



Inclusive $|V_{cb}|$ from q^2 moments of $B \rightarrow X_c \ell \nu_\ell$ decays

ICHEP 2022

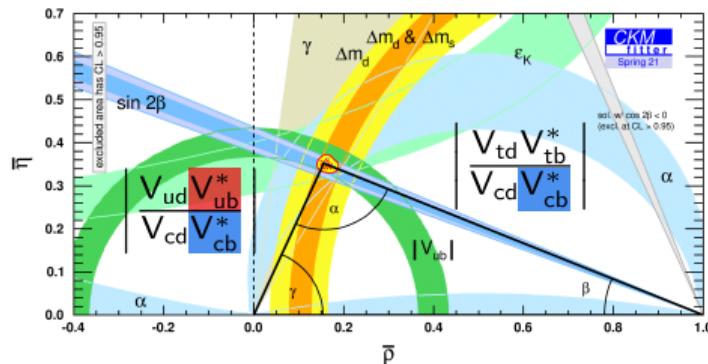
Maximilian Welsch

in collaboration with F. Bernlochner, E. Perrson, M. Fael, K. Vos, K. Olschewsky, R. van Tonder

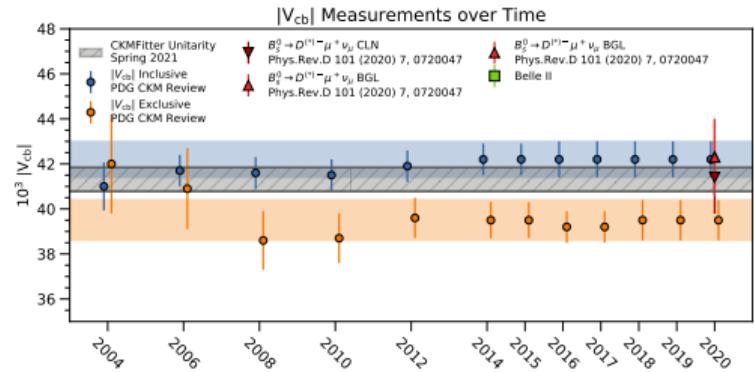
July 8, 2022

based on: [arXiv:2205.10274](https://arxiv.org/abs/2205.10274)

Why is V_{cb} important?



$|V_{cb}|$ (and $|V_{ub}|$) crucial to CKM unitarity tests:
→ both sides of the UT are constrained by $|V_{cb}|$



$\sim 1\text{--}3\sigma$ tensions between inclusive and exclusive determinations from B -meson decays

Inclusive Determination of $|V_{cb}|$ - Prerequisites

- Inclusive decays: hadronic final-state not specified:

$$X_c = \{D, D^*, D^{**}, D^{(*)}\pi\pi, \dots\}$$

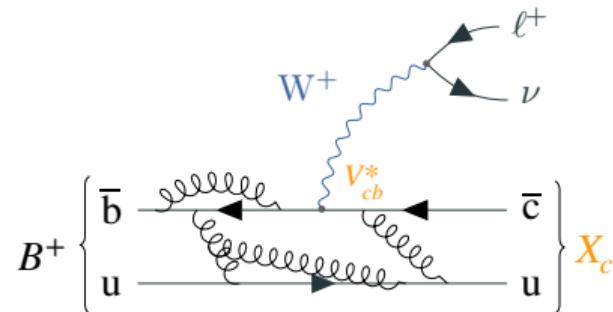
- Heavy Quark Expansion (HQE): operator product expansion in $1/m_b$

$$d\Gamma = d\Gamma^{(0)} + d\Gamma^{(\pi)} \frac{\mu_\pi^2}{m_b^2} + d\Gamma^{(G)} \frac{\mu_G^2}{m_b^2} + d\Gamma^{(D)} \frac{\rho_D^3}{m_b^3} + \dots$$

