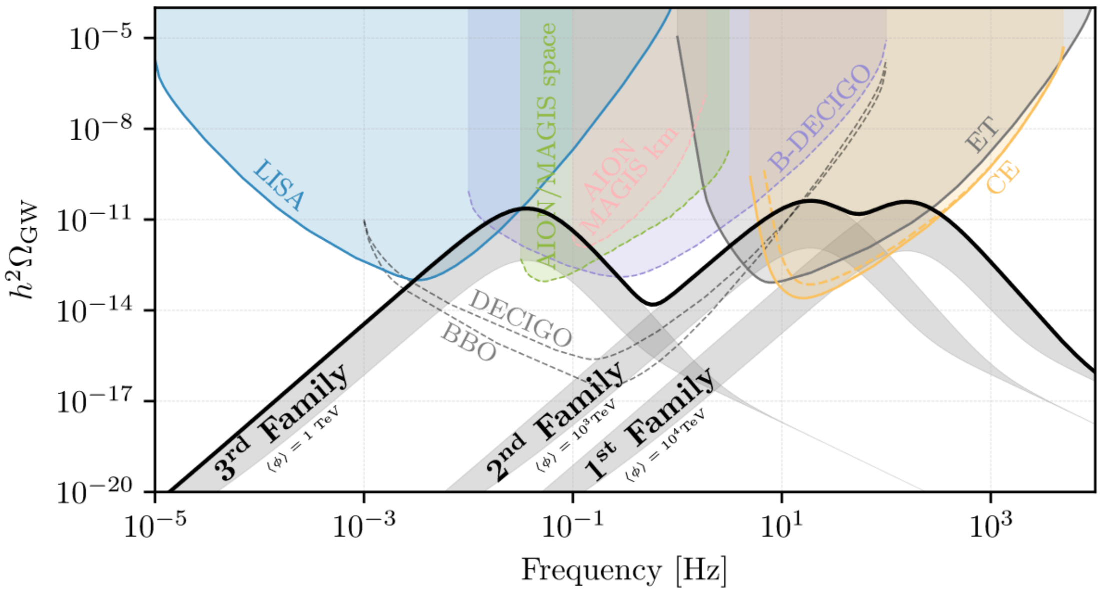
Some future directions

- Low scale EWFU
  - Flavour-dependent forces – B anomalies etc?
  - Phenomenological analysis: compute **lower bounds on scales**
  - How much tuning in scalar sector?



- Neutrino masses

- Cosmology
  - EWFU predicts **monopole** production, since $\pi_2(SU(4) \times Sp(6)_L \times Sp(6)_R/SM) = \mathbb{Z}$. Dilute by taking $\Lambda_R > \Lambda_{\text{inflation}}$
  - **Gravitational wave** production in early Universe: stochastic multi-peaked GW signal. An alternative probe of EWFU, even if the SSB scales are very high



Greljo, Opferkuch, Stefanek, 2019

A tale of scales

$$SU(4)\times Sp(6)_L\times Sp(6)_R$$

$\Lambda_L,\ \Lambda_R$ $\langle S_L\rangle \sim (\mathbf{1},\mathbf{14},\mathbf{1})$ $\langle S_R\rangle \sim (\bar{\mathbf{4}},\mathbf{1},\mathbf{6})$

Λ_H [Heavy Higgses integrated out]

$$SU(3)\times \prod_i SU(2)_{L,i}\times Sp(4)_{R,12}\times U(1)_R$$

$\epsilon\Lambda_H$ $\langle \Phi_L\rangle \sim (\mathbf{1},\mathbf{14},\mathbf{1})$ $\langle \Phi_R\rangle \sim (\mathbf{1},\mathbf{1},\mathbf{14})$

$$SU(3)\times SU(2)_L\times U(1)_Y$$

Arbitrary scale separation; can be small to reduce tuning / m_h^2 sensitivity

Small scale separation, $\mathcal{O}(10^{-1}-10^{-2})$, fixed by Yukawa hierarchy

"Distance to New Physics": can be as low as experimentally allowed by flavour bounds

Gauge flavour unification: the embedding

Let $g = \begin{pmatrix} \alpha & -\beta^* \\ \beta & \alpha^* \end{pmatrix} \in Sp(2) = SU(2)$, and $o \in SO(n)$. The embedding is

