



The effect of light sea quark symmetry breaking on polarized nucleus and sum rules

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INTRODUCTION

Deep Inelastic Scattering (DIS) of leptons from nucleons is one of the new methods of understanding the complex internal structure of nucleons in current and future lepton and hadron colliders. Regarding this, the experimental groups such as E142, E143, COMPASS, HERMES, and JLAB have published their experimental results from huge particle accelerators [1–6]. The polarized structure functions of nucleons and nuclei, which are usually determined by the empirical results of these experiments at high energies, provide useful information about spin distribution on partons of nucleon and nucleons within the nucleus and assess the different models for understanding their structures [7]. In the simplest image of the ^3He nucleus, all nucleons are in the S state wherein two protons with opposite spins exist, so their spins in the asymmetry are completely nullified and the nucleus polarization is determined solely by neutron spin. Therefore, the use of ^3He targets in DIS experiments of leptons from the polarized target is common and is considered as the alternative target of the neutron. The same thing happens for ^3H by replacing neutrons with protons. However, in more precise calculations and by considering other components of the three-particle wave, such as S' and D states, the protons spins are no longer nullified in the ^3He structure function and must be considered. Also it is notable that knowing the distribution of momentum and the energy of the electron scattered off the nucleon limits the probability of obtaining information about the nucleon structure function via scatterings off the nucleus targets [8].

The main purpose of these experiments is to assess the quantity of the nucleons spin fractions carried by the quark and gluon. In most of the verified phenomenological models, the nucleon spin fractions carried by the sea quarks are considered equal, and symmetry breaking is not considered, i.e. $\delta\bar{u} = \delta\bar{d} = \delta\bar{s}$ [9–49]. However, in some of the more precise models, where both flavor SU(2) and SU(3) symmetry breaking is taken into consideration, the nucleon spin fraction carried by light sea quarks are considered unequal as $\delta\bar{u} \neq \delta\bar{d} \neq \delta\bar{s}$ [50–55]. In the current research, both of the mentioned phenomenological models are applied and the polarized structure functions of nucleons and nuclei calculated through the results of these two models are compared. In the analysis of the NAMY21 model [16], the polarized deep inelastic scattering data are used, and the polarized parton distribution functions of protons, neutrons, and deuterons are calculated in NLO approximation, disregarding symmetry breaking. In the second phenomenological model AKS14 [54], the asymmetry data of inclusive and semi-inclusive polarized deep inelastic scattering are used and both flavor SU(2) and SU(3) symmetry breaking are taken into consideration. The Pegasus software package [56] was used for both of these analyses and fitting was performed on the experimental data. The number of experimental data is 863 for NAMY21 [16] analysis and 1149 for AKS14 [54] analysis. Finally, the polarized parton distribution functions are calculated in NLO approximation. In the current article, after calculating the momentum of the polarized structure functions created by the parton distribution functions of the two aforementioned models in the Mellin transform, the DGLAP equations [7] are solved. Then, using Jacobi polynomials, the polarized structure functions of nucleons are calculated in the Bjorken x variable space. Finally, the polarized structure functions of the light nucleuses of Helium-3 (^3He) and tritium (^3H) are extracted

BJORKEN SUM RULE

$$\eta \equiv \frac{g_A|_{\text{triton}}}{g_A} = \frac{\int_0^1 [g_1^3\text{H}(x, Q^2) - g_1^3\text{He}(x, Q^2)] dx}{\int_0^1 [g_1^p(x, Q^2) - g_1^n(x, Q^2)] dx} = 0.956 \pm 0.004.$$

In our calculations, we obtained a value of 0.936376 for NAMY21 [16] model and 0.954312 for AKS14 [54]. This result shows that the analysis considering symmetry breaking yields a value of the ratio η closer to Bjorken sum rule than the value of analysis disregarding symmetry breaking.

