COVER ALL YOUR BASES!

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Even without knowing, you might over-reliant using Wilks' theorem to determine your confidence limits…

This is **mostly** fine, unless … unless you are close to the boundary of your configuration space

LET'S BREAK WILKS'

EFFECTIVE FIELD THEORIES (EFTS)

Parametrize **New Physics** in terms of **Wilson coefficients** *unresolved* by the scale of the experiment you are conducting:

e.g. cross section; New Physics

$$\sigma(f;c) \propto \left| \mathcal{M}_{\rm SM}(f) + \sum_{c_\alpha \in c} c_\alpha \mathcal{M}_{\rm NP}^{(c_\alpha)}(f) \right|^2 ,$$

$$= s + \sum_\alpha c_\alpha l_\alpha + \sum_{\alpha,\beta} c_\alpha c_\beta t_{\alpha,\beta} + \sum_\alpha c_\alpha^2 n_\alpha$$

s, l, t, n = constant, linear, mixed, quadratic terms

Can enter with **linear** or **quadratic** dependence into your observable of choice

WILKS' THEOREM

Wilks' theorem states that the **profile likelihood ratio** (**PLR**) test-statistics is distributed like a χ^2**-distribution**

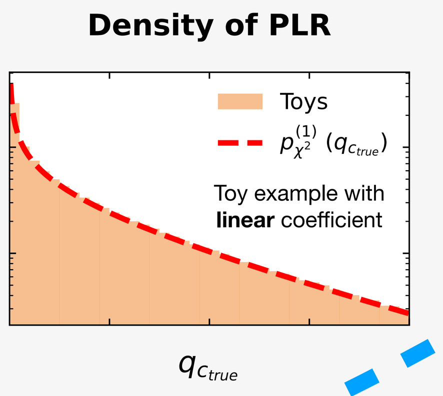
$p_x(x|c)$ = your likelihood evaluated under observation x, c value of Wilson coefficient

