Soft photon bremsstrahlung at next-to-leading power

Domenico Bonocore, Anna Kulesza

Institut für Theoretische Physik, Westfälische Wilhelms-Universität Münster, Germany

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An experimental conundrum

Theoretical descriptions of soft photon emission spectra typically rely on a formula based on the **Leading Power (LP)** eikonal approximation, where the photon momentum $k \to 0$:

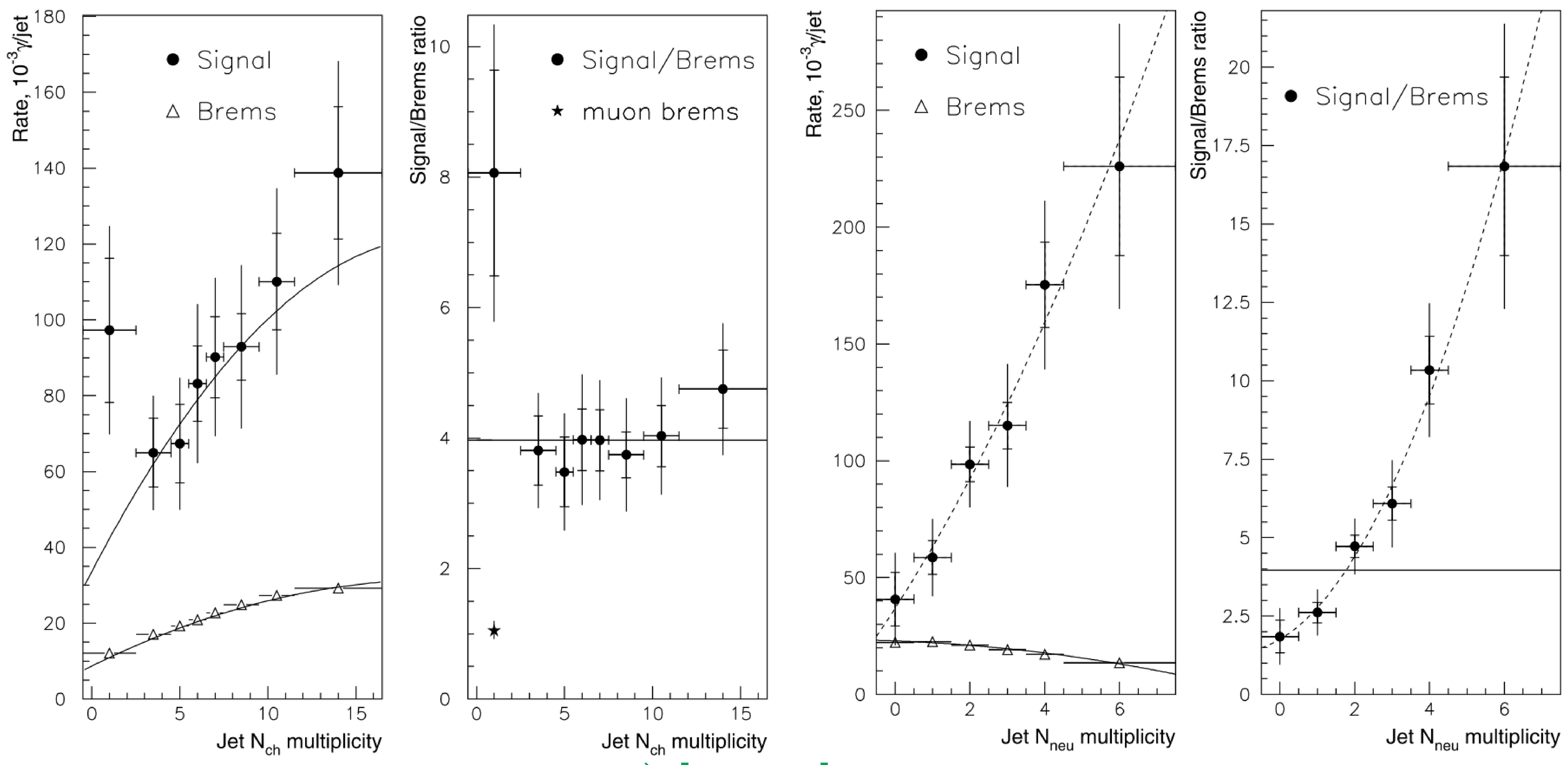
$$\frac{d\sigma_{\rm LP}}{d^3k} = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega_k}\int d^3p_3 \cdots \int d^3p_n \left(\sum_{i,j=1}^{n} -\eta_i\eta_j \frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}\right) d\sigma_H(p_1,\dots,p_n)$$

LP formula is universal, with no loop corrections, insensitive to spin and recoil, in agreement with classical power spectrum $\frac{d\sigma}{d\omega_k} \sim \frac{1}{\omega_k}$.

