A parton branching with transverse momentum dependent splitting functions

ICHEP 2022

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arXiv:2205.15873

Motivation

- Development of Monte Carlo (MC) generators is essential for
  - experimental analyses at current colliders
  - planning of future experimental programs such as HL-LHC, EIC, FCC
- Recently, several studies have started to investigated the impact of the physics of transverse momentum dependent (TMD) parton distributions on MC, e.g.
  - parton branching (PB) formulation of TMD evolution [arXiv:1708.03279]
  - TMD perturbative resummation and NNLO matching [arXiv:1805:05916]
  - multi-jet merging with TMD parton showers [arXiv:2107.01224]
  - ...
- We investigated transverse momentum dependence at level of partonic splitting functions - an aspect not explored so far in PB MC

Strategy

- We use TMD Splitting functions defined through high-energy factorization [arXiv:hep-ph/9405388]
- We extend the PB approach [arXiv:1704.01757, arXiv:1708.03279], using "unitarity", to introduce TMD splitting kernels and new TMD Sudakov form factors
- First step toward a full generator that extends PB approach to the small-x phase space

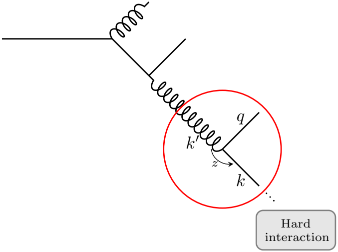
TMD splitting functions

$$P_{qg}\left(\alpha_s, z, k'_\perp, \tilde{q}_\perp\right) = \frac{\alpha_s T_F}{2\pi} \frac{\tilde{q}_\perp^2 z(1-z)}{(\tilde{q}_\perp^2 + z(1-z)k_\perp'^2)^2} \times$$

[arXiv:hep-ph/9405388]

$$\left[\frac{\tilde{q}_\perp^2}{z(1-z)} + 4(1-2z)\tilde{q}_\perp \cdot k'_\perp - 4\frac{(\tilde{q}_\perp \cdot k'_\perp)^2}{k_\perp'^2} + 4z(1-z)k_\perp'^2\right]$$

$$\tilde{q}_\perp = k_\perp - zk'_\perp$$



For $k_\perp'^2 \ll k_\perp^2$ after angular averaging:
DGLAP splitting function

For $k_\perp'^2 \sim \mathcal{O}(k_\perp^2)$:
Series expansion $(k_\perp'^2/\tilde{q}_\perp^2)^n$
Resummation $\ln(1/z)$

Other partonic channels studied in [1511.08439, 1607.01507, 1711.04587]
The splitting functions are positive definite and interpolate consistently between the collinear limit and the high-energy limit

Unitarity

TMD evolution equations: $\tilde{\mathcal{A}}$: Momentum weighted TMD

$$\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2) = \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu_0^2) - \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp} F_a(\mu'^2_\perp, k_\perp^2)\tilde{\mathcal{A}}_a(x, k_\perp^2, \mu'^2_\perp)\Theta(\mu'^2_\perp - \mu_0^2)\Theta(\mu^2 - \mu'^2_\perp) +$$

$$+ \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp} \int_x^{z_M} dz \tilde{P}^R_{ab}(z, k_\perp + (1-z)\mu'_\perp, \mu'_\perp)\tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2\right)\Theta(\mu'^2_\perp - \mu_0^2)\Theta(\mu^2 - \mu'^2_\perp)$$

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- Real emissions

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- Real emissions
- Virtual/Non-resolvable emissions

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- Real emissions
- Virtual/Non-resolvable emissions
  $\Rightarrow$ Fix with momentum conservation:

$$0 = \sum_a \int_0^1 dx \int dk_\perp^2 \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2) - \sum_a \int_0^1 dx \int dk_\perp^2 \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu_0^2).$$

$$\Rightarrow$$

$$F_a(\mu'^2, k_\perp^2) = \sum_b \int_0^{z_M} dz\, z\bar{P}^R_{ba}(z, k_\perp^2, \mu'^2).$$

$\bar{P}^R_{ba}(z, k_\perp^2, \mu'^2)$: Angular averaged TMD splitting functions

Unitarity (2)

Introduce TMD Sudakov form factors:

$$\Delta_a(\mu^2, k_\perp^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\; z\bar{P}_{ba}^R(z, k_\perp^2, \mu'^2)\right)$$