EFREMOV-LEADER-TERYAEV (ELT) SUM RULE

The Efremov-Leader-Teryaev (ELT) sum rule can be obtained via integrating the valence part of g_1 and g_2 structure functions over x variable of Bjorken in the limit $m_q \rightarrow 0$ [84]

$$\int_0^1 x [g_1^V(x) + 2g_2^V(x)] dx = 0. \quad (27)$$

where $g_{1(2)}^V$ denotes the valence quark contributions to $g_{1(2)}$. When symmetry of light sea quarks is considered and they are assumed to carry equal fraction of spin in protons and neutrons ELT sum rule can be written as

$$\int_0^1 x [g_1^p(x) - g_1^n(x) + 2(g_2^p(x) - g_2^n(x))] dx = 0. \quad (28)$$

Considering light sea quarks symmetry breaking the ELT sum rule is derived directly from Eq. 27. The value of left hand side of above equation is obtained -0.011 ± 0.008 from E155[85] analysis at $Q^2 = 5 \text{ GeV}^2$. This value is obtained 0.01017 ± 0.00004 and -0.030763 ± 0.0004071 from NAMY21 and AKS14 respectively. It seems the consideration of symmetry breaking makes the results negative and closer to E155 analysis. Also it is concluded that disregarding the symmetry of $\delta\bar{u}$, $\delta\bar{d}$ and $\delta\bar{s}$ can be effective on the distance of EFM sum rule from zero.

CONCLUSIONS

Figures 5, 6, and 7 compare polarized structure functions of proton, neutron, and deuteron from AKS14 and NAMY21 with E143 experimental data at $Q^2 = 2 \text{ GeV}^2$. According to the figures, when symmetry breaking is taken into consideration by a model, the extracted results are better than when symmetry breaking is disregarded. Figure 9 shows $g_1^{^3\text{He}}$ polarized structure function from E142 and JLAB experimental data and compares them with the same extractions from three analyses of NAMY21, AKS14 and TKA17 in NLO and NNLO approximations. As shown in the figures, the polarized nuclei structure function extracted from AKS14 model, with breaking the symmetry of light sea quarks, passes more data than other models, specially at $0.1 \leq x \leq 0.4$ region. Figure 10 compares $g_1^{^3\text{H}}$ structure function extracted from three mentioned models with HERMES experimental data. Again the extracted nuclei polarized structure function from AKS14 model passes more data points than the other models considering the symmetry of light sea quarks. In Fig 11, the $x^2 g_1^{^3\text{He}}$ structure function is compared with E142, JLAB04, JLAB03 and JLAB16 experimental data in NLO and NNLO approximations. The results look better at small x for AKS14 model with symmetry breaking consideration. Observing that the peaks and valleys of light sea quarks spin distribution graphs of AKS14 model are in the range of $x \leq 0.4$, their effects in these areas will be more significant. This prediction is also proven by the results of the present article in most of the figures. In Figs 12 and 13, the $g_2^{^3\text{He}}$ structure func-

In conclusion, the presented results demonstrates that a detailed description of the spin structure of the nucleon, nuclei, sum rules and Lorentz color force components of polarized structure functions often could be achieved when both flavor SU(2) and SU(3) symmetry breaking are considered and three flavour of light sea quarks do not carry the same fraction of nucleon spin in the analysis.

POLARIZED STRUCTURE FUNCTIONS OF NUCLEONS

After determining the moment of structure functions, they can be recreated in the Bjorken x space utilizing the advantage of Jacobi polynomials in separating the dependence of structure function to x and Q^2 [11]:

$$x g_1(x, Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{\text{max}}} a_n(Q^2) \Theta_n^{\alpha, \beta}(x), \quad (6)$$

Based on the orthogonality condition the Jacobi moments, $a_n(Q^2)$, can be obtained as [61–65]:

$$a_n(Q^2) = \int_0^1 dx x g_1(x, Q^2) \Theta_n^{\alpha, \beta}(x) = \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) \mathcal{M}[x g_1, j+2](Q^2), \quad (9)$$

