Complete One-loop Matching of the Type-I Seesaw Model onto the Standard Model Effective Field Theory

Di Zhang (Presenter: zhangdi@ihep.ac.cn), Shun Zhou

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

I. Seesaw Effective Field Theory

Seesaw mechanisms are the simplest and the most natural ways to explain **tiny neutrino masses**, and may also elegantly account for **the matter-antimatter asymmetry of the Universe**.

➢ **What are the seesaw effective field theories (SEFTs)?**

Seesaw mechanisms extend the SM with some heavy degrees of freedom, e.g., the right-handed neutrinos in the type-I seesaw mechanism, which can be integrated out from the models. Then, the obtained effective field theories with a series of higher-dimensional (d > 4) operators denote the SEFTs.

➢ **Why are the SEFTs at the one-loop level necessary?**

1. To **simply eliminate large logs** appearing in the loop calculations with several mass scales via the matching and RG-running procedures.
2. To **consistently study the low-energy consequences** of heavy degrees of freedom in the seesaw models up to the one-loop level.

In this poster, we will perform the **complete one-loop matching** of the type-I seesaw model, and achieve all threshold corrections to the SM couplings, operators up to dimension-six and the associated Wilson coefficients (WCs), which constitute the type-I SEFT, i.e., **SEFT-I**.

II. Matching between UV Theory and EFT

The main idea is to equate the **one-light-particle-irreducible effective action** (i.e., $\Gamma_{\rm L,UV}$) in the UV theory with the **one-particle-irreducible effective action** (i.e., $\Gamma_{\rm EFT}$) in the low-energy EFT at the matching scale μ, namely,

$$\Gamma_{\rm L,UV}\left[\phi_{\rm B}\right] = \Gamma_{\rm EFT}\left[\phi_{\rm B}\right]$$

Thus , the UV theory and EFT will have the **same infrared behaviors**.

1. Matching at the tree level

$$\mathcal{L}^{\rm tree}_{\rm EFT}\left[\phi_{\rm B}\right] = \mathcal{L}_{\rm UV}\left[\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right], \phi_{\rm B}\right]$$

where $\widehat{\Phi}_{\rm c}\left[\phi_{\rm B}\right]$ are the localized solution of the classical equations of motion (EOMs) of heavy fields $\Phi_{\rm B}$, i.e.,

$$\left.\frac{\delta \mathcal{L}_{\rm UV}\left[\Phi,\phi\right]}{\delta\Phi}\right|_{\Phi=\Phi_{\rm c}\left[\phi_{\rm B}\right],\, \phi=\phi_{\rm B}} = 0 \;.$$

2. Matching at the one-loop level

$$\int {\rm d}^d x \mathcal{L}^{\rm 1-loop}_{\rm EFT}\left[\phi\right] = \left.\frac{\rm i}{2}{\rm STr}\ln\left(-\boldsymbol{K}\right)\right|_{\rm hard} - \left.\frac{\rm i}{2}\sum_{n=1}^{\infty}\frac{1}{n}{\rm STr}\left[\left(\boldsymbol{K}^{-1}\boldsymbol{X}\right)^n\right]\right|_{\rm hard}$$

in which $\boldsymbol{K}$ and $\boldsymbol{X}$ are the inverse-propagator matrix and the interaction matrix, respectively, and can be extracted via the following formula:

$$\left.\delta^2\mathcal{L}_{\rm UV}\right|_{\Phi=\widehat{\Phi}_{\rm c}[\phi]} = 2\,\mathcal{L}_{\rm UV}\left[\varphi+\delta\varphi\right]\big|_{\Phi=\widehat{\Phi}_{\rm c}[\phi]} \supset \delta\overline{\varphi}_i\left(\boldsymbol{K}_i\delta_{ij}-\boldsymbol{X}_{ij}\right)\delta\varphi_j$$

III. Results in the Warsaw Basis: Threshold Corrections

The Lagrangian for the type-I seesaw mechanism:

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + \overline{N_{\rm R}}{\rm i}\partial\!\!\!/ N_{\rm R} - \left(\frac{1}{2}\overline{N^c_{\rm R}}MN_{\rm R} + \overline{\ell_{\rm L}}Y_\nu\widetilde{H}N_{\rm R} + {\rm h.c.}\right)$$