$$q(x;c) = -2\ln\left[\frac{p_x(x|c)}{\max_{c'} p_x(x|c')}\right]$$

$$= \chi^2(x;c) - \min_{c'} \chi^2(x;c')$$

In Gaussian limit

$p_{\chi^2}^{(1)}(q_{c_{\rm true}})$ = χ^2-distribution with 1 degree of freedom



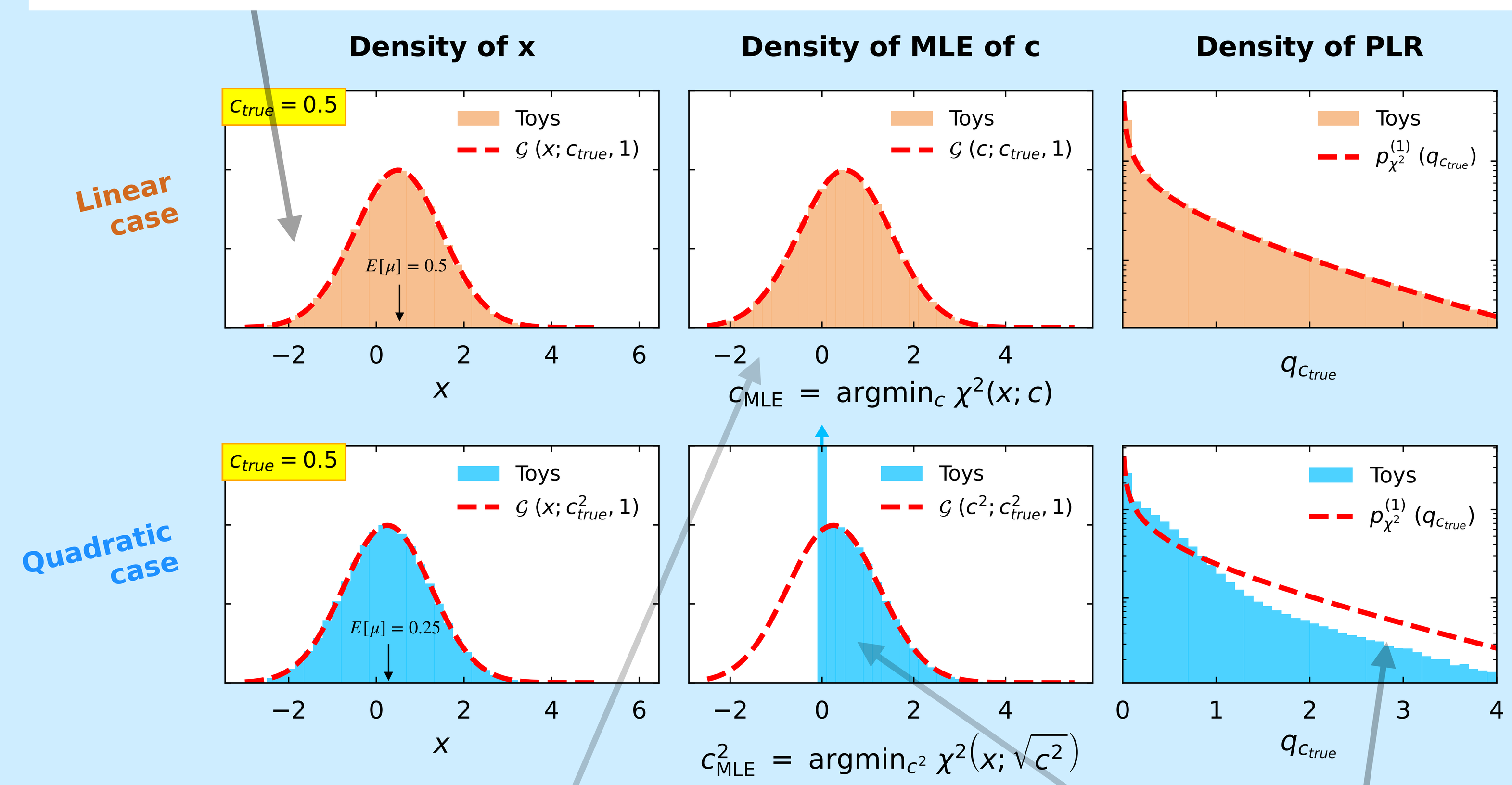
Let's assume you are carrying out a new physics search using an EFT approach with either **linear** or **quadratic** terms using an **observation** x

interference of NP with SM dominates — $\mu(c) = c$

pure new physics dominates — $\mu(c) = c^2$

Distributed as a Gaussian with width 1 — $x \sim \mathcal{G}(\mu, \Sigma = 1)$

The observations are distributed ~ $E[x]$ and we note that they can take **positive** and **negative** values



The maximum likelihood estimators though look very different: for the **linear term** we can find always a c value to describe **negative observations**

But for **quadratic terms** any fluctuation below $x = 0$ leads to $c^2_{\rm MLE} = 0$

Since the optimization step **reaches the boundary of its parameter space, Wilks's theorem is violated** and your **statistical coverage** is **compromised**