$$d\Gamma^{(i)} = \sum d\Gamma^{(i,n)} \left(\frac{\alpha_s}{\pi} \right)^n$$

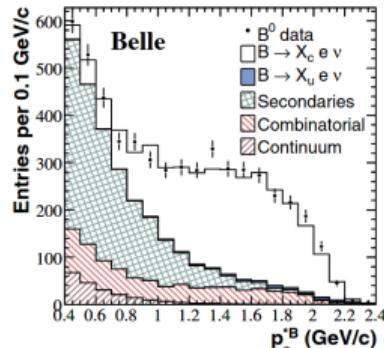
- Wilson coefficients $d\Gamma^{(i,n)}$ are calculated perturbatively
- HQE parameters $\mu_\pi^2, \mu_G^2, \rho_D^3, \dots$ are non-perturbative \rightarrow have to be determined from measurements
- Proliferation of non-perturbative parameters at higher orders in $1/m_b$

$$4 \text{ up to } \mathcal{O}(1/m_b^3) \rightarrow 13 \text{ up to } \mathcal{O}(1/m_b^4) \rightarrow 31 \text{ up to } \mathcal{O}(1/m_b^5)$$

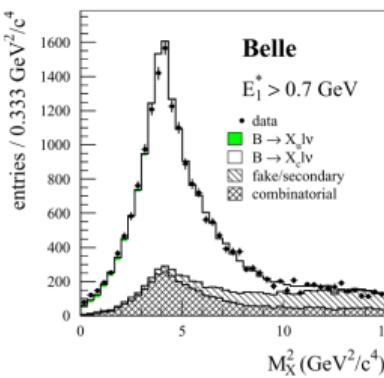


Inclusive Determination of $|V_{cb}|$ - Traditional Method

Phys.Rev.D 75 032001



Phys.Rev.D 75 032005



- Inclusive observables M can be expressed with the HQE

- Spectral moments:

$$\langle x^n \rangle_{E_\ell > E_{\text{cut}}} = \frac{\int_{E_\ell > E_{\text{cut}}} dx x^n \frac{d\Gamma}{dx}}{\int_{E_\ell > E_{\text{cut}}} dx \frac{d\Gamma}{dx}}$$

- Partial rate:

$$R^*(E_\ell > E_{\text{cut}}) = \frac{\int_{E_\ell > E_{\text{cut}}} dE_\ell \frac{d\Gamma}{dE_\ell}}{\int dE_\ell \frac{d\Gamma}{dE_\ell}}$$

- Use existing measurements of $\langle M_x \rangle$, $\langle E_\ell \rangle$, R^* determined as functions of lower cuts on $E_\ell > E_{\text{cut}}$ Gambino, Schwanda, PRD 89 (2014) 014022
 - Recent update: $|V_{cb}| = (42.16 \pm 0.51) \times 10^{-3}$ Bordone, Capdevila, Gambino Phys.Lett.B 822 136679

- Includes terms up to $1/m_b^3$ (including higher order terms has also been explored Gambino, Healey, Turczyk Phys.Lett.B 763 60-65)
 - α_s^3 for total rate and kinetic mass Fael, Schonwald, Steinhauser Phys.Rev.D 104 016003

The alternative method to determine $|V_{cb}|$

- Reparameterization invariance (RPI): links different orders in $1/m_b$ in the HQE
→ reduces number of independent parameters Mannel, Vos JHEP 1806 (2018) 115
- Total inclusive rate: reduction from 13 to 8 parameters at $\mathcal{O}(1/m_b^4)$
- RPI observables: Fael, Mannel, Vos JHEP 02 (2019) 177
 - Ratio between rate with and without a cut

$$R^* \left(q^2 > q_{\text{cut}}^2 \right) = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}}{\int dq^2 \frac{d\Gamma}{dq^2}}$$

- Spectral moments of the q^2 distribution

$$\left\langle q^{2n} \right\rangle_{q^2 > q_{\text{cut}}^2} = \frac{\int_{q^2 > q_{\text{cut}}^2} dq^2 q^{2n} \frac{d\Gamma}{dq^2}}{\int_{q^2 > q_{\text{cut}}^2} dq^2 \frac{d\Gamma}{dq^2}}$$

- Hadronic mass m_X and lepton energy E_ℓ moments are not RPI
- E_ℓ cuts are not RPI, but a q^2 cut is RPI

The alternative method to determine $|V_{cb}|$

$$R^* \left(q^2 > q_{\text{cut}}^2 \right) \& \left\langle q^{2n} \right\rangle_{q^2 > q_{\text{cut}}^2}$$

