Non-unitary Leptonic Flavor Mixing and CP Violation in Neutrino-antineutrino Oscillations

Yilin Wang[1,2,*], Shun Zhou[1,2]

1. Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
2. School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

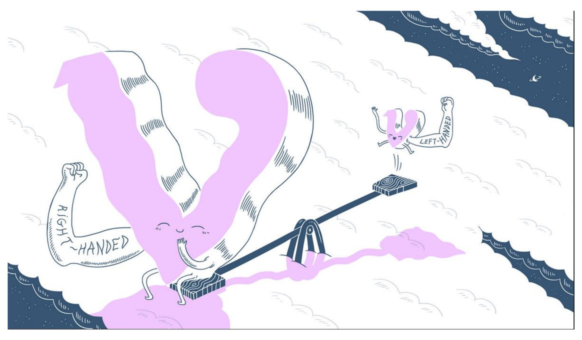
*Speaker: Yilin Wang (Email: wangyilin@ihep.ac.cn)

ICHEP 2022 BOLOGNA

Base on: Phys. Lett. B 824 (2022) 136797.

I. Motivation

Neutrino oscillation experiments have provided us with evidence that neutrinos are actually massive[1]. In order to accommodate tiny neutrino masses, one can naturally extend the Standard Model (**SM**) by introducing **three right-handed neutrino singlets**, which called type-I seesaw model.



Neutrino oscillations → imply → Neutrino masses ← Seesaw mechanism ← Type-I seesaw → Non-unitary leptonic flavor mixing

Due to the mixing between light and heavy Majorana neutrinos, leptonic flavor mixing matrix becomes non-unitary.
If neutrino obtains Majorana nature, it is necessary to determine the two Majorana CP-violating phases.

Neutrinoless double beta decay[2] → Majorana phases ← Neutrino-antineutrino oscillations[3]

- How different the CP violation with a non-unitary mixing matrix is from that with a unitary one?
- How large the deviations can be in light of the latest experimental bounds on unitarity violation?

II. Non-unitary Flavor Mixing

After spontaneous breaking of the SM gauge symmetry, the 6×6 neutrino mass matrix can be diagonalized by a unitary matrix as

Non-unitary ← V

$$\begin{pmatrix} \boxed{V} & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} \mathbf{0} & M_{\rm D} \\ M_{\rm D}^{\rm T} & M_{\rm R} \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_{\rm R} \end{pmatrix} ,$$

Frist we give the exact relation of the two parametrizations of non-unitary matrix [4],

Hermitian × unitary — **QR Factorization** — **lower-triangular × unitary**

$$V = \begin{pmatrix} 1-\eta_{ee} & -\eta_{e\mu} & -\eta_{e\tau} \\ -\eta^*_{e\mu} & 1-\eta_{\mu\mu} & -\eta_{\mu\tau} \\ -\eta^*_{e\tau} & -\eta^*_{\mu\tau} & 1-\eta_{\tau\tau} \end{pmatrix} \cdot V' \quad \Longleftrightarrow \quad V = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} \cdot \widetilde{V}$$

complex (α_{21}, α_{31}, α_{32})

$$\mathbf{1}-\eta \approx \begin{pmatrix} 1-\eta_{ee} & 0 & 0 \\ -2\eta^*_{e\mu} & 1-\eta_{\mu\mu} & 0 \\ -2\eta^*_{e\tau} & -2\eta^*_{\mu\tau} & 1-\eta_{\tau\tau} \end{pmatrix} \cdot \begin{pmatrix} 1 & -\eta_{e\mu} & -\eta_{e\tau} \\ +\eta^*_{e\mu} & 1 & -\eta_{\mu\tau} \\ +\eta^*_{e\tau} & +\eta^*_{\mu\tau} & 1 \end{pmatrix} .$$

Non-unitarity of the leptonic flavor mixing matrix brings in **extra sources of CP violation**, which can be probed in future long-baseline accelerator neutrino oscillation experiments.