Now by substituting Eq. (9) into Eq. (6) the polarized structure function $x g_1(x, Q^2)$, based on Jacobi polynomial expansion method, can be constructed. Therefore the following expression for $x g_1(x, Q^2)$ would be obtained

$$x g_1(x, Q^2) = x^\beta (1-x)^\alpha \sum_{n=0}^{N_{\text{max}}} \Theta_n^{\alpha, \beta}(x) \times \sum_{j=0}^n c_j^{(n)}(\alpha, \beta) \mathcal{M}[x g_1, j+2](Q^2). \quad (10)$$

The polarized parton distribution function for valance quarks, sea quarks and gluon in the NAMY21 model is chosen according to the following parametrization:

$$x \delta q(x, Q_0^2) = \mathcal{N}_q \eta_q x^{\alpha_q} (1-x)^{\beta_q} (1 + c_q x^{0.5} + d_q x). \quad (11)$$

In the AKS14 model, the polarized parton distribution function, due to the presence of more data for controlling the middle parts of the Bjorken variable, was chosen as below:

$$x \delta q = \mathcal{N}_q \eta_q x^{\alpha_q} (1-x)^{\beta_q} (1 + c_q x^{0.5} + d_q x), \quad (13)$$

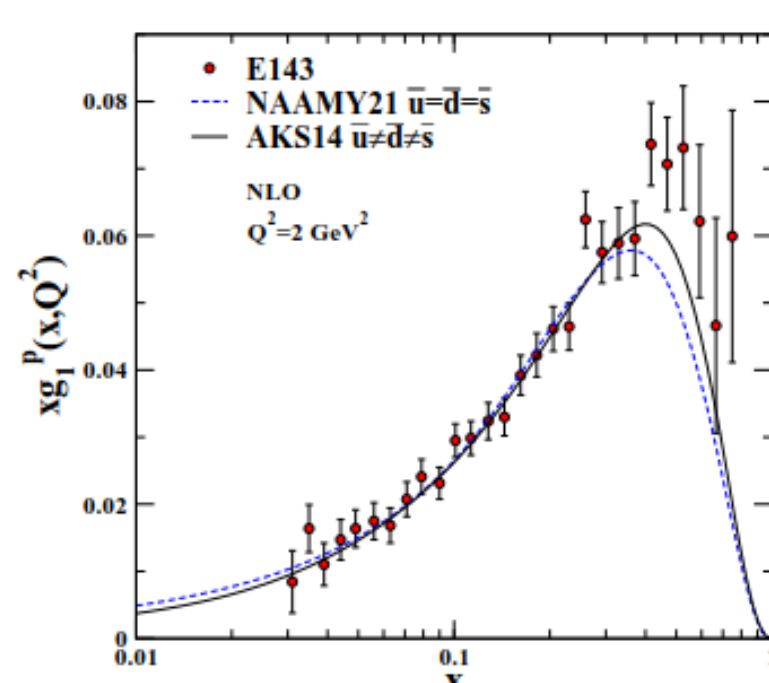


Figure 5: Proton polarized structure function at $Q^2 = 2 \text{ GeV}^2$, compared with E143 experimental data [2].

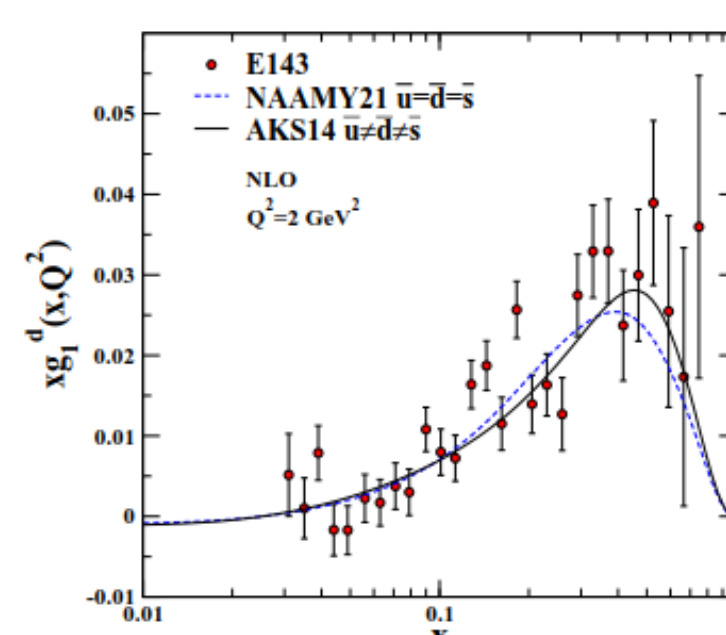


Figure 7: Deuteron polarized structure function at $Q^2 = 2 \text{ GeV}^2$, compared with E143 experimental data [2].