↓

$$\mu_3, \mu_G, \tilde{\rho}_D, r_E, r_G, s_E, s_B, s_{qB}, m_b, m_c \quad (\text{at } \mathcal{O}(1/m_b^4))$$

↓

$$\begin{aligned} \mathcal{B}(B \rightarrow X_c \ell \bar{\nu}_\ell) \propto \frac{|V_{cb}|^2}{\tau_B} & \left[\Gamma_{\mu_3} \mu_3 + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\tilde{\rho}_D} \frac{\tilde{\rho}_D^3}{m_b^3} \right. \\ & + \Gamma_{r_E} \frac{r_E^4}{m_b^4} + \Gamma_{r_G} \frac{r_G^4}{m_b^4} + \Gamma_{s_B} \frac{s_B^4}{m_b^4} + \Gamma_{s_E} \frac{s_E^4}{m_b^4} + \Gamma_{s_{qB}} \left. \frac{s_{qB}^4}{m_b^4} \right] \end{aligned}$$

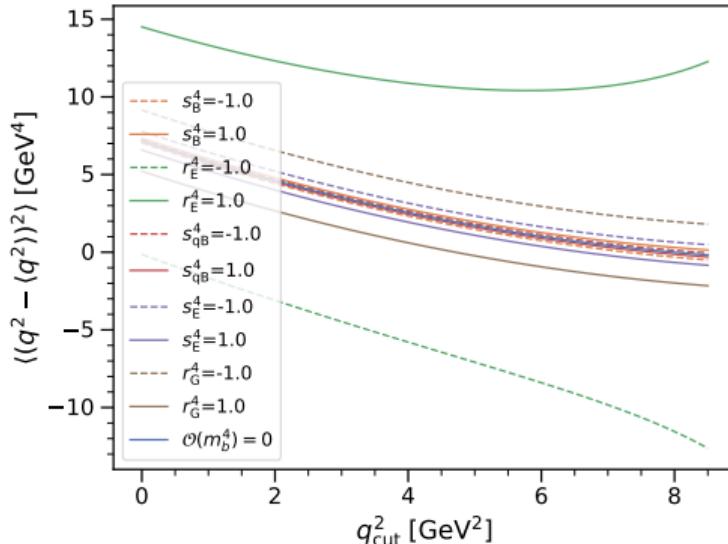
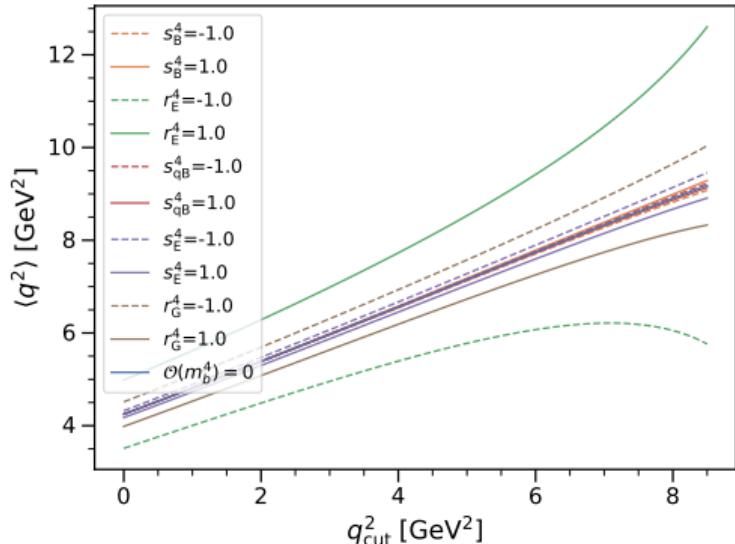
↓

$$|V_{cb}| = ?$$

$$2M_B \mu_3 = \langle B | \bar{b}_v b_v | B \rangle = 2M_B \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b} \right)$$

Fael, Mannel, Vos JHEP 02 (2019) 177

Sensitivity to $\mathcal{O}(1/m_b^4)$ Terms



→ highest sensitivity to r_E^4 and r_G^4 , other parameters small impact on moments and due to sub-leading contributions to the total rate on $|V_{cb}|$