AND NOW LET'S FIX IT

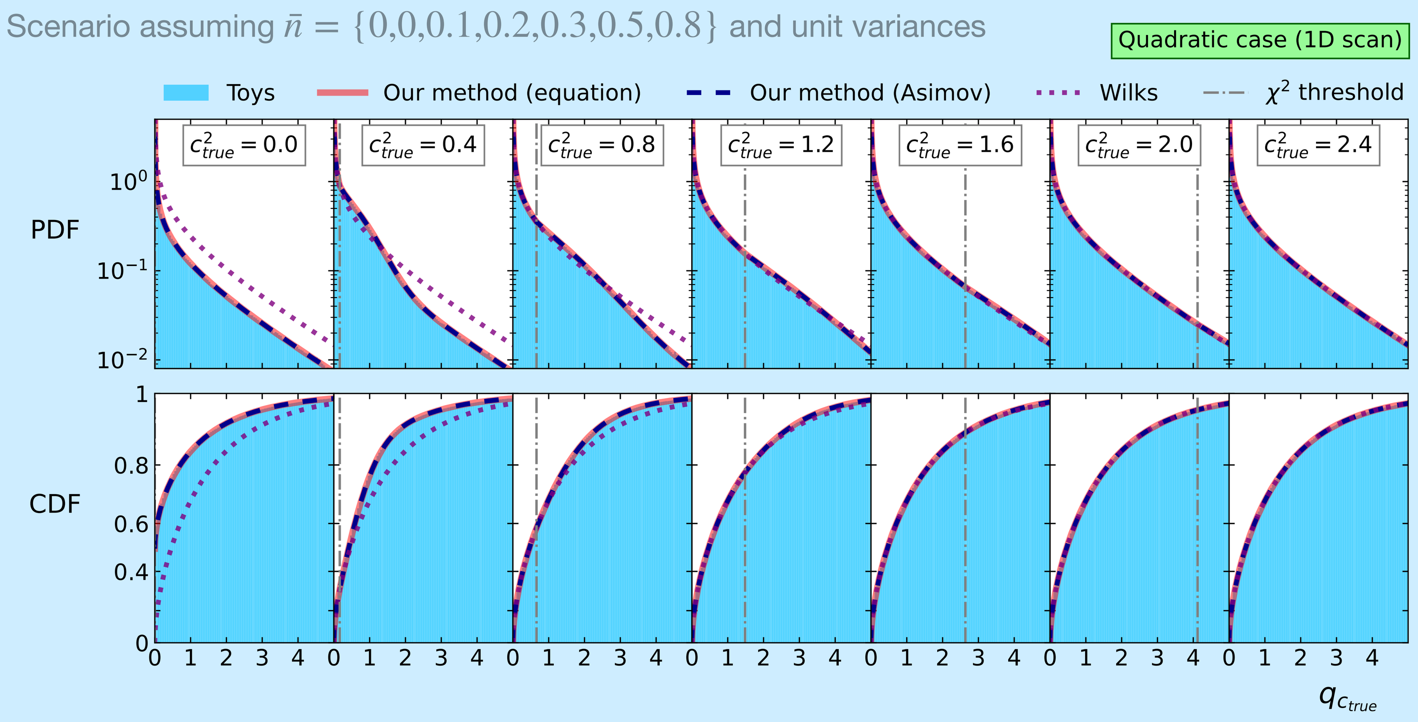
For the **quadratic case** this can actually easily be fixed: A short calculation shows that the **correct distribution** for the test statistics is: $\bar{N}^2 = \sum_i \bar{n}_i^2$

$$p_q(q_{c_{\rm true}}) = \begin{cases} p_{\chi^2}^{(1)}(q_{c_{\rm true}}) & q_{c_{\rm true}} < \bar{N}^2 c^4_{\rm true} \\ \frac{1}{2} p_{\chi^2}^{(1)}(q_{c_{\rm true}}) + \mathcal{N}(q_{c_{\rm true}}; -\bar{N}^2 c^4_{\rm true}, 2\bar{N} c^2_{\rm true}) & q_{c_{\rm true}} \geq \bar{N}^2 c^4_{\rm true} \end{cases}$$

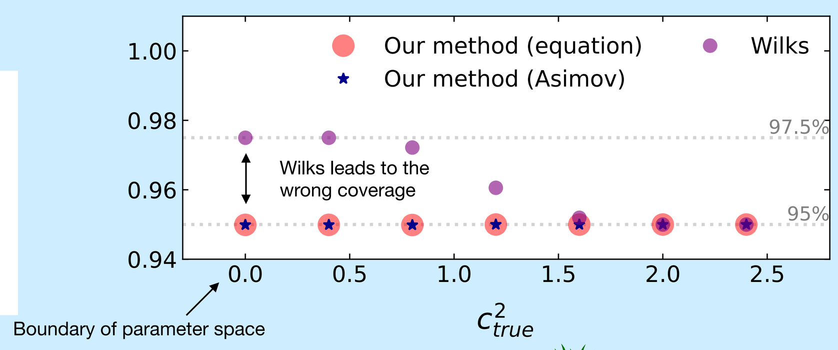
Wilks' piece; New piece

We can even now **scan different values** of $c^2_{\rm true}$ to **progress** from **regions** for which **Wilks is completely adequate to** the **boundary of the parameter space**