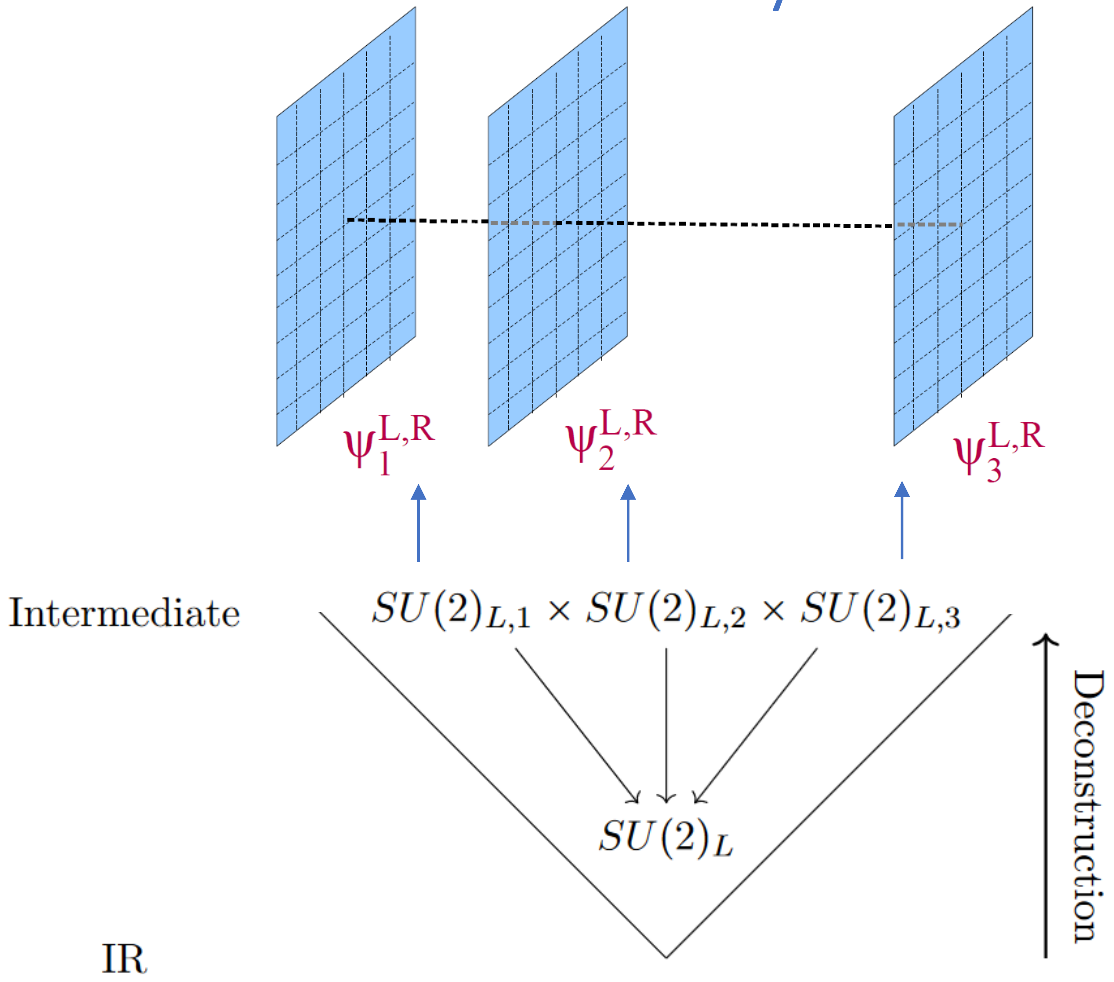
$$i(g, o) = \begin{pmatrix} \alpha.o & -\beta^*.o \\ \beta.o & \alpha^*.o \end{pmatrix} \in Sp(2n)\text{, acts on } (u_1, \dots, u_n, d_1, \dots, d_n)^T$$

