High precision calculations for the MUonE experiment

ICHEP 2022, Bologna

Ettore Budassi

08/07/2022

¹University of Pavia and INFN Pavia

In collaboration with: C. M. Carloni Calame, M. Chiesa, C. L. Del Pio, S. M. Hasan, G. Montagna, O. Nicrosini and F. Piccinini
The muon $g - 2$ & Spacelike approach
Starting point: $g_\mu - 2$ & Theoretical Approaches

$$a_{\mu}^{SM} \times 10^{11} = 116591810(43)$$
$$a_{\mu}^{EXP} \times 10^{11} = 116592061(41)$$

$$a_{\mu}^{QED} \times 10^{11} = 116584718.931(104)$$
$$a_{\mu}^{EW} \times 10^{11} = 153.6(1.0)$$
$$a_{\mu}^{HLbL} \times 10^{11} = 92(18)$$
$$a_{\mu}^{HVP} \times 10^{11} = 6845(40)$$

$$a_{\mu}^{SM} \times 10^{11} = 116591810(43)$$

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Timelike & Spacelike approaches

Timelike approach:

$$a^\text{HLO}_\mu = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \left[ \int_{m_\pi^2}^{\infty} ds \frac{K(s)R^\text{had}(s)}{s^2} \right];$$

$$R^\text{had}(s) = \frac{\sigma(e^+e^- \rightarrow \text{had, } s)}{\frac{4}{3} \pi \alpha^2 \frac{s}{s}}$$

Spacelike approach:

$$a^\text{HLO}_\mu = \frac{\alpha}{\pi} \int_0^1 dx (1 - x) \Delta\alpha^\text{had} [t(x)];$$

$$t(x) = \frac{x^2 m_\mu^2}{x - 1} < 0$$


**Going spacelike**

- $\Delta\alpha_{\text{had}}$: measured in a single experiment with a spacelike process.
- A high-precision experiment is needed: 10 ppm.

$\mu e$ scattering on a low $Z$ target is an ideal process:
- pure $t$-channel process
- M2 muon beam ($E_\mu \approx 160$ GeV) is available at CERN
- $\sqrt{s} \approx 0.4$ GeV and $-0.143 < t < 0$ GeV$^2$. We can cover 87% of the integral with data. We can then extrapolate up to $x \to 1$.

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State of the art of $\mu e \rightarrow \mu e$ scattering calculations

**Theory work**

- M. Alacevich, C. M. Carloni Calame et al., JHEP 02 (2019) 155;
- M. Fael, JHEP02 (2019) 027;
- C.M. Carloni Calame et al., Towards muon-electron scattering at NNLO, JHEP 11 (2020) 028;
- E. Budassi et al., JHEP 11 (2021) 098;
- E. Balzani, S. Laporta, M. Passera, arXiv: 2112.05704 [hep-ph] [S. Laporta's Talk];

**Numerical implementations for $\mu e$ scattering NLO and NNLO: State of the Art**

- **MESMER** (Monte Carlo Event Generator)
- **McMULE** (Monte Carlo Integrator)

**Possible “Contamination” from New Physics**

Towards muon-electron scattering at NNLO in QED
Photonic NNLO corrections: exact contributions

- Virtual NNLO photonic contributions are included exactly for electron or muon leg emission. 2-loop QED vertex from factors taken from Mastrolia and Remiddi.

- 1-loop corrections to real photon emission exactly included: e.g. pentagon diagrams.

- Double real emission included exactly.

Photonic NNLO corrections: approximated contributions

- Of the 2-loop virtual diagrams with a virtual photon insertion on top of NLO boxes, only the IR part is included exactly (YFS).
- The non-IR remnants are approximate.
- All photonic NNLO effects weigh at most some % at the Phase Space boundaries.
- Work is in progress for the full $\mu e \rightarrow \mu e$ at NNLO (Padova&PSI).