However, it is in **poor agreement with data** when hadrons are present [1]:

Experiment	Energy	Photon k_t	exp/th
K^+p (BEBC,1984)	70 GeV	< 60 MeV	4.0 ± 0.8
K^+p (EHS, 1993)	250 GeV	< 40 MeV	6.4 ± 1.6
π^+p (EHS, 1997)	250 GeV	< 40 MeV	6.9 ± 1.3
π^-p (OMEGA, 1997)	280 GeV	< 10 MeV	7.9 ± 1.4
π^+p (OMEGA, 2002)	280 GeV	< 20 MeV	5.3 ± 0.9
pp (OMEGA, 2002)	450 GeV	< 20 MeV	4.1 ± 0.8
$e^+e^- \to$ hadrons (DELPHI, 2010)	91 GeV	< 60 MeV	~ 4.0
$e^+e^- \to \mu^+\mu^-$ (DELPHI, 2008)	91 GeV	< 60 MeV	~ 1.0

$e^+e^- \to$ hadrons [3], photon range: 200 MeV $< \omega_k <$ 1 GeV

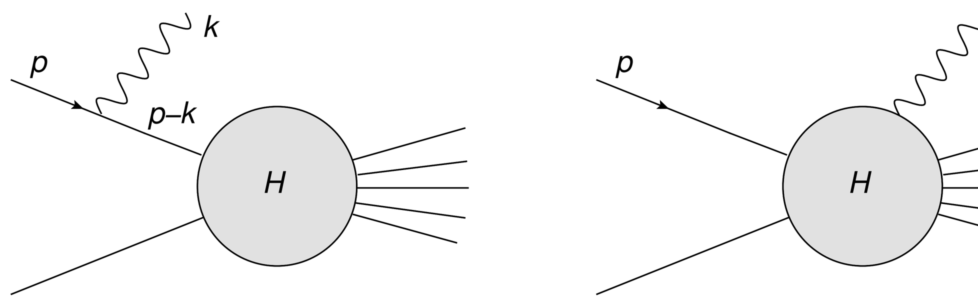


Future measurements with the **ALICE** detector are under consideration [2].

Idea: building on recent progress in threshold resummation (where the soft boson is an *undetected* gluon) [4, 5, 6], investigate the impact of Next-to-Leading Power (NLP) corrections to the strict $k \to 0$ limit in the case of a *detected* photon.

NLP via LBK theorem

At the **tree-level**, NLP soft emissions are governed by **next-to-soft theorems**, first studied in QED by Low, Burnett and Kroll (LBK) [7, 8]:



$$\mathcal{A}_{n+1} = (\mathcal{S}_{LP} + \mathcal{S}_{NLP})\,\mathcal{A}_n\ ,$$

$$\mathcal{S}_{LP} = \sum_{i=1}^{n} q_i \frac{\epsilon^*(k)\cdot p_i}{p_i\cdot k}\ , \qquad \mathcal{S}_{NLP} = \sum_{i=1}^{n} q_i \frac{\epsilon^*_\mu(k)k_\nu(S^{\mu\nu}+L^{\mu\nu})}{p_i\cdot k}\ .$$

Note the coupling with the spin generator $S^{\mu\nu}$ and the orbital angular momentum $L^{\mu\nu}$ of the hard emitter. Hence, NLP soft emissions are sensitive to **spin** and **recoil** of the hard emitter.

At the cross-section level, for numerical purposes, it is convenient to express LBK theorem with **shifted kinematics**:

$$|\mathcal{A}(p_1,\dots,p_n,k)|^2 = \underbrace{\left(\sum_{i,j=1}^{n} -\eta_i\eta_j \frac{p_i\cdot p_j}{p_i\cdot k\, p_j\cdot k}\right)}_{\text{LP factor!}} |\mathcal{A}(p_1+\delta p_1,\dots,p_n+\delta p_n)|^2$$

where shifts are defined as

$$\delta p_\ell^\mu = \left(\sum_{i,j=1}^{n} \eta_i\eta_j \frac{p_i\cdot p_j}{p_i\cdot k\, p_j\cdot k}\right)^{-1} \sum_{m=1}^{n}\left(\eta_m\eta_\ell \frac{(p_m)_\nu G_\ell^{\mu\nu}}{p_m\cdot k}\right)$$

and

$$G_i^{\mu\nu} = g^{\mu\nu} - \frac{(2p_i-k)^\mu k^\nu}{2p_i\cdot k} = g^{\mu\nu} - \frac{p_i^\mu k^\nu}{p_i\cdot k} + \mathcal{O}(k)$$