Scenario assuming $\bar{n} = \{0,0.0.1,0.2,0.3,0.5,0.8\}$ and unit variances

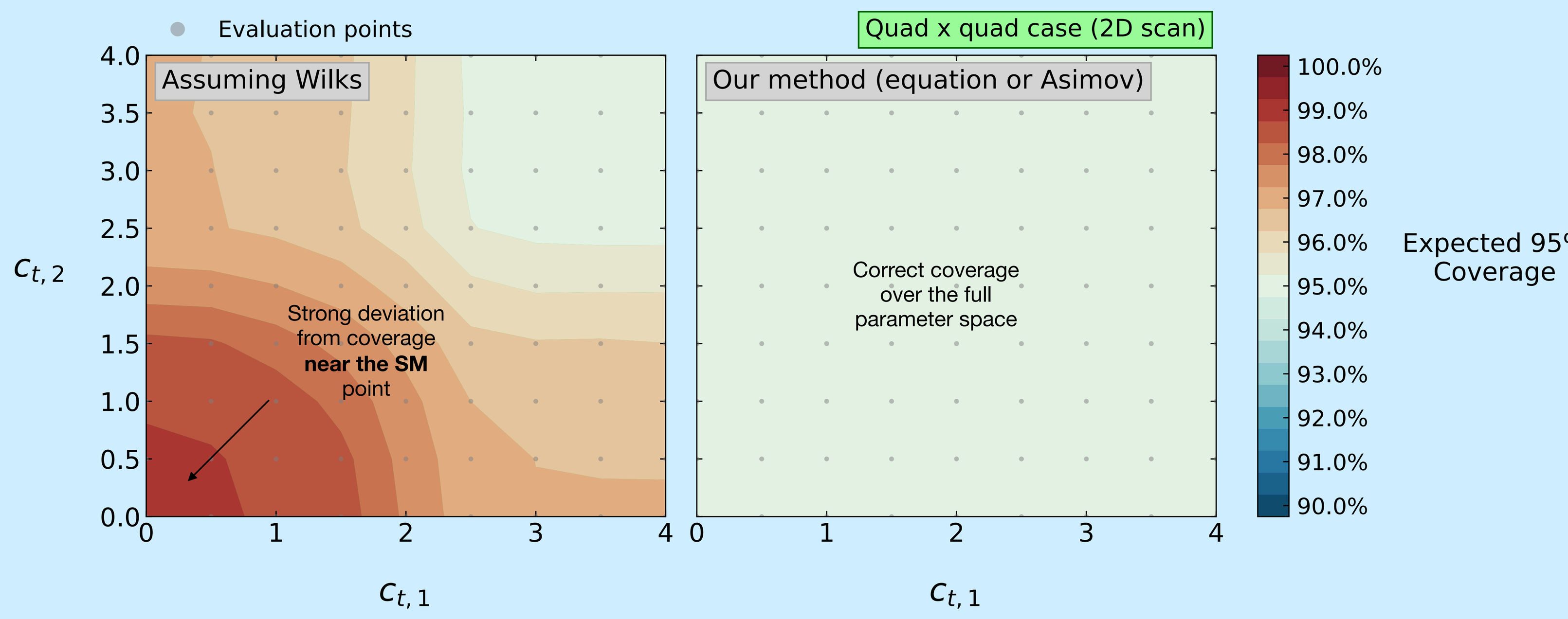


We can investigate how **expected 95% coverage** differs when **using Wilks** versus our method and versus the assumed value of $c^2_{\rm true}$