So $Sp(6)_L$ does not act "block diagonally" on flavours:

$$Sp(6)_L \ni U = \begin{pmatrix} 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \\ 1 & . & . & 1 & . & . \\ . & 2 & . & . & 2 & . \\ . & . & 3 & . & . & 3 \end{pmatrix}$$

but it *does* contain a flavour-deconstructed subgroup $[SU(2)_L]^3$

Deconstructed flavour symmetries

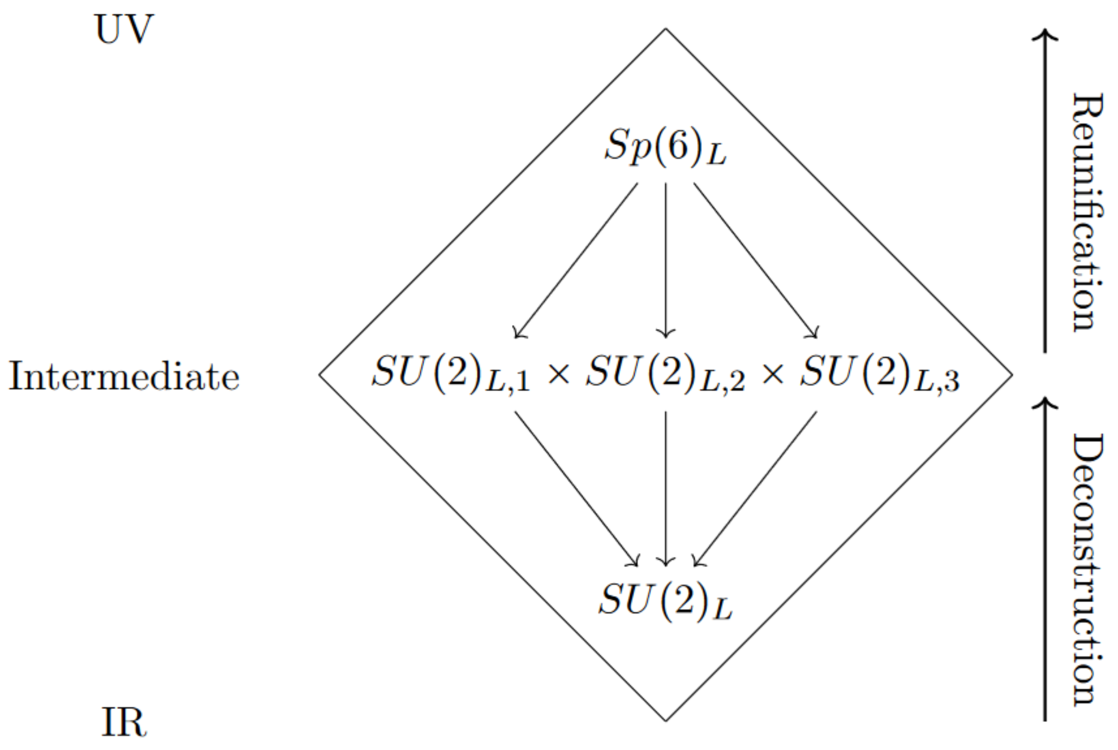


Deconstructed gauge groups have been used in flavour model building e.g. $G = \prod_i^3 PS_i$ for B-anomalies + fermion masses.

A relic of 5d physics?

Bordone, Cornella, Fuentes-Martín, Isidori, 1712.01368
Bordone, Cornella, Fuentes-Martín, Isidori, 1805.09328
Fuentes-Martín, Isidori, Pagès, Stefanek, 2012.10492
Fuentes-Martín, Isidori, Lizana, Selimovic, Stefanek, 2203.01952

Deconstructed flavour symmetries



Deconstructed gauge groups have been used in flavour model building e.g. $G = \prod_i^3 PS_i$ for B-anomalies + fermion masses.

