Partial wave analysis of the $\tau \to 3\pi\nu$ decay at Belle

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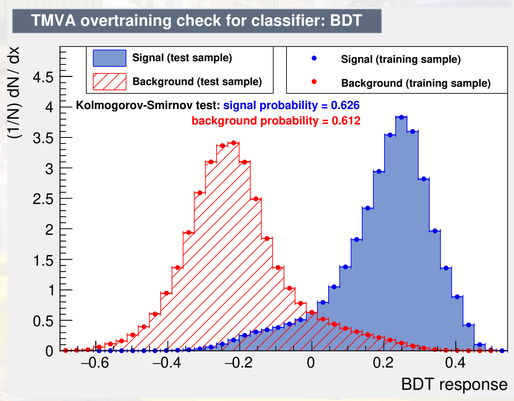
Motivation

- Searches for New Physics: need to know various hadron form-factors
- Light meson spectra usually can't be obtained from the first-principle QCD
- Disagreements in measurements of $a_1(1260)$ parameters in PDG not resolved
- New unexpected narrow peak $a_1(1420)$ discovered recently by COMPASS
- Existing models in TAUOLA show significant discrepancy with data
- Belle detector recorded up to 50 million $\tau^\mp \to \pi^\mp\pi^\mp\pi^\pm\nu_\tau$ decays at KEKB
- KEKB collider is an asymmetric electron-positron collider, $\sqrt{s} \approx 10$ GeV

$\tau^\mp \to \pi^\mp\pi^\mp\pi^\pm\nu_\tau$: selection criteria

- Four central tracks: $p_\perp > 0.1$ GeV
- Photons: $E > 0.04$ GeV
- Topology: $3+1$ tracks
- BDT($\tau^+\tau^-$) response above zero
- Signal side PID
- K_S-veto in signal hemisphere: $\left|m_{2\pi} - m_{K_S}\right| < 0.012$ GeV
- π^0-veto in signal hemisphere



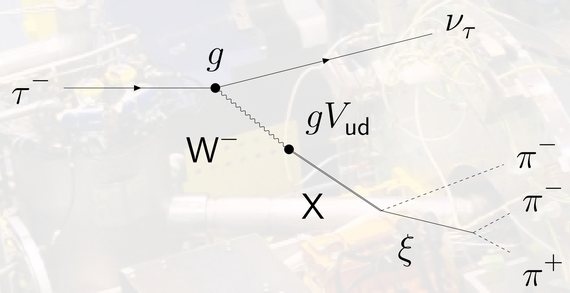
Efficiency	31.5%
Purity	87.2%

Major background componets:
$\tau^\mp \to \pi^\mp\pi^\mp\pi^\pm N\pi^0\nu_\tau, N \geq 1$
$e^+e^- \to q\bar{q}$
$\tau^\mp \to K^\mp\pi^\pm\pi^\mp\nu_\tau$

Decay amplitude and fitting method

$$\mathcal{M}_{if} = \frac{G_F}{\sqrt{2}} V_{ud} \underbrace{\left\langle \nu_\tau \middle| \bar{u}(l')\gamma^\mu\left(1-\gamma^5\right)u(l) \middle| \tau \right\rangle}_{\text{lepton current}} \underbrace{\left\langle \pi^-\pi^-\pi^+ \middle| J_{\text{had},\mu} \middle| 0 \right\rangle}_{\text{hadron current}},$$

- Four-body decay: 7 kinematic variables: $m_{3\pi}$, $s_1 = m_{23}^2$, $s_2 = m_{13}^2$, helicity angle $\cos\theta$ of the hadron system in the tauon rest frame, and three other angles in the hadron system
- Intensity is averaged over one of the angles of the hadron system because tauon direction can't be measured



- J^μ_{had} decomposes into partial waves [1]:

Partial wave	$m(\xi)$ [GeV]	$\Gamma(\xi)$ [GeV]	Threshold [GeV]
$1^+[\sigma\pi]_P$	Broad $[\pi\pi]_S$-wave component parametrization		
$1^+[f_0(980)\pi]_P$	0.990	0.07	1.18
$1^+[f_0(1500)\pi]_P$	1.504	0.109	1.7
$1^+[\rho(770)\pi]_S$	0.769	0.1509	—
$1^+[\rho(770)\pi]_D$	"	"	—
$1^+[\rho(1450)\pi]_S$	1.465	0.40	1.6
$1^+[\rho(1450)\pi]_D$	"	"	1.6
$1^+[f_2(1270)\pi]_P$	1.2755	0.1867	1.22
$0^-[\sigma\pi]_S$			—
$0^-[f_0(980)\pi]_S$			1.18
$0^-[\rho(770)\pi]_P$			—
$1^-[\rho(770)\pi]_P$	Spin exotic		