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NNLO Lepton Pair Contributions: Virtual

\[ d\sigma_{N_f}^{\alpha^2} = d\sigma_{\text{virt}}^{\alpha^2} + d\sigma_{\gamma}^{\alpha^2} + d\sigma_{\text{real}}^{\alpha^2} \]

- Integration over \( z \) is performed numerically with MC techniques.
- **Master Integral** techniques for a subset of such diagrams to cross-check results.
- Interplay between real photon radiation and leptonic loop insertions.
- IR divergences are cancelled by a sub-set of the virtual contributions.

\[ \frac{-ig_{\mu\nu}}{q^2 + i\epsilon} \to -ig_{\mu\nu} \left( \frac{\alpha}{3\pi} \right) \int_{4m^2_\ell}^{\infty} \frac{dz}{z} \times \frac{1}{q^2 - z + i\epsilon} \left( 1 + \frac{4m^2_\ell}{2z} \right) \sqrt{1 - \frac{4m^2_\ell}{z}}. \]

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\]
NNLO Lepton Pair Contributions: Real

\[ d\sigma^{\alpha^2}_{N_f} = d\sigma^{\alpha^2}_{\text{virt}} + d\sigma^{\alpha^2}_{\gamma} + d\sigma^{\alpha^2}_{\text{real}} \]

- \(2 \rightarrow 4\) LIPS.
- The QED matrix elements have been calculated with FORM and cross-checked with RECOLA.
- Cuts: a set of elasticity cuts must be imposed to reduce a potentially large background

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B. Ruijl, T. Ueda and J. Vermaseren, FORM version 4.2.

Real NNLO Lepton Pair Contributions: Results

Take-Home Message and Outlook

- Important efforts to develop NNLO fixed-order Monte Carlo event generators for $\mu e$ scattering.
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- We studied NNLO Photonic corrections exactly except for a subset (YFS). Effects weigh some % at PS boundaries (small $\vartheta_e$, large $|t_{ee}|$).

- NNLO $e^+e^-$ Real pair production could be a potential background: effects are controlled if cuts are applied.
- $\mu e \rightarrow \mu e \pi^0$ has been studied as a possible background process (C. L. Del Pio’s poster).
- Higher-order QED corrections must be included to reach the required precision, e.g. by matching a QED Parton Shower with exact NNLO matrix elements.
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• NNLO Virtual Lepton Pair contributions weigh $10^{-4}$ to $10^{-3}$. $e^+e^-$ emission is dominant w.r.t. $\mu^+\mu^-$.
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Backup slides
Backup: NLO EW corrections

![Graphs showing NLO EW corrections for different variables](image_url)
Backup: $\Delta \alpha$
At NLO for virtual box diagrams, YFS misses terms of order:

$$\frac{\alpha}{\pi} \ln \frac{m_{\mu}^2}{m_e^2} \simeq 0.025.$$ 

Therefore, for NNLO boxes YFS is expected to be accurate up to terms of order:

$$\left( \frac{\alpha}{\pi} \right)^2 \ln^2 \frac{m_{\mu}^2}{m_e^2} \simeq 6 \times 10^{-4}.$$ 

Improving the accuracy requires the inclusion of exact NNLO boxes, at least their leading terms in $m_e$. 
Backup: YFS @ NLO

\[ \frac{100 \times (d\sigma_{\text{approx.}}^{\text{NLO}} - d\sigma_{\text{LO}})}{d\sigma_{\text{LO}}} \]

\[ t_{ee} \text{ (GeV}^2) \]
\[ \vartheta_e \text{ (mrad)} \]
Backup: \( \mu e \rightarrow \mu e \pi_0 \)

- \( \mu^+ \mu^- \) production is negligible without cuts & goes to zero with acceptance cuts.
- Hadronic production: \( \pi^+ \pi^- \) production and \( \pi^0 \) production.
- \( \pi^+ \pi^- \) is more suppressed than \( \mu^+ \mu^- \) production since \( m_\pi > m_\mu \).

\[ \mathcal{L}_{\text{int}} = \frac{g}{2!} F_{\mu\nu} \tilde{F}_{\mu\nu}. \]