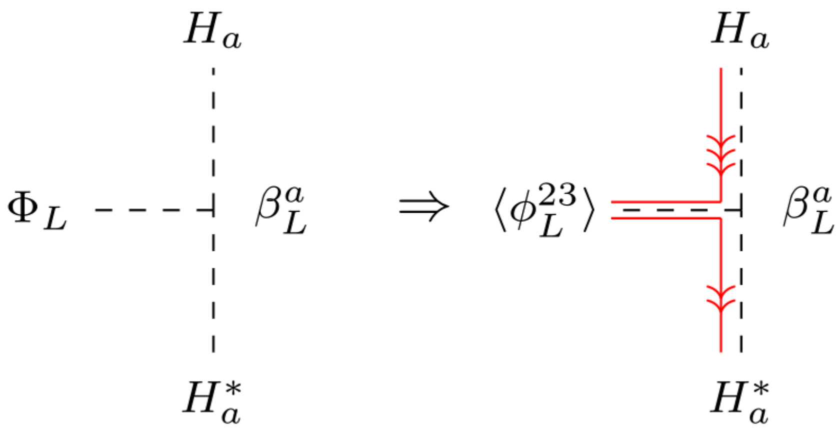
Here, "gauge-flavour unification" provides a natural 4d explanation of such a flavour-deconstructed gauge symmetry.

Terms in the scalar potential

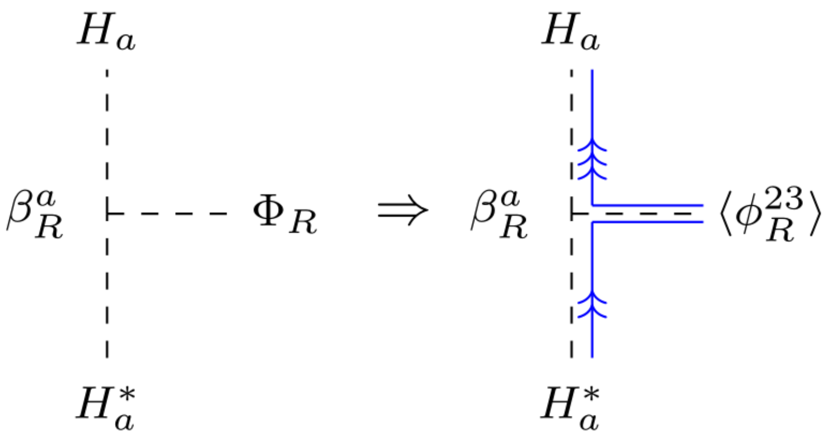
All the required EFT operators are already generated in our model, by integrating out the heavy components of $H_{1,15}$; if we include (renormalizable) interactions in the scalar potential.

Scalar Interactions

Cubics:



x1 $\mathbb{R}$ coupling (per a)



x1 $\mathbb{C}$ coupling (per a)