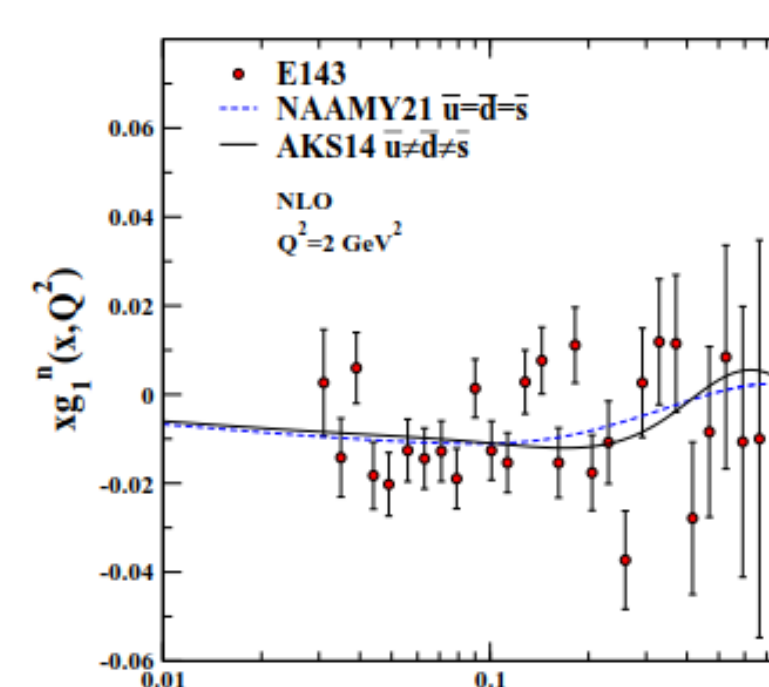


Figure 6: Neutron polarized structure function at $Q^2 = 2 \text{ GeV}^2$, compared with E143 experimental data [2].

POLARIZED STRUCTURE FUNCTIONS OF NUCLEI

For nuclei, the polarized structure function equation will be written as:

$$g_1^{^3\text{He}} = \int_x^3 \frac{dy}{y} \Delta f_{^3\text{He}}^n(y) g_1^n(x/y) + 2 \int_x^3 \frac{dy}{y} \Delta f_{^3\text{He}}^p(y) g_1^p(x/y) - 0.014 (g_1^p(x) - 4g_1^n(x)), \quad (18)$$

and

$$g_1^{^3\text{H}} = 2 \int_x^3 \frac{dy}{y} \Delta f_{^3\text{H}}^p(y) g_1^p(x/y) + \int_x^3 \frac{dy}{y} \Delta f_{^3\text{H}}^n(y) g_1^n(x/y) + 0.014 (g_1^p(x) - 4g_1^n(x)). \quad (19)$$

the calculation of their polarized structure function is performed using the contribution of proton and neutron polarized structure function in addition to the spin-dependent nucleon light-cone momentum distributions $\Delta f_{^3\text{He}}^n$ and $\Delta f_{^3\text{H}}^n$