The fit procedure

- Extract $|V_{cb}|$ and HQE parameters with simultaneous fit to $\mathcal{B}(B \rightarrow X_c \ell \nu_\ell)$ and q^2 moment measurements by Belle Phys.Rev.D 104 112011 and Belle II arXiv:2205.10274

$$\chi^2(|V_{cb}|, \boldsymbol{\theta}) = \frac{(\mathcal{B} - \Gamma(|V_{cb}|, \boldsymbol{\theta})\tau_B)^2}{\sigma_{\mathcal{B}}^2 + \sigma_\Gamma^2} + (\mathbf{q}(\boldsymbol{\theta}) - \mathbf{q}_{\text{meas}})^\top \mathbf{C}^{-1} (\mathbf{q}(\boldsymbol{\theta}) - \mathbf{q}_{\text{meas}}) + \sum_{i=1}^4 \frac{(\theta_i - \tilde{\theta}_i)^2}{\sigma_{\tilde{\theta}_i}^2}$$

- Fit parameters $\boldsymbol{\theta} = \{m_b^{\text{kin}}, \bar{m}_c, \mu_\pi^2, \mu_G^2, \rho_D^3, r_E^4, r_G^4\}$
- External constraints $\tilde{\boldsymbol{\theta}} = \{m_b^{\text{kin}}, \bar{m}_c, \mu_\pi^2, \mu_G^2\}$
- Covariance matrix $\mathbf{C} = \mathbf{C}_{\text{stat}} + \mathbf{C}_{\text{sys}} + \mathbf{C}_{\text{theo}}$

Theory Covariance Matrix

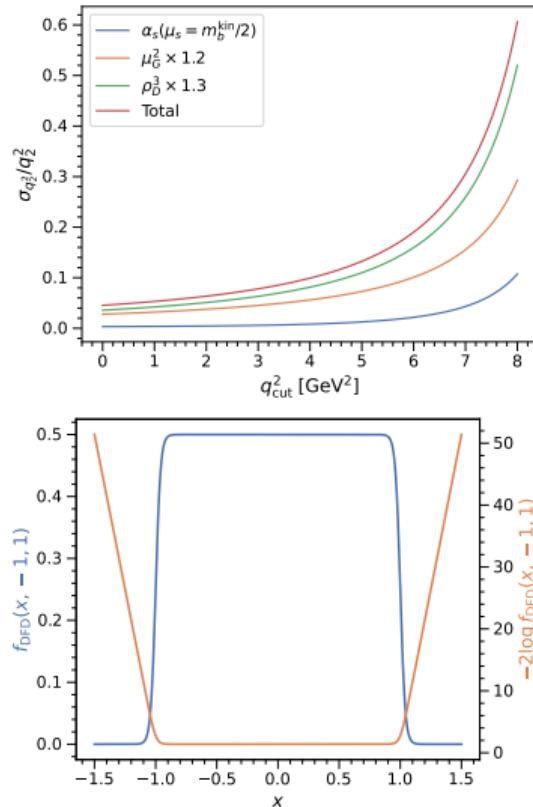
To account for missing higher-order:

- Perturbative corrections: variation of $\alpha_s(\mu_s)$ between $m_b^{\text{kin}}/2 < \mu_s < 2m_b^{\text{kin}}$
- $1/m_b$ corrections: variation of ρ_D^3 by 30%
- $\alpha_s/m_b^{2,3}$ corrections: variation of μ_G^2 by 20%

Flexible theory covariance matrix $C_{\text{theo}}(\rho_{\text{cut}}, \rho_{\text{mom}})$:

- ρ_{cut} : correlation between same order moments with different lower q^2 cut
- ρ_{mom} : correlation between different order moments and different lower q^2 cut

Default Fit: add ρ_{cut} and ρ_{mom} as nuisance parameters to the χ^2 function



$B \rightarrow X_c \ell \nu_\ell$ Branching Fraction

- Branching Fraction (BF) $\mathcal{B}(B \rightarrow X_c \ell \nu_\ell)$ is instrumental to determine $|V_{cb}|$
- Available measurements: partial BF $\mathcal{B}_{\text{cut}}(B \rightarrow X_c \ell \nu_\ell)(E_\ell > E_{\text{cut}})$ or the total $B \rightarrow X \ell \nu_\ell$ BF
- Our average:

$$\mathcal{B}(B \rightarrow X_c \ell \nu_\ell) = (10.48 \pm 0.13)\%$$

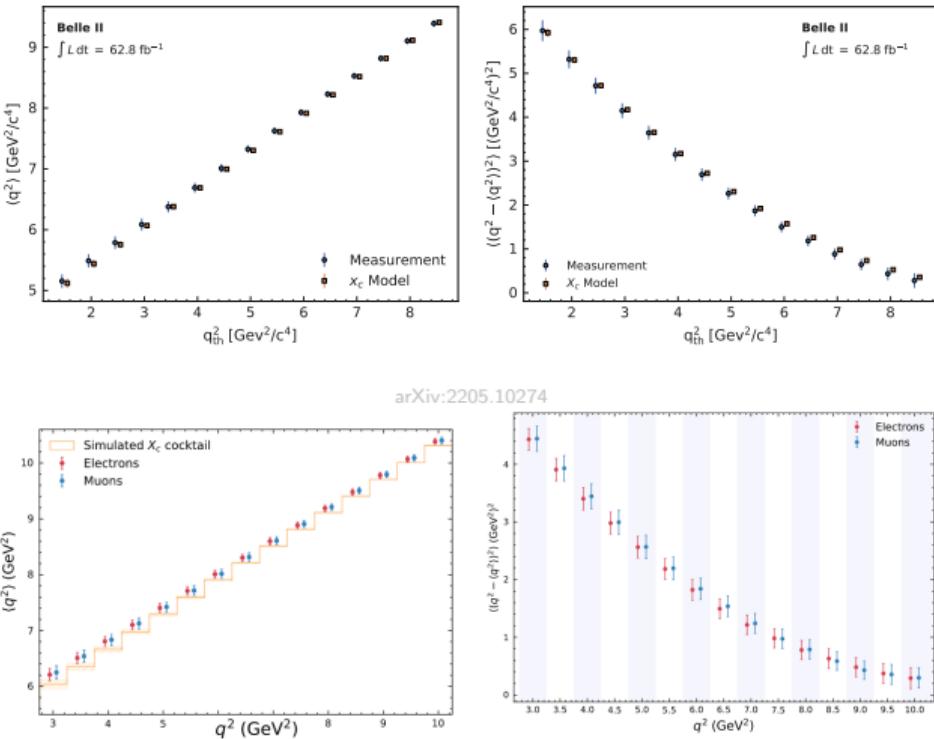
based on CLEO Phys.Rev.D 70 032003, Belle Phys.Rev.D 75 032001, and BaBar Phys.Rev.D 95 072001, Phys.Rev.D 81 032003 measurements

- Differs compared to $\mathcal{B}(B \rightarrow X_c \ell \nu_\ell) = (10.66 \pm 0.15)\%$ obtained by Bordone, Capdevila, Gambino Phys.Lett.B 822

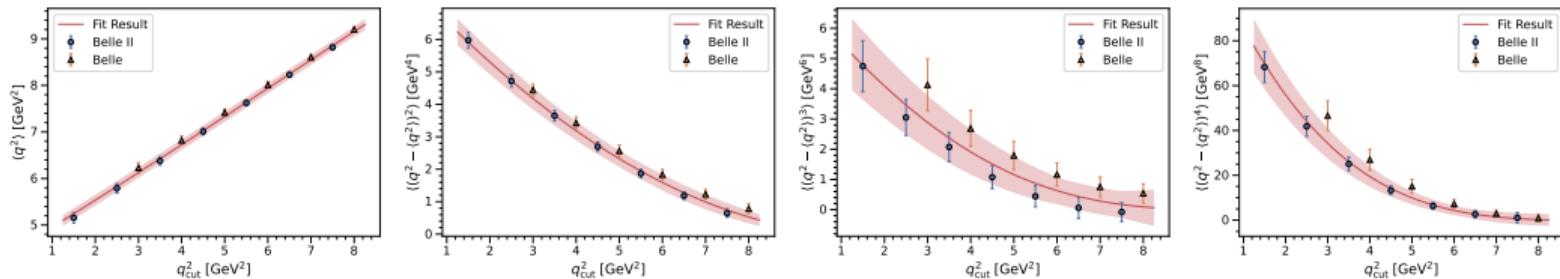
136679

Measurements of q^2 Moments

- Raw and central q^2 moments measured by Belle and Belle II
- Highly correlated measurements → use subsets
- Average Belle result over lepton flavor
- Systematic uncertainties regarding $B \rightarrow X_c \ell \nu_\ell$ modeling fully correlated (BF & form factors)