Conventional partial wave analysis

- Fit: maximizing unbinned extended log-likelihood function
- Signal intensity:

$$\mathcal{I} = \sum_{w,v} \mathcal{C}_w \mathcal{C}_v^* \overline{\mathcal{L}_{\mu\nu}} j_w^\mu \left(j_v^\nu\right)^* \propto \left|\mathcal{M}_{fi}\right|^2$$

- Isobar shapes are fixed
- Fit in slices of $m_{3\pi}$ (no a_1 shape assumptions)
- $N_{\text{partial waves}}$ free complex parameters $\mathcal{C}_w$ for each slice

References

[1] F. Krinner and S. Paul, arXiv:2107.04295

[2] A. Mathad, D. O'Hanlon, A. Poluektov, and R. Rabadan, arXiv:1902.01452

MC studies

Input-output test fit to MC Truth:

Wave component	Input	Output	Δ/σ
$\mathfrak{Re}\ 1^+[\sigma\pi]_P$	-0.0500	-0.0571 ± 0.0053	1.34
$\mathfrak{Im}\ 1^+[\sigma\pi]_P$	0.0866	0.0825 ± 0.0066	0.62
$\mathfrak{Re}\ 1^+[f_0(980)\pi]_P$	0.0500	0.0506 ± 0.0023	0.26
$\mathfrak{Im}\ 1^+[f_0(980)\pi]_P$	-0.0866	-0.0814 ± 0.0027	1.93
$\mathfrak{Re}\ 1^+[\rho(770)\pi]_S$	0.7000	0.7000 ± 0.0034	Ref. wave
$\mathfrak{Im}\ 1^+[\rho(770)\pi]_S$		Set to zero in fit	
$\mathfrak{Re}\ 1^+[\rho(1450)\pi]_S$	0.0500	0.0510 ± 0.0048	0.21
$\mathfrak{Im}\ 1^+[\rho(1450)\pi]_S$	0.0866	0.0874 ± 0.0068	0.12
$\mathfrak{Re}\ 1^+[\rho(770)\pi]_D$	-0.1000	-0.0947 ± 0.0032	1.66
$\mathfrak{Im}\ 1^+[\rho(770)\pi]_D$	0.1732	0.1714 ± 0.0045	0.40
$\mathfrak{Re}\ 1^+[\rho(1450)\pi]_D$	-0.0500	-0.0530 ± 0.0043	0.70
$\mathfrak{Im}\ 1^+[\rho(1450)\pi]_D$	0.0000	-0.0039 ± 0.005	0.78
$\mathfrak{Re}\ 1^+[f_2(1270)\pi]_P$	0.0966	0.0953 ± 0.0032	0.41
$\mathfrak{Im}\ 1^+[f_2(1270)\pi]_P$	0.0259	0.0247 ± 0.0033	0.36

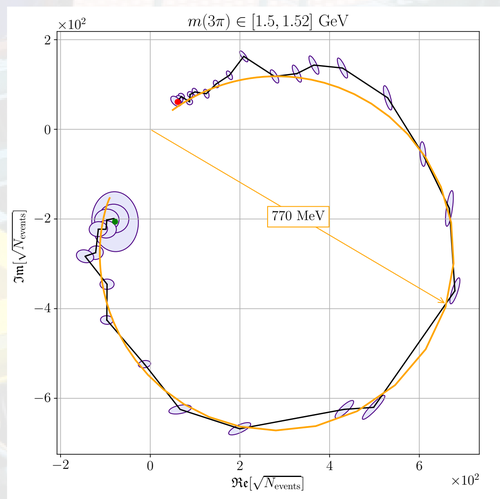
Freed-isobar partial wave analysis

Substitute isobar shape with a complex-valued step function to reduce model dependence:

$$\sum_{w\ \text{freed}} \mathcal{C}_{w\,\text{freed}} \Theta_w(s), \quad \Theta_w(s) = \begin{cases} 1 & \text{if } s \text{ in the } m_{2\pi} \text{ bin } w \\ 0 & \text{otherwise} \end{cases}$$