\( \pi_0 \) production is well under the experimental resolution of \( \sim 10 \) ppm.
Backup: Real NNLO Lepton Pair Contributions: More Results

|e + μ rad + periph|^2 [μ^+]  
|e + μ rad + periph|^2 [μ^-]  
|e + μ rad|^2 [μ^+]  
|e + μ rad|^2 [μ^-]  

with identical particles, 
symmetric cuts

\[\theta_e, \theta_\mu > 0.2 \, \text{mrad}, \xi < 3.5 \, \text{mrad}, \delta < 0.2 \, \text{mrad}\]
Backup: Muon pair production

• $\mu e \rightarrow \mu e + \mu^+ \mu^-$ contributions are well below 10 ppm without cuts.

• By imposing standard (symmetrical) cuts, the process is kinematically forbidden.

Elasticity curve can be parametrised as follows:

\[ \theta_\mu (\theta_e) = \arctan \left[ \frac{2m_e r \cos \theta_e \sin \theta_e}{E^i_\mu - r (rE^i_\mu + 2m_e) \cos^2 \theta_e} \right], \]

where \( r \) is defined as:

\[ r = \sqrt{(E^i_\mu)^2 - m^2_\mu} \]

\[ \frac{E^i_\mu + m_e}{E^i_\mu + m_e} \]

and \( E^i_\mu \) is the incident muon energy in the laboratory reference frame.
• **Basic acceptance cuts**

• When we have 4 particles in the final state we require that only 2 are detected \((E_i > 200\) MeV and \(\vartheta_i < 100\) mrad).

On top of it, we added 3 selection cuts to select elastic events:

• **cut 1**: \(\vartheta_e > 0.2\) mrad and \(\vartheta_\mu > 0.2\) mrad

• **cut 2**: \(\xi = |\pi - |\phi_e - \phi_\mu|| < \xi_c = 3.5\) mrad

• **cut 3**: Elasticity distance \(\delta < \delta_c = 0.2\) mrad. \(\delta\) is defined as the distance from the elastic curve:

\[
\delta = \min_{\theta_e} \sqrt{(\theta_e - \theta^0_e)^2 + (\theta_\mu(\theta_e) - \theta^0_\mu)^2}.
\]

\[ \Delta_{i}^{\text{NNLO}} = \frac{d\sigma_{i}^{\text{NNLO}}}{d\sigma_{i}^{\text{LO}}} \times 100 \]

Backup: Real NNLO Lepton Pair Contributions: Results

NLO contributions:

\[ \sigma_{\text{NLO}} = \sigma_{2\rightarrow2} + \sigma_{2\rightarrow3} \]

- Leading Order and NLO virtual contributions:

\[ \sigma_{2\rightarrow2} = \sigma_{\text{LO}} + \sigma_{\text{NLO}}^{v} = \frac{1}{F} \int d\Phi_{2} \left\{ |\mathcal{M}_{\text{LO}}|^{2} + 2 \text{Re} \left[ \mathcal{M}_{\text{LO}}^{\dagger} \mathcal{M}_{\text{NLO}}^{v}(\lambda) \right] \right\} \]

- NLO Real contributions:

\[ \sigma_{2\rightarrow3} = \frac{1}{F} \left( \int_{\lambda < E_{\gamma} < \Delta E} d\Phi_{3} |\mathcal{M}_{\text{NLO}}^{\gamma}|^{2} + \int_{E_{\gamma} > \Delta E} d\Phi_{3} |\mathcal{M}_{\text{NLO}}^{\gamma}|^{2} \right) \]

- Same strategy used at NNLO
\[ K_{\text{NNLO}} = \frac{d\sigma_{N_f}^{\alpha^2}}{d\sigma_{\text{LO}}} \]

Backup: MUonE Apparatus

- Measure $\mu$ and $e$ angles with very high precision.
- Modular: 40 tracking stations with silicon detectors built for HL-LHC upgrade.
- ECAL and $\mu$ filter downstream.
- Precision required on the differential cross sections: 10 ppm (!)