Results - First Determination of $|V_{cb}|$



First $|V_{cb}|$ determination using q^2 moments:

$$\begin{aligned} |V_{cb}| &= (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\text{Exp.}} \pm 0.17|_{\text{Theo.}} \pm 0.34|_{\text{Constr.}}) \times 10^{-3} \\ &= (41.69 \pm 0.59) \times 10^{-3} \end{aligned}$$

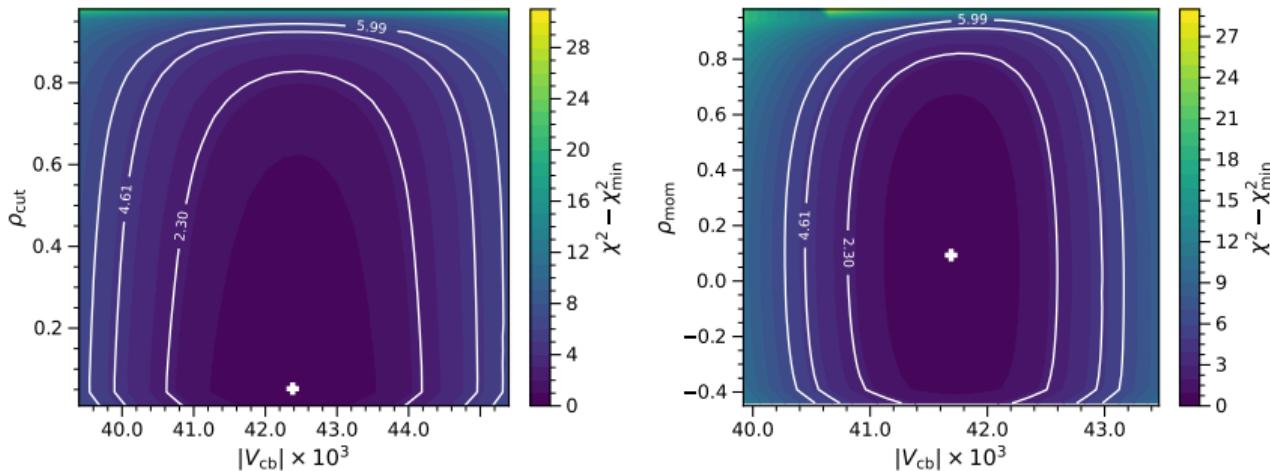
Include additional uncertainty by varying s_{qB}^4 , s_E^4 , and s_B^4 by $\pm 1 \text{ GeV}^4$:

$$|V_{cb}| = (41.69 \pm 0.59|_{\text{fit}} \pm 0.23|_{\text{h.o.}}) \times 10^{-3} = (41.69 \pm 0.63) \times 10^{-3}$$

$$|V_{cb}| = (42.16 \pm 0.51) \times 10^{-3} \quad \text{Bordone, Capdevila, Gambino Phys.Lett.B 822 136679}$$

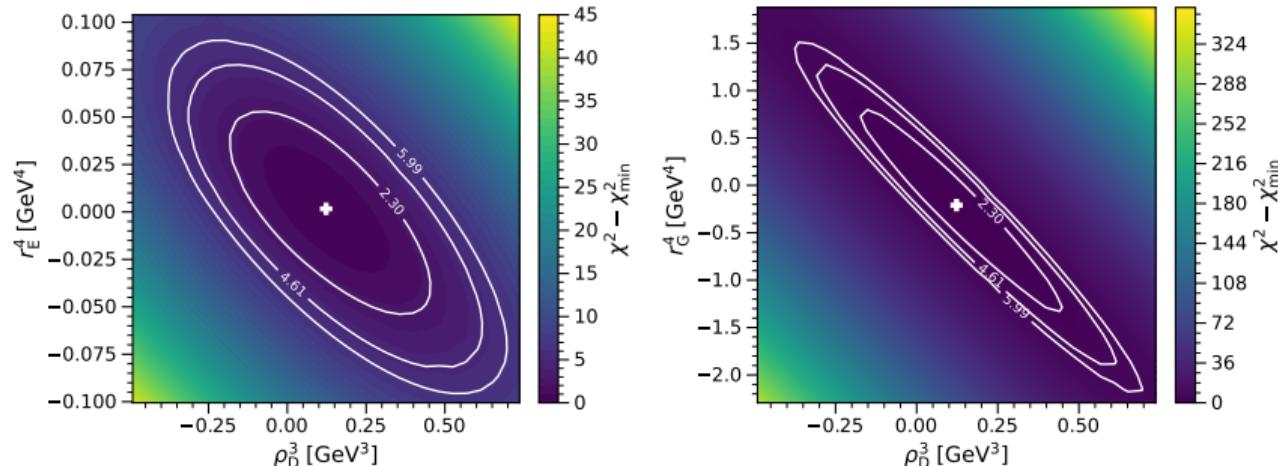
$$|V_{cb}| = (39.10 \pm 0.50) \times 10^{-3} \quad \text{arXiv:2206.07501}$$