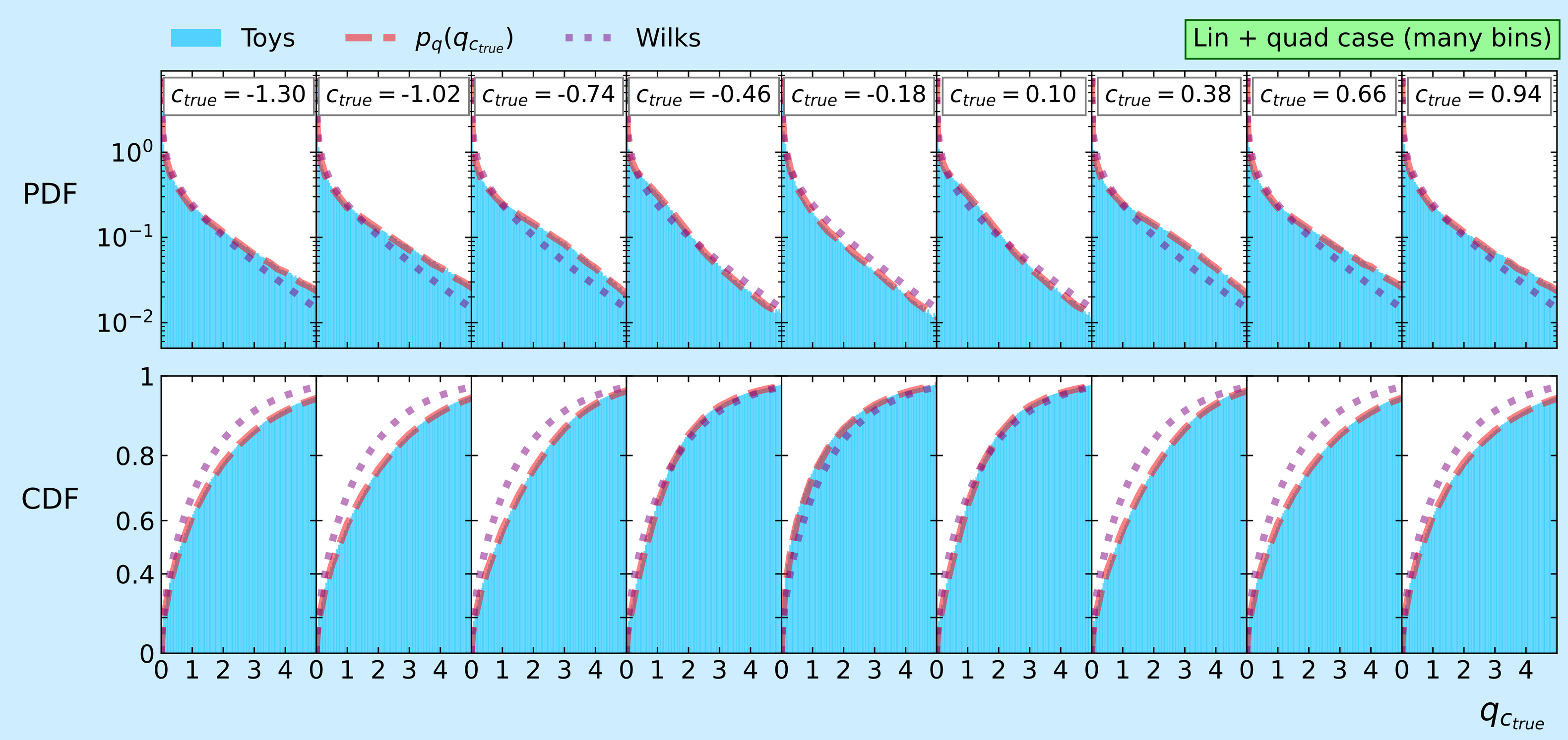


ONTO MORE COMPLICATED CASES:

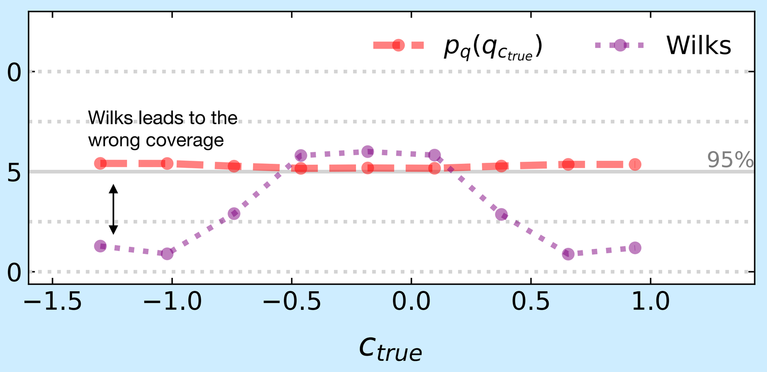
We also investigated and derived the correct distributions for **2D scans** of **two quadratic** Wilson coefficients:



Or the full **quadratic + linear dependence** involving many bins:



With our method one can obtain the **correct 95% coverage**, versus **Wilks** shows as expected a deviation



CONCLUSION

The power and rigor of modern high-energy physics analysis are defined by **both** the **quality of the experimental measurement** *and* **the quality of the statistical analysis** performed on it**. We spend much time and money on performing world-leading measurements,** and should also invest in ensuring that the statistical analysis is as powerful and rigorous as possible.

We **demonstrate** what **can go wrong** in **EFT fits**, and propose asymptotic solutions to recover the **correct coverage properties.**