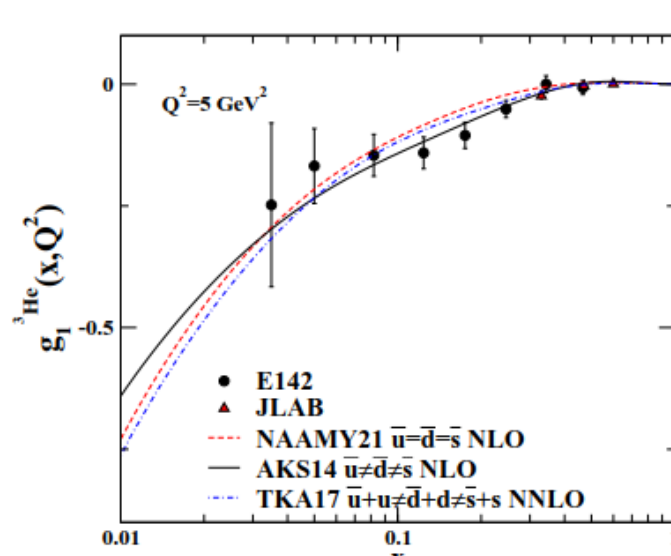


Figure 9: $g_1^{^3\text{He}}$ polarized structure functions in NLO and NNLO approximation from Refs. [16, 54] and [50] compared with E142 [1], JLAB01 [78], JLAB03 [79] and JLAB16 [6] experimental data.

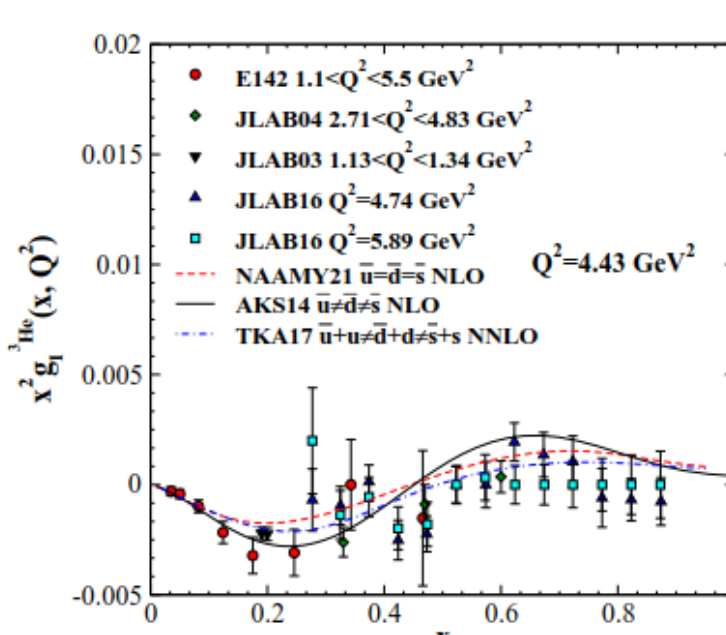


Figure 11: $x^2 g_1^{^3\text{He}}$ polarized structure functions in NLO and NNLO approximation from Refs. [16, 54] and [50] compared with E142 [1], JLAB01 [78], JLAB03 [79] and JLAB16 [6] experimental data.

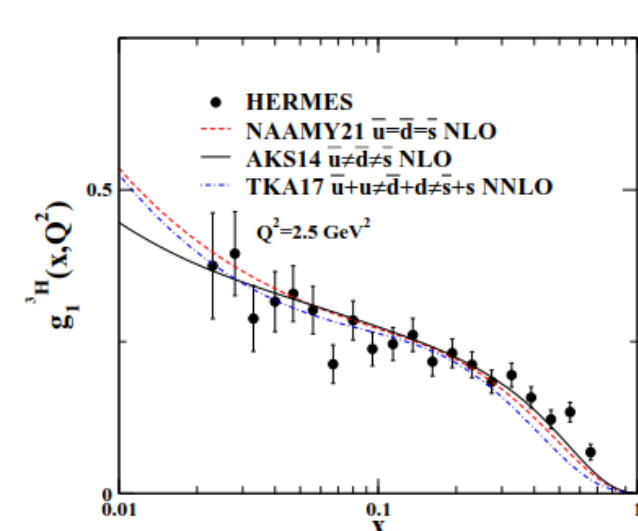


Figure 10: $g_1^{^3\text{H}}$ polarized structure functions in NLO and NNLO approximation from Refs. [16, 54] and [50] compared with HERMES [4] experimental data.

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