$\Lambda_L,\ \Lambda_R$

Λ_H

$\epsilon\Lambda_H$

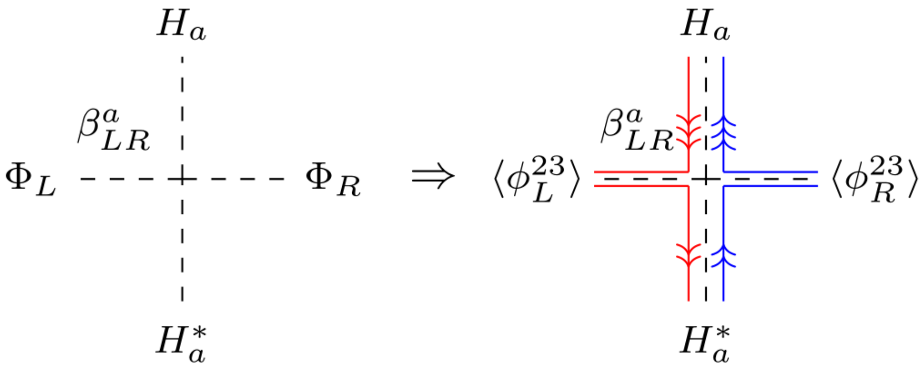
v

Terms in the scalar potential

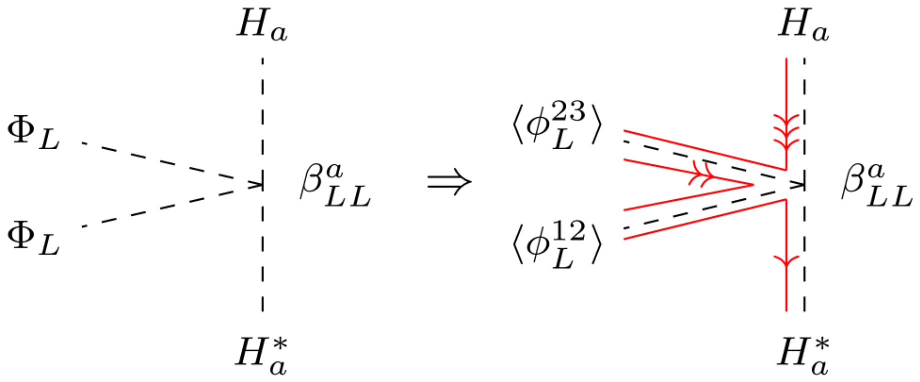
All the required EFT operators are already generated in our model, by integrating out the heavy components of $H_{1,15}$; if we include (renormalizable) interactions in the scalar potential.

Scalar Interactions

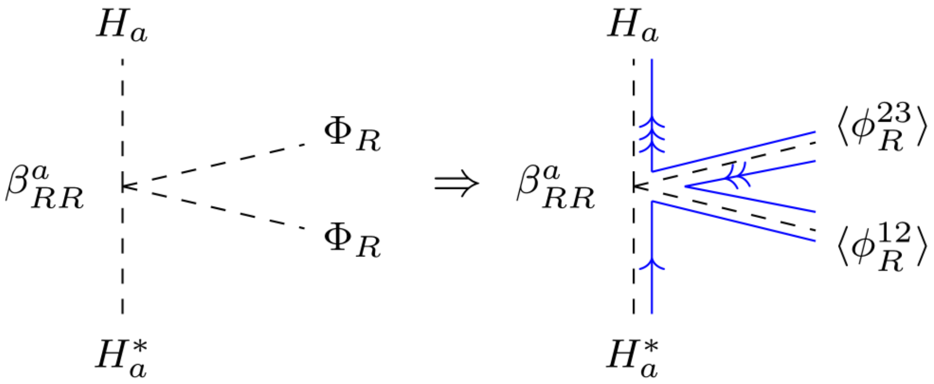
Quartics:



x1 $\mathbb{C}$ coupling (per a)



x1 $\mathbb{R}$ coupling (per a)



x1 $\mathbb{C}$ and x2 $\mathbb{R}$ couplings (per a)