The effective couplings with **threshold corrections** from the one-loop matching:

$$m^2_{\rm eff} = m^2 - \frac{1}{(4\pi)^2}\left(Y^\dagger_\nu Y_\nu\right)_{ii}\left[\frac{m^2}{2}\left(1+2L_i\right) + \frac{m^4}{3M_i^2} - 2M_i^2\left(1+L_i\right)\right] ,$$

$$\begin{aligned}\lambda_{\rm eff} = \lambda &+ \frac{1}{(4\pi)^2}\left\{\left(Y^\dagger_\nu Y_\nu\right)_{ii}\left[-\lambda\left(1+2L_i\right) - \frac{4\lambda m^2}{3M_i^2} + \frac{g_2^2 m^2}{18M_i^2}\left(5+6L_i\right)\right] - \frac{m^2}{2M_i^2}\left(Y^\dagger_\nu Y_l Y^\dagger_l Y_\nu\right)_{ii}\left(1+2L_i\right)\right.\\ &+ \left(Y^\dagger_\nu Y_\nu\right)_{ik}\left(Y^\dagger_\nu Y_\nu\right)_{ik}\frac{m^2M_i^2\left(1+2L_k\right) - m^2M_k^2\left(1+2L_i\right) - 2M_i^2M_k^2L_{ik}}{2M_iM_k\left(M_i^2-M_k^2\right)}\\ &\left.+ \left(Y^\dagger_\nu Y_\nu\right)_{ik}\left(Y^\dagger_\nu Y_\nu\right)_{ki}\frac{m^2L_{ik} + M_i^2\left(1+L_i\right) - M_k^2\left(1+L_k\right)}{M_i^2-M_k^2}\right\} ,\end{aligned}$$

$$\left(Y^{\rm eff}_l\right)_{\alpha\beta} = \left(Y_l\right)_{\alpha\beta} - \frac{1}{(4\pi)^2}\left\{\left(Y_l\right)_{\alpha\beta}\left(Y^\dagger_\nu Y_\nu\right)_{ii}\left[\frac{1}{4}\left(1+2L_i\right) + \frac{m^2}{3M_i^2}\right] - \frac{1}{8}\left(Y_\nu\right)_{\alpha i}\left(Y^\dagger_\nu Y_l\right)_{i\beta}\left[5 + \frac{6m^2}{M_i^2} + 2\left(3+\frac{2m^2}{M_i^2}\right)L_i\right]\right\} ,$$

$$\left(Y^{\rm eff}_{\rm u}\right)_{\alpha\beta} = \left(Y_{\rm u}\right)_{\alpha\beta} - \frac{1}{(4\pi)^2}\left(Y_{\rm u}\right)_{\alpha\beta}\left(Y^\dagger_\nu Y_\nu\right)_{ii}\left[\frac{1}{4}\left(1+2L_i\right) + \frac{m^2}{3M_i^2}\right] ,$$

$$\left(Y^{\rm eff}_{\rm d}\right)_{\alpha\beta} = \left(Y_{\rm d}\right)_{\alpha\beta} - \frac{1}{(4\pi)^2}\left(Y_{\rm d}\right)_{\alpha\beta}\left(Y^\dagger_\nu Y_\nu\right)_{ii}\left[\frac{1}{4}\left(1+2L_i\right) + \frac{m^2}{3M_i^2}\right] . \qquad L_i \equiv \ln\left(\frac{\mu^2}{M_i^2}\right) + \frac{1}{\varepsilon} - \gamma_{\rm E} + \ln\left(4\pi\right) , \quad L_{ij} \equiv \ln\left(\frac{M_i^2}{M_j^2}\right)$$