Rewrite the evolution equation:

$$\tilde{\mathcal{A}}_a\left(x, k_\perp^2, \mu^2\right) = \Delta_a\left(\mu^2, k_\perp^2\right)\tilde{\mathcal{A}}_a\left(x, k_\perp^2, \mu_0^2\right) + \sum_b \int \frac{d^2\mu'_\perp}{\pi\mu'^2_\perp}\Theta(\mu'^2_\perp - \mu_0^2)\Theta(\mu^2 - \mu'^2_\perp)$$

$$\times \int_x^{z_M} \mathrm{d}z\, \frac{\Delta_a\left(\mu^2, k_\perp^2\right)}{\Delta_a\left(\mu'^2_\perp, k_\perp^2\right)}\tilde{P}_{ab}^R\left(z, k_\perp + (1-z)\mu'_\perp, \mu'_\perp\right)\tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu'^2_\perp\right)$$

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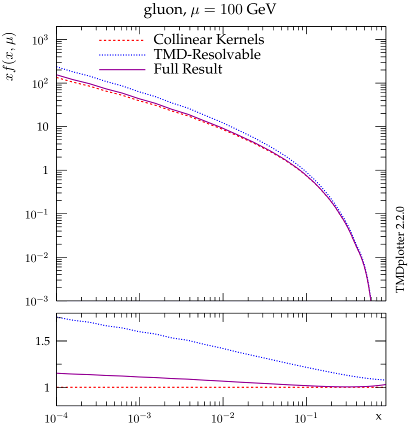
$$\times \int_x^{z_M} \mathrm{d}z \frac{\Delta_a\left(\mu^2, k_\perp^2\right)}{\Delta_a\left(\mu_\perp'^2, k_\perp^2\right)} \tilde{P}_{ab}^R\left(z, k_\perp + (1-z)\mu'_\perp, \mu'_\perp\right) \tilde{\mathcal{A}}_b\left(\frac{x}{z}, (k_\perp + (1-z)\mu'_\perp)^2, \mu_\perp'^2\right)$$

Equation has similar structure to other Parton Branching equations [arXiv:1704.01757, arXiv:1708.03279] $\rightarrow$ similar MC

Except for scale generation according to TMD Sudakov form factor: VETO algorithm [arXiv:hep-ph/0603175]

Numerical results: integrated TMDs



Take fixed starting distribution at scale μ_0.

Compare evolved integrated TMDs:
Purple curve: Full result
Red dashed curve: Evolution with collinear kernels

Significant differences especially for low x, not washed out after integration over $k_\perp$

- Differences between red and purple due to dynamical effects from TMD splitting functions

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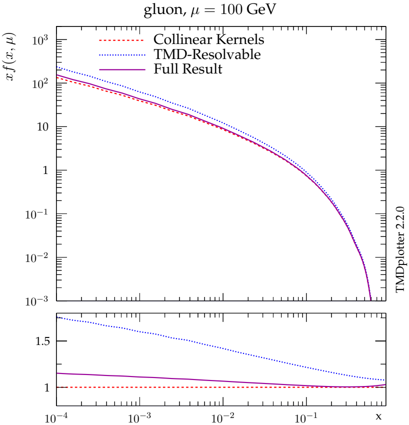

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- Large differences between Full result and TMD-Resolvable due to violation of momentum conservation in TMD-Resolvable

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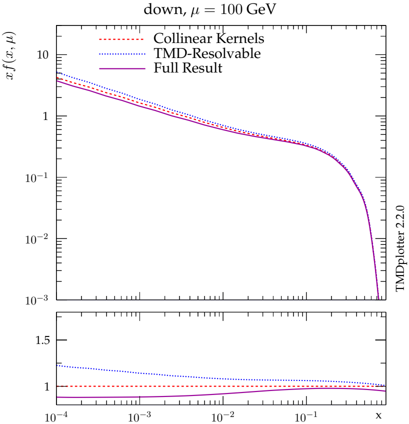