Results - Correlation Parameters vs. $|V_{cb}|$



- Correlation parameters describing theory correlation between moments included as nuisance parameters in the fit
- Profile over a large amount of possible correlation scenarios
- Contributes to uncertainty of $|V_{cb}|$

Results - HQE Parameters



$$r_D^3 = (0.12 \pm 0.20) \text{ GeV}^3, \quad r_E^4 = (0.02 \pm 0.34) \text{ GeV}^4, \quad r_G^4 = (-0.21 \pm 0.69) \times 10^{-1} \text{ GeV}^4$$

- HQE parameters highly correlated
- Interesting to extract, but large uncertainties

Summary & Conclusion

Summary

- First determination of $|V_{cb}|$ using q^2 moments from inclusive $B \rightarrow X_c \ell \nu_\ell$ decays
 $|V_{cb}| = (41.69 \pm 0.59) \times 10^{-3}$
- 2 out of 5 $\mathcal{O}(1/m_b^4)$ HQE parameters included in the fit
- Independent cross-check of previous inclusive $|V_{cb}|$ determinations based on new method and data
- Difference between determinations based on different input BFs \rightarrow need for new $B \rightarrow X_c \ell \nu_\ell$ BF measurements (as functions of q^2 cuts)

Conclusion

- Analysis extension by inclusion of higher-order perturbative corrections and inclusion of lepton energy and hadronic mass moments
- Study of forward-backward asymmetry Turczyk JHEP 04 (2016) 131 or differences of partial moments Herren arXiv:2205.03427
- future improvements may allow to push the inclusive $|V_{cb}|$ precision to below 1%

BACKUP

Non-Perturbative Matrix Elements

- 1: $-2M_B\mu_3 = \langle B | \bar{b}_v b_v | B \rangle = 2M_B \left(1 - \frac{\mu_\pi^2 - \mu_G^2}{2m_b} \right)$
- $1/m_b^2$: $-2M_B\mu_G^2 = \langle B | \bar{b}_v (-i\sigma^{\mu\nu}) iD_\mu iD_\nu b_v | B \rangle$
- $1/m_b^3$: $-2M_B\tilde{\rho}_D^3 = \frac{1}{2} \left\langle B \left| \bar{b}_v \left[iD_\mu, \left[\left(ivD + \frac{(iD)^2}{m_b} \right), iD^\mu \right] \right] b_v \right| B \right\rangle$
- $1/m_b^4$:
 - $2M_B r_G^4 \equiv \frac{1}{2} \left\langle B \left| \bar{b}_v [iD_\mu, iD_\nu] [iD^\mu, iD^\nu] b_v \right| B \right\rangle \propto \langle \vec{E}^2 - \vec{B}^2 \rangle$
 - $2M_B r_E^4 \equiv \frac{1}{2} \left\langle B \left| \bar{b}_v [ivD, iD_\mu] [ivD, iD^\mu] b_v \right| B \right\rangle \propto \langle \vec{E}^2 \rangle$
 - $2M_B s_B^4 \equiv \frac{1}{2} \left\langle B \left| \bar{b}_v [iD_\mu, iD_\alpha] [iD^\mu, iD_\beta] (-i\sigma^{\alpha\beta}) b_v \right| B \right\rangle \propto \langle \vec{\sigma} \cdot \vec{B} \times \vec{B} \rangle$
 - $2M_B s_E^4 \equiv \frac{1}{2} \left\langle B \left| \bar{b}_v [ivD, iD_\alpha] [ivD, iD_\beta] (-i\sigma^{\alpha\beta}) b_v \right| B \right\rangle \propto \langle \vec{\sigma} \cdot \vec{E} \times \vec{E} \rangle$
 - $2M_B s_{qB}^4 \equiv \frac{1}{2} \left\langle B \left| \bar{b}_v [iD_\mu, [iD^\mu, [iD_\alpha, iD_\beta]]] (-i\sigma^{\alpha\beta}) b_v \right| B \right\rangle \propto \langle \square \vec{\sigma} \cdot \vec{B} \rangle$.