IV. Results in the Warsaw Basis: Dim-5 and Dim-6 Operators

X^2H^2		ψ^2DH^2		Four-quark	
$\mathcal{O}_{HB}$	$B_{\mu\nu}B^{\mu\nu}H^\dagger H$	$\mathcal{O}^{(1)\alpha\beta}_{HQ}$	$\left(\overline{Q_{\alpha\rm L}}\gamma^\mu Q_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{QU}$	$\left(\overline{Q_{\alpha\rm L}}\gamma^\mu Q_{\beta\rm L}\right)\left(\overline{U_{\gamma\rm R}}\gamma_\mu U_{\lambda\rm R}\right)$
$\mathcal{O}_{HW}$	$W^I_{\mu\nu}W^{I\mu\nu}H^\dagger H$	$\mathcal{O}^{(3)\alpha\beta}_{HQ}$	$\left(\overline{Q_{\alpha\rm L}}\gamma^\mu\tau^I Q_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H\right)$	$\mathcal{O}^{(8)\alpha\beta\gamma\lambda}_{QU}$	$\left(\overline{Q_{\alpha\rm L}}\gamma^\mu T^A Q_{\beta\rm L}\right)\left(\overline{U_{\gamma\rm R}}\gamma_\mu T^A U_{\lambda\rm R}\right)$
$\mathcal{O}_{HWB}$	$W^I_{\mu\nu}B^{\mu\nu}\left(H^\dagger\tau^I H\right)$	$\mathcal{O}^{\alpha\beta}_{HU}$	$\left(\overline{U_{\alpha\rm R}}\gamma^\mu U_{\beta\rm R}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{Qd}$	$\left(\overline{Q_{\alpha\rm L}}\gamma^\mu Q_{\beta\rm L}\right)\left(\overline{D_{\gamma\rm R}}\gamma_\mu D_{\lambda\rm R}\right)$
H^4D^2		$\mathcal{O}^{\alpha\beta}_{Hd}$	$\left(\overline{D_{\alpha\rm R}}\gamma^\mu D_{\beta\rm R}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}^{(8)\alpha\beta\gamma\lambda}_{Qd}$	$\left(\overline{Q_{\alpha\rm L}}\gamma^\mu T^A Q_{\beta\rm L}\right)\left(\overline{D_{\gamma\rm R}}\gamma_\mu T^A D_{\lambda\rm R}\right)$
$\mathcal{O}_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$	$\mathcal{O}^{(1)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{QUQd}$	$\left(\overline{Q^a_{\alpha\rm L}}U_{\beta\rm R}\right)\epsilon^{ab}\left(\overline{Q^b_{\gamma\rm L}}D_{\lambda\rm R}\right)$
$\mathcal{O}_{HD}$	$\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D^\mu H\right)$	$\mathcal{O}^{(3)\alpha\beta}_{H\ell}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu\tau^I \ell_{\beta\rm L}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}^I_\mu H\right)$	Four-lepton	
H^6		$\mathcal{O}^{\alpha\beta}_{He}$	$\left(\overline{E_{\alpha\rm R}}\gamma^\mu E_{\beta\rm R}\right)\left(H^\dagger {\rm i}\overleftrightarrow{D}_\mu H\right)$	$\mathcal{O}^{\alpha\beta\gamma\beta}_{\ell\ell}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L}\right)\left(\overline{\ell_{\gamma\rm L}}\gamma_\mu \ell_{\lambda\rm L}\right)$
$\mathcal{O}_H$	$\left(H^\dagger H\right)^3$	ψ^2H^3		$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell e}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L}\right)\left(\overline{E_{\gamma\rm R}}\gamma_\mu E_{\lambda\rm R}\right)$
ψ^2XH		$\mathcal{O}^{\alpha\beta}_{UH}$	$\left(\overline{Q_{\alpha\rm L}}\widetilde{H}U_{\beta\rm R}\right)\left(H^\dagger H\right)$		
$\mathcal{O}^{\alpha\beta}_{eB}$	$\left(\overline{\ell_{\alpha\rm L}}\sigma^{\mu\nu}E_{\beta\rm R}\right)HB_{\mu\nu}$	$\mathcal{O}^{\alpha\beta}_{dH}$	$\left(\overline{Q_{\alpha\rm L}}HD_{\beta\rm R}\right)\left(H^\dagger H\right)$		
$\mathcal{O}^{\alpha\beta}_{eW}$	$\left(\overline{\ell_{\alpha\rm L}}\sigma^{\mu\nu}E_{\beta\rm R}\right)\tau^I HW^I_{\mu\nu}$	$\mathcal{O}^{\alpha\beta}_{eH}$	$\left(\overline{\ell_{\alpha\rm L}}HE_{\beta\rm R}\right)\left(H^\dagger H\right)$		
Semi-leptonic					
$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{\ell Q}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L}\right)\left(\overline{Q_{\gamma\rm L}}\gamma_\mu Q_{\lambda\rm L}\right)$	$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell U}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L}\right)\left(\overline{U_{\gamma\rm R}}\gamma_\mu U_{\lambda\rm R}\right)$	$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell edQ}$	$\left(\overline{\ell_{\alpha\rm L}}E_{\beta\rm R}\right)\left(\overline{D_{\gamma\rm R}}Q_{\lambda\rm L}\right)$
$\mathcal{O}^{(3)\alpha\beta\gamma\lambda}_{\ell Q}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu\tau^I \ell_{\beta\rm L}\right)\left(\overline{Q_{\gamma\rm L}}\gamma_\mu\tau^I Q_{\lambda\rm L}\right)$	$\mathcal{O}^{\alpha\beta\gamma\lambda}_{\ell d}$	$\left(\overline{\ell_{\alpha\rm L}}\gamma^\mu \ell_{\beta\rm L}\right)\left(D_{\gamma\rm R}\gamma_\mu D_{\lambda\rm R}\right)$	$\mathcal{O}^{(1)\alpha\beta\gamma\lambda}_{\ell eQU}$	$\left(\overline{\ell^a_{\alpha\rm L}}E_{\beta\rm R}\right)\epsilon^{ab}\left(\overline{Q^b_{\gamma\rm L}}U_{\lambda\rm R}\right)$