Fit results of the freed $1^+[1^{--}\pi]_S$ PWA on MC Truth:

Parameter	Value	Uncert.
$m(\xi)$ [GeV]	0.76982	0.00057
$\Gamma(\xi)$ [GeV]	0.15516	0.00132
$\mathfrak{Re}\ \mathcal{C}\{1^+[\rho\pi]_S\}$	-78.58	1.73
$\mathfrak{Im}\ \mathcal{C}\{1^+[\rho\pi]_S\}$	-133.16	0.94

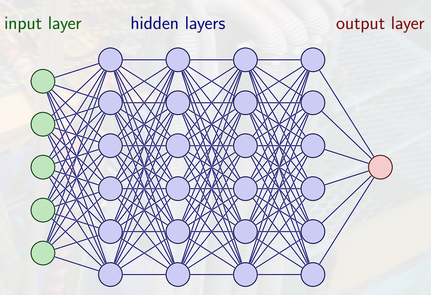


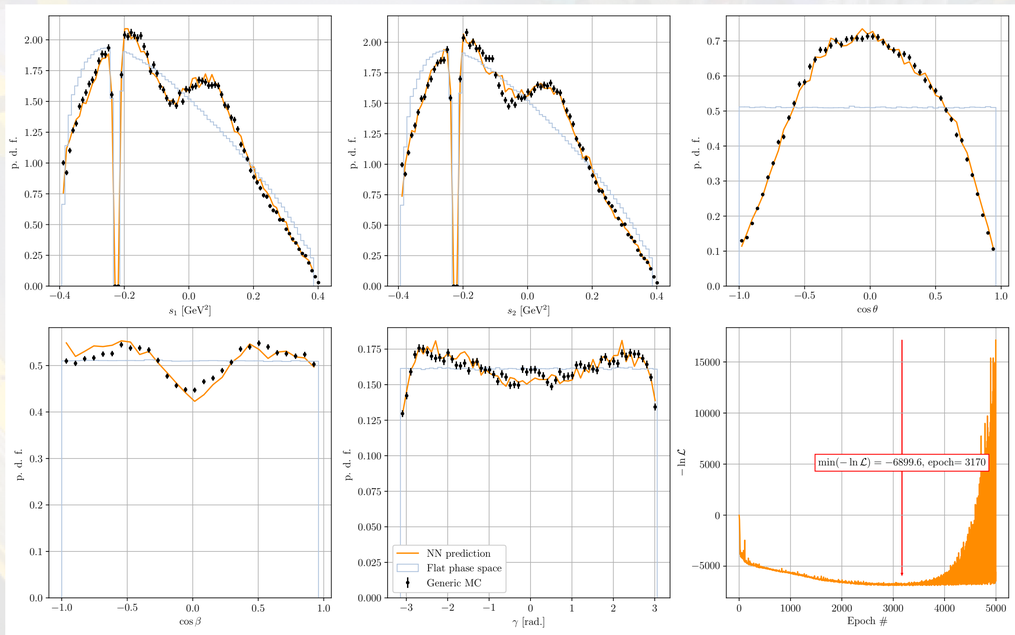
Compare fixed PWA and isobar fit parameters with input:

Wave	M_{in}	M_{out}	ϕ_{in}	ϕ_{out}	Extracted from:
$[f_0(980)\pi]_P$	0.10	0.10	$-1/3\pi$	-0.31π	PWA
$[\rho(770)\pi]_S$	0.70	0.70	0	0.00π	Isobar fit
$[\rho(770)\pi]_D$	0.92	0.94	$2/3\pi$	0.67π	PWA
$[f_0(1270)\pi]_P$	0.53	0.51	$1/12\pi$	0.10π	PWA

Background description

- Background shape in 6 kinematic variables can't be described with templated histogramms
- Train 5D neural networks on background MC [2] in slices of $m(3\pi)$
- NN topology: 4 hidden 40×40 layers





Conclusion

- $\tau \to 3\pi\nu_\tau$ PWA has been verified in both cases with fixed and freed isobars
- Large data sample collected by Belle and detailed analysis technique let us significantly improve the model of $\tau \to 3\pi\nu_\tau$ decay

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