$\Lambda_L,\ \Lambda_R$

Λ_H

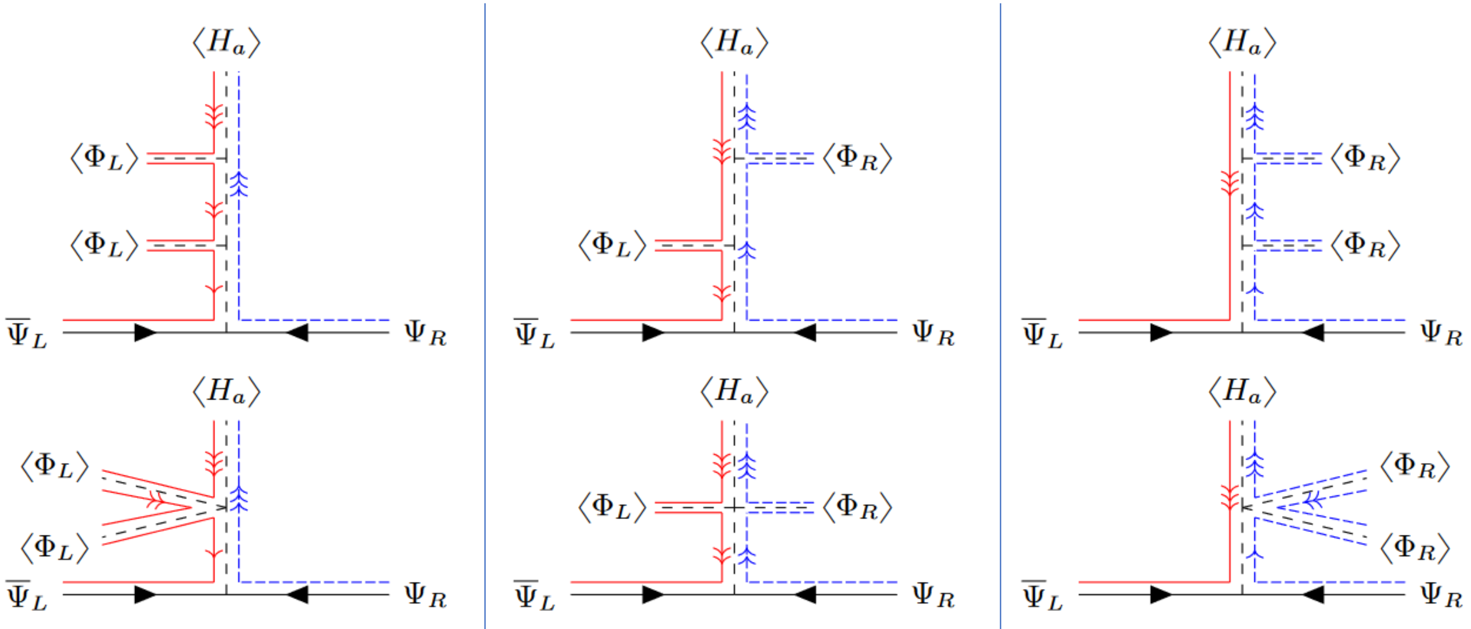
$\epsilon\Lambda_H$

v

EFT: light fermion Yukawas

Dimension 6: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi^2$

$$\begin{pmatrix} & & \times \\ & \times & \\ \times & & \end{pmatrix}$$

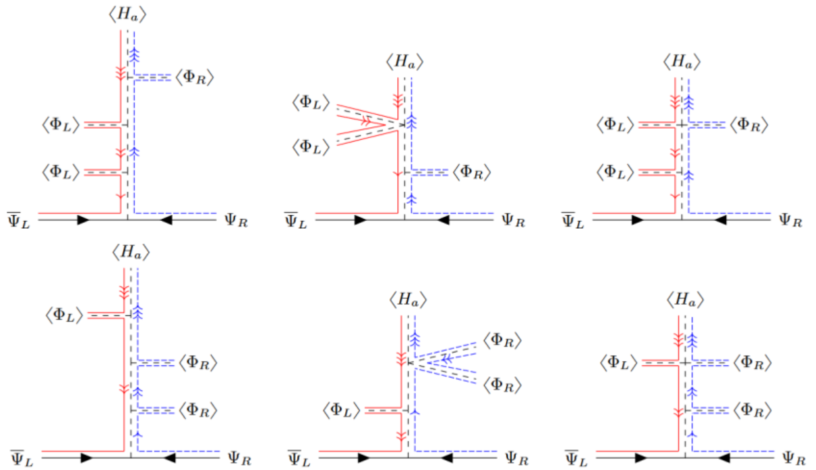


$Y_{13}^{u,d,e}$ $Y_{22}^{u,d,e}$ $Y_{31}^{u,d,e}$

EFT: light fermion Yukawas

Dimension 7: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi^3$

$$\begin{pmatrix} & \times & \\ \times & & \\ & & \end{pmatrix}$$

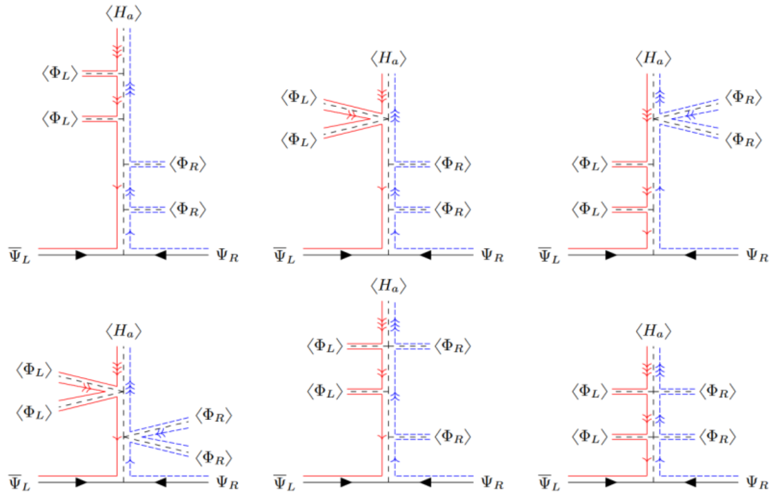


$Y_{12}^{u,d,e}$

$Y_{21}^{u,d,e}$

Dimension 8: $\mathcal{O} \sim \overline{\psi_L} H \psi_R \phi^4$

$$\begin{pmatrix} \times & & \\ & & \\ & & \end{pmatrix}$$



$Y_{11}^{u,d,e}$

Properties of our CKM model

Our CKM is not a general unitary matrix. Like Wolfenstein, it satisfies

$$|V_{ud}| = |V_{cs}|, \qquad |V_{ts}| = |V_{cb}|, \qquad |V_{ud}| = 1 - \frac{1}{2}|V_{us}|^2$$