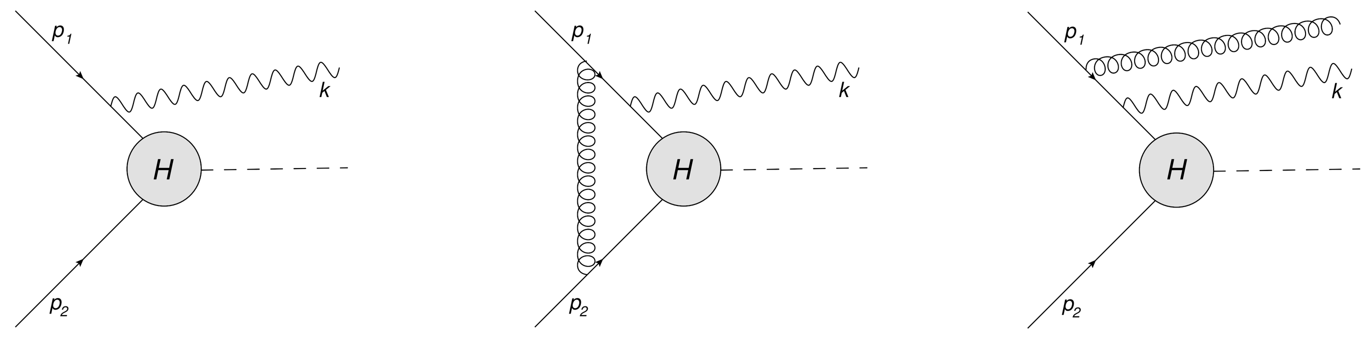
Shifted kinematics allows to efficiently implement LBK theorem in the bremsstrahlung cross-section:

$$\frac{d\sigma_{\text{NLP-tree}}}{d^3k} = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega_k}\int d^3p_3\cdots\int d^3p_n \left(\sum_{i,j=1}^{n} -\eta_i\eta_j\frac{p_i\cdot p_j}{(p_i\cdot k)(p_j\cdot k)}\right)$$
$$(1-(1-z)R_{12})\ d\sigma_H(p_1+\delta p_1,\dots,p_n+\delta p_n)$$

- **Correction of order** $(\omega_k)^0 \sim 1$ **to LP spectrum**

NLP with QCD loop corrections

At NLP, soft theorems receive **loop** corrections. Virtual collinear effects are captured by **radiative jet functions** J^μ [9].



In particular, for a process with a single quark-antiquark pair in the **massless limit** (such as $e^+e^- \to q\bar{q}\gamma$), the leading correction to the LBK contribution comes from the quark radiative jet, which in dimensional regularization (with $d = 4-2\varepsilon$ and $\bar{\mu}$ the $\overline{\rm MS}$ scale) reads at one-loop [4]

$$J^{\mu(1)} = \left(\frac{\bar{\mu}^2}{2p\cdot k}\right)^{\varepsilon}\left[\left(\frac{2}{\varepsilon}+4+8\varepsilon\right)\left(\frac{n\cdot k}{p\cdot k}\frac{p^\mu}{p\cdot n} - \frac{n^\mu}{p\cdot n}\right) - (1+2\varepsilon)\frac{ik_\alpha S^{\alpha\mu}}{p\cdot k}\right.$$
$$\left. + \left(\frac{1}{\varepsilon}-\frac{1}{2}-3\varepsilon\right)\frac{k^\mu}{p\cdot k} + (1+3\varepsilon)\left(\frac{\gamma^\mu \not{n}}{p\cdot n} - \frac{p^\mu}{p\cdot k}\frac{\not{k}\not{n}}{p\cdot n}\right)\right] + \mathcal{O}(\varepsilon^2, k)\ .$$

Thus, the next-to-soft theorem receives a **logarithmic correction**:

$$\left(\sum_i \epsilon^*_\mu(k)\, q_i\, J_i^{\mu(1)}\right)\mathcal{A}_n = \frac{2}{p_1\cdot p_2}\left[\sum_{ij}\left(\frac{1}{\varepsilon} + \log\left(\frac{\bar{\mu}^2}{2p_i\cdot k}\right)\right) q_j\, p_i\cdot k\, \frac{p_j\cdot\epsilon}{p_j\cdot k}\right]\mathcal{A}_n$$

The soft photon bremsstrahlung at $\mathcal{O}(\alpha_s)$ becomes

$$\frac{d\sigma_{\rm NLP}}{d^3k} = \frac{d\sigma_{\text{NLP-tree}}}{d^3k} + \frac{\alpha_s}{4\pi}\frac{d\sigma_{\text{NLP-J}}}{d^3k}\ ,$$

where

$$\frac{d\sigma_{\text{NLP-J}}}{d^3k} = \frac{\alpha}{(2\pi)^2}\frac{1}{\omega_k}\int d^3p_3\dots d^3p_n\left(\sum_{i=1}^{2}\eta_i\frac{8\log\left(\frac{\bar{\mu}^2}{2p_i\cdot k}\right)}{p_i\cdot k}\right) d\sigma_H(p_n)$$

- **Correction of order** $\alpha_s \log\left(\frac{\bar{\mu}^2}{2p_i\cdot k}\right)$ **to LP spectrum, i.e. particularly enhanced for small** ω_k **and small** k_t

References

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