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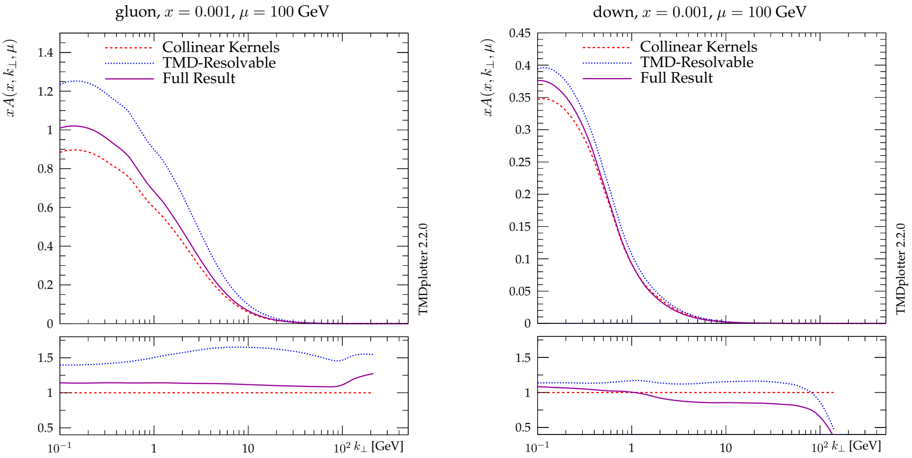

Differences in whole $k_\perp$-region

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Momentum conservation check

μ^2 (GeV^2)	Full Result		
	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.996	0.997
10^3	0.994	0.992	0.994
10^4	0.991	0.987	0.991
10^5	0.984	0.978	0.983

μ^2 (GeV^2)	TMD-Resolvable		
	$\alpha_s(\mu^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.029	1.038	1.000
10	1.087	1.139	1.007
10^2	1.156	1.304	1.045
10^3	1.195	1.413	1.091
10^4	1.219	1.478	1.129
10^5	1.229	1.507	1.148

μ^2 (GeV^2)	Collinear Kernels		
	$\alpha_s(\mu^2)$ fix. z_M	$\alpha_s(q_\perp^2)$, fix. z_M	$\alpha_s(q_\perp^2)$, dyn. z_M
3	1.000	1.000	1.000
10	0.999	0.999	0.999
10^2	0.997	0.997	0.997
10^3	0.995	0.993	0.995
10^4	0.992	0.989	0.992
10^5	0.986	0.981	0.984

In table:
$\sum_a \int_{x_0}^1 dx \int dk_\perp^2 \, \tilde{\mathcal{A}}_a(x, k_\perp^2, \mu^2)$
$x_0 = 10^{-5}$

Studied for different scales of α_s, soft gluon resolution scales z_M

As expected: Our full result and the result with collinear kernels conserve momentum. When we use TMD splitting function only in resolvable branchings, there is violation of momentum conservation.

Conclusions

- First parton branching approach that takes into account both z and $k_\perp$-dependence of splitting functions
- These TMD splitting functions have well-prescribed collinear and high-energy limits
- We introduced new TMD Sudakov form factors
- Our approach describes resolvable and non-resolvable branchings
- We presented its MC implementation
  Applied it to obtain numerical results on the evolution of TMDs and to verify numerical momentum conservation

Outlook:

- Fitted TMDs with our method
- A full MC generator that incorporates the method
- Including CCFM phase space

Back-up

Using TMD Sudakov in a MC

Scale of a branching is generated according to Sudakov:

$$R = 1 - \frac{\Delta_a(\mu_i^2)}{\Delta_a(\mu_{i-1}^2)} \Leftrightarrow \mu_i^2 = \Delta_a^{-1}((1-R)\Delta_a(\mu_{i-1}^2))$$

VETO algorithm: [arXiv:hep-ph/0603175]

$\Delta_a(\mu^2, k_\perp^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2} (d\mu'^2/\mu'^2) F_a(\mu'^2, k_\perp^2)\right)$ [1]

Select g'_a, such that $g'_a(\mu^2) \geq F_a(\mu^2, k_\perp^2)$ [2]

1. start with $j = 0$, $p_{j=0}^2 = \mu_{i-1}^2$,
2. add one to j. Select $p_j^2 > p_{j-1}^2$ according to $R = 1 - \exp\left(-\int_{p_{j-1}^2}^{p_j^2} (dp'^2/p'^2) g'_a(p'^2)\right)$,
3. if $F(p_j^2, k_\perp^2)/g'(p_j^2) \leq (\text{newly generated})\ \mathrm{R}$ go to 2,
4. else: $\mu_i^2 = p_j^2$ is the generated scale.

[1] $F_a(\mu^2, k_\perp^2) = \sum_b \int_0^{z_M} dz\ z\ \bar{P}_{ba}^R(z, k_\perp^2, \mu'^2)$

[2] $g'_a(\mu^2) = \sum_b \int_0^{z_M} dz\ z\ (P_{ba}^R(z, \mu^2) + h_{ba}(z, \mu^2))$

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