at leading order. Also, Jarlskog invariant satisfies

$$4J^2 = 2|V_{us}V_{cb}|^2(|V_{ub}|^2 + |V_{td}|^2) + 2|V_{ub}V_{td}|^2 - |V_{td}|^4 - |V_{ub}|^4 - |V_{us}V_{cb}|^4$$

which implies CP-violating phase $\delta_{13} \approx 1.25$ radians.

All these relations agree well with data.

Also, $V_{td} = -V_{ub}^* + (V_{us}V_{cb})^*$.

<u>The upshot of these relations:</u>

If, in our model, we can fit V_{us}, V_{cb}, and V_{ub} to be arbitrary $\mathbb{C}$-numbers, then can freely fit $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, $|V_{td}|$ to their central experimental values, and the rest of CKM is in close agreement.

Fitting quark masses and mixings

Indeed there is enough freedom in the model to freely fit the coefficients of [all as $\mathbb{C}$-numbers]

- x9 masses (quarks and charged leptons)
- V_{us}, V_{cb}, and V_{ub}

<u>Sketch of how this works:</u>

1. Fit $\{m_t, m_b, m_\tau, V_{cb}\}$ from $\{y_1, y_{15}, \overline{y_1}, \overline{y_{15}}\}$, for any* values of $(\beta_L^1, \beta_L^{15})$

$$m_t \approx (y_1\overline{v}_1 + \overline{y}_1 v_1^*) + (y_{15}\overline{v}_{15} + \overline{y}_{15}v_{15}^*),$$
$$m_b \approx (y_1 v_1 + \overline{y}_1\overline{v}_1^*) + (y_{15}v_{15} + \overline{y}_{15}\overline{v}_{15}^*),$$
$$m_\tau \approx (y_1 v_1 + \overline{y}_1\overline{v}_1^*) - 3(y_{15}v_{15} + \overline{y}_{15}\overline{v}_{15}^*),$$

$$V_{cb} = \frac{\lambda^2}{2}\left\{\frac{\beta_L^1}{y_b}(y_1 v_1 + \overline{y}_1\overline{v}_1^*) + \frac{\beta_L^{15}}{y_b}(y_{15}v_{15} + \overline{y}_{15}\overline{v}_{15}^*) - \frac{\beta_L^1}{y_t^*}(\overline{y}_1^* v_1 + y_1^*\overline{v}_1^*) - \frac{\beta_L^{15}}{y_t^*}(\overline{y}_{15}^* v_{15} + y_{15}^*\overline{v}_{15}^*)\right\},$$

2. Fit $\{m_c, m_s, m_\mu, V_{us}, V_{ub}\}$ from $\{\beta_R^1, \beta_{LR}^1, \beta_{LL}^1, \beta_{LL}^{15}, w_{23}, \overline{w_{23}}\}$ …
3. Fit $\{m_u, m_d, m_e\}$ from $\{\beta_{RR}^1, w_{12}, \overline{w_{12}}\}$

The hierarchies are "in-built" from the dependence on $\epsilon_{L,R}^{12,23}$

Recall

$$\langle\Phi_R\rangle = \underbrace{\Lambda_H\epsilon_R^{23}w_{23}c_2 \wedge c_6}_{\phi_R^{23}} + \underbrace{\Lambda_H\epsilon_R^{23}\overline{w}_{23}c_3 \wedge c_5}_{\overline{\phi}_R^{23}} + \underbrace{\Lambda_H\epsilon_R^{12}(w_{12}c_1 \wedge c_5 + \overline{w}_{12}c_2 \wedge c_4)}_{\phi_R^{12}}$$

*Away from a small set of points