Up to $1/m_b^4$: 8 parameters versus previous 13

Towards a new extraction of $|V_{cb}|$

Γ	tree	α_s	α_s^2	α_s^3	$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓	✓	✓	Partonic	✓	✓		
μ_G^2	✓	✓			μ_G^2	✓		✓	
ρ_D^3	✓	✓			ρ_D^3	✓		✓	
$1/m_b^4$	✓				$1/m_b^4$	✓			
$m_b^{\text{kin}}/\overline{m}_c$		✓	✓	✓					

- Predictions for raw and central moments available [here](#)
- α_s^3 corrections to partonic rate included
- kinetic scheme for bottom quark mass
- kinetic and \overline{MS} scheme for charm quark mass
- *flexible* covariance matrix

$B \rightarrow X_c \ell \bar{\nu}_\ell$ Branching Fraction

	$\mathcal{B}(B \rightarrow X \ell \bar{\nu}_\ell) (\%)$	$\mathcal{B}(B \rightarrow X_c \ell \bar{\nu}_\ell) (\%)$	In Average
Belle [1] $E_\ell > 0.6$ GeV	-	10.54 ± 0.31	✓
Belle [1] $E_\ell > 0.4$ GeV	-	10.58 ± 0.32	
CLEO [2] incl.	10.91 ± 0.26	10.72 ± 0.26	
CLEO [2] $E_\ell > 0.6$	10.69 ± 0.25	10.50 ± 0.25	✓
BaBar [3] incl.	10.34 ± 0.26	10.15 ± 0.26	✓
BaBar SL [4] $E_\ell > 0.6$ GeV	-	10.68 ± 0.24	✓
Our Average	-	10.48 ± 0.13	
Average Belle [1] & BaBar [4] $(E_\ell > 0.6$ GeV)	-	10.63 ± 0.19	

Table 1: Available measurements of the inclusive $B \rightarrow X \ell \bar{\nu}_\ell$ and $B \rightarrow X_c \ell \bar{\nu}_\ell$ branching fractions, extrapolated to the full region using the correction factors in $\Delta(E_\ell > 0.6 \text{ GeV}) = 1.047 \pm 0.004$ and $\Delta(E_\ell > 0.4 \text{ GeV}) = 1.014 \pm 0.001$. The χ^2 of our average with respect to the included measurements is 2.2, corresponding to a p-value of 52%. We do not include [5], as the analysis does not quote a partial branching fraction corrected for FSR radiation.

References i

- [1] P. Urquijo et al.
Moments of the electron energy spectrum and partial branching fraction of $B \rightarrow X_c e \nu$ decays at Belle.
Phys. Rev. D, 75:032001, 2007.
- [2] A. H. Mahmood et al.
Measurement of the B-meson inclusive semileptonic branching fraction and electron energy moments.
Phys. Rev. D, 70:032003, 2004.
- [3] J. P. Lees et al.
Measurement of the inclusive electron spectrum from B meson decays and determination of $|V_{ub}|$.
Phys. Rev. D, 95(7):072001, 2017.

References ii

- [4] Bernard Aubert et al.
Measurement and interpretation of moments in inclusive semileptonic decays $\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$.
Phys. Rev. D, 81:032003, 2010.
- [5] Bernard Aubert et al.
Measurement of the ratio $\mathcal{B}(B^+ \rightarrow X e \nu)/\mathcal{B}(B^0 \rightarrow X e \nu)$.
Phys. Rev. D, 74:091105, 2006.