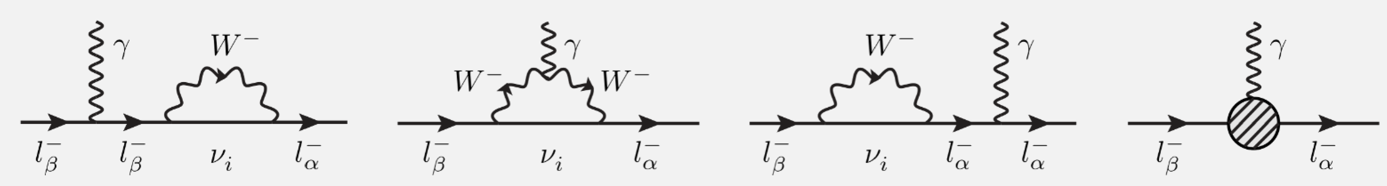
Table 1. All dim-6 operators induced by the type-I seesaw model at the one-loop level in the Warsaw basis.

The complete Lagrangian of the SEFT-I up to dimension-six and $\mathcal{O}\left(M^{-2}\right)$:

$$\begin{aligned}\mathcal{L}_{\rm SEFT-I} = \mathcal{L}_{\rm SM}&\left(m^2\to m^2_{\rm eff}, \lambda\to\lambda_{\rm eff}, Y_l\to Y^{\rm eff}_l, Y_{\rm u}\to Y^{\rm eff}_{\rm u}, Y_{\rm d}\to Y^{\rm eff}_{\rm d}\right)\\ &+ \left[\frac{1}{2}\left(C^{(5)}_{\rm eff}\right)_{\alpha\beta}\mathcal{O}^{(5)}_{\alpha\beta} + {\rm h.c.}\right] + \frac{1}{4}\left(C^{(6)}_{\rm tree}\right)_{\alpha\beta}\left[\mathcal{O}^{(1)\alpha\beta}_{H\ell} - \mathcal{O}^{(3)\alpha\beta}_{H\ell}\right] + \sum_i C_i\mathcal{O}_i\end{aligned}$$

1. $\mathcal{O}^{(5)}_{\alpha\beta}$ are dim-5 Weinberg operator, whose WC is given by
$$\begin{aligned}\left(C^{(5)}_{\rm eff}\right)_{\alpha\beta} = \left(C^{(5)}_{\rm tree}\right)_{\alpha\beta} - \frac{1}{(4\pi)^2}&\left\{\frac{1}{2}\left(C^{(5)}_{\rm tree}\right)_{\alpha\beta}\left(Y^\dagger_\nu Y_\nu\right)_{ii}\left(1+2L_i\right) + \frac{1}{8}\left(Y_\nu\right)_{\alpha i}\left(Y^\dagger_\nu C^{(5)}_{\rm tree}\right)_{i\beta}\left(3+2L_i\right)\right.\\ &\left.+ \frac{1}{8}\left(C^{(5)}_{\rm tree}\right)_{\alpha\gamma}\left(Y_\nu\right)_{\beta i}\left(Y^\dagger_\nu\right)_{i\gamma}\left(3+2L_i\right) - \left[2\lambda\left(1+L_i\right) + \frac{g_1^2+g_2^2}{4}\left(1+3L_i\right)\right]\left(Y_\nu\right)_{\alpha i}M_i^{-1}\left(Y^{\rm T}_\nu\right)_{i\beta}\right\}\end{aligned}$$
with $C^{(5)}_{\rm tree} = Y_\nu M^{-1}Y^{\rm T}_\nu$.
2. $C^{(6)}_{\rm tree} = Y_\nu M^{-2}Y^\dagger_\nu$ is the tree-level contribution to the WCs of $\mathcal{O}^{(1)\alpha\beta}_{H\ell}$ and $\mathcal{O}^{(3)\alpha\beta}_{H\ell}$
3. $\mathcal{O}_i$ and C_i are the dim-6 operators listed in Table 1 and the corresponding WCs at the one-loop level, such as
$$C^{\alpha\beta}_{eB} = \frac{g_1}{24(4\pi)^2}\left(Y_\nu M^{-2}Y^\dagger_\nu Y_l\right)_{\alpha\beta} \qquad C^{\alpha\beta\gamma\lambda}_{\ell U} = \frac{1}{8(4\pi)^2}\left(Y_\nu\right)_{\alpha i}M_i^{-2}\left(Y^\dagger_\nu\right)_{i\beta}\left[\frac{2g_1^2}{27}\delta^{\gamma\lambda}\left(11+6L_i\right) - \left(Y^\dagger_{\rm u}Y_{\rm u}\right)_{\gamma\lambda}\left(3+2L_i\right)\right]$$
$$C^{\alpha\beta}_{eW} = \frac{5g_2}{24(4\pi)^2}\left(Y_\nu M^{-2}Y^\dagger_\nu Y_l\right)_{\alpha\beta} \qquad C^{\alpha\beta\gamma\lambda}_{\ell d} = \frac{1}{8(4\pi)^2}\left(Y_\nu\right)_{\alpha i}M_i^{-2}\left(Y^\dagger_\nu\right)_{i\beta}\left[-\frac{g_1^2}{27}\delta^{\gamma\lambda}\left(11+6L_i\right) + \left(Y^\dagger_{\rm d}Y_{\rm d}\right)_{\gamma\lambda}\left(3+2L_i\right)\right]$$

V. Radiative Decays of Charged Leptons in the SEFT-I



(a) (b) (c) (d)

1. Diagrams **(a)-(c)** contributed by the **tree-level** part of $\mathcal{O}^{(5)}_{\alpha\beta}$, $\mathcal{O}^{(1)\alpha\beta}_{H\ell}$ and $\mathcal{O}^{(3)\alpha\beta}_{H\ell}$
$${\rm i}\mathcal{M}_{abc} = \frac{-{\rm i}eg_2^2}{2(4\pi)^2M_W^2}\sum_{i=1}^{3}U_{\alpha i}U^*_{\beta i}\left(-\frac{5}{6}+\frac{m_i^2}{4M_W^2}\right)\left[\epsilon^*_\mu\overline{u}\left(p_2\right){\rm i}\sigma^{\mu\nu}q_\nu\left(m_\alpha P_{\rm L}+m_\beta P_{\rm R}\right)u\left(p_1\right)\right]$$
2. Diagrams **(d)** coming from the **one-loop** operators $\mathcal{O}^{\alpha\beta}_{eB}$ and $\mathcal{O}^{\alpha\beta}_{eW}$
$${\rm i}\mathcal{M}_d = \frac{{\rm i}eg_2^2}{6(4\pi)^2M_W^2}\left(RR^\dagger\right)_{\alpha\beta}\left[\epsilon^*_\mu\overline{u}\left(p_2\right){\rm i}\sigma^{\mu\nu}q_\nu\left(m_\alpha P_{\rm L}+m_\beta P_{\rm R}\right)u\left(p_1\right)\right]$$
3. The ratio of the decay width of $l^-_\beta\to l^-_\alpha+\gamma$ to that of $l^-_\beta\to l^-_\alpha+\overline{\nu}_\alpha+\nu_\beta$
$$\frac{\Gamma\left(l^-_\beta\to l^-_\alpha+\gamma\right)}{\Gamma\left(l^-_\beta\to l^-_\alpha+\overline{\nu}_\alpha+\nu_\beta\right)} \simeq \frac{3\alpha_{\rm em}}{2\pi}\left|-\frac{5}{6}\left(UU^\dagger\right)_{\alpha\beta} - \frac{1}{3}\left(RR^\dagger\right)_{\alpha\beta}\right|^2 = \frac{3\alpha_{\rm em}}{8\pi}\left|\left(RR^\dagger\right)_{\alpha\beta}\right|^2$$

Consistent with the results in the full type-I seesaw model

VI. Summary

1. The **complete one-loop matching** of the **type-I seesaw model** onto SMEFT has been carried out, and **31 independent dim-6 operators** in the Warsaw basis have been obtained up to the **one-loop level**.
2. The one-loop matching can induce threshold corrections to the SM couplings, and these effective couplings are the very matching conditions for two-loop RGEs.
3. The contributions from the one-loop operators to some one-loop processes must be included for **self-consistent calculations** in the SEFT.

This poster is based on the paper: **D. Zhang and S. Zhou, JHEP 09 (2021) 163**