III. Neutrino-Antineutrino Oscillations

Flavor eigenstates ← $|\nu_\alpha\rangle$; $|\nu_i\rangle$ → Mass eigenstates

$$\boxed{|\nu_\alpha\rangle} = \frac{1}{\sqrt{(VV^\dagger)_{\alpha\alpha}}} \sum_i V^*_{\alpha i} \boxed{|\nu_i\rangle} ,$$

one can calculate the neutrino-antineutrino oscillation amplitudes. After calculating the probabilities, the **CP asymmetries** for neutrino-antineutrino oscillations turn out to be

$$\mathcal{A}_{\alpha\beta} \equiv \frac{P(\nu_\alpha \to \overline{\nu}_\beta) - P(\overline{\nu}_\alpha \to \nu_\beta)}{P(\nu_\alpha \to \overline{\nu}_\beta) + P(\overline{\nu}_\alpha \to \nu_\beta)} = \frac{2\sum\limits_{i<j} m_i m_j \boxed{\mathcal{V}^{ij}_{\alpha\beta}} \sin F_{ji}}{|\langle m\rangle_{\alpha\beta}|^2 - 4\sum\limits_{i<j} m_i m_j \mathcal{C}^{ij}_{\alpha\beta} \sin^2 \frac{F_{ji}}{2}} ,$$

where we define:

$$F_{ji} \equiv \Delta m^2_{ji} L/(2E) , \quad \mathcal{C}^{ij}_{\alpha\beta} \equiv {\rm Re}\left[V_{\alpha i} V_{\beta i} V^*_{\alpha j} V^*_{\beta j}\right] , \quad \boxed{\mathcal{V}^{ij}_{\alpha\beta} \equiv {\rm Im}\left[V_{\alpha i} V_{\beta i} V^*_{\alpha j} V^*_{\beta j}\right]} .$$

References:

[1] Z. z. Xing, Phys. Rept. 854, 1-147 (2020).
[2] W. H. Furry, Phys.Rev. 56, 1184-1193 (1939).
[3] B. Pontecorvo, Sov. Phys. JETP 6, 429 (1957).
[4] Y. Wang and S. Zhou, Phys.Lett.B 824 (2022).

IV. CP Asymmetries

In the so-called minimal seesaw model, as the lightest neutrino is massless, there exists only one Majorana-type CP-violating phase σ. Then we define the working observable

$$\varepsilon_{\alpha\beta} \equiv \frac{\mathcal{A}_{\alpha\beta} - \boxed{\widetilde{\mathcal{A}}_{\alpha\beta}}}{\widetilde{\mathcal{A}}_{\alpha\beta}} \times 100\% ,$$

Unitary CPA → $\widetilde{\mathcal{A}}_{\alpha\beta}$

With the appropriate inputs[4], $\varepsilon_{\alpha\beta}$ obtain the following ranges in the NO case.

Normal Ordering	$\delta = 195°,\ \sigma = 0°$		$\delta = 195°,\ \sigma = 45°$	
	$\varepsilon^{\rm U}_{\alpha\beta}$	$\varepsilon^{\rm L}_{\alpha\beta}$	$\varepsilon^{\rm U}_{\alpha\beta}$	$\varepsilon^{\rm L}_{\alpha\beta}$
$\alpha,\ \beta = e,\ e$	0%	0%	0%	0%
$\alpha,\ \beta = e,\ \mu$	−0.008974%	+0.008974%	−0.001717%	+0.001717%
$\alpha,\ \beta = e,\ \tau$	+1.946%	−1.948%	+0.2681%	−0.2698%
$\alpha,\ \beta = \mu,\ \mu$	+0.09932%	−0.09932%	+0.005116%	−0.005116%
$\alpha,\ \beta = \mu,\ \tau$	−206.8%	+206.8%	−0.4555%	+0.4564%
$\alpha,\ \beta = \tau,\ \tau$	−19.39%	+19.40%	+0.9050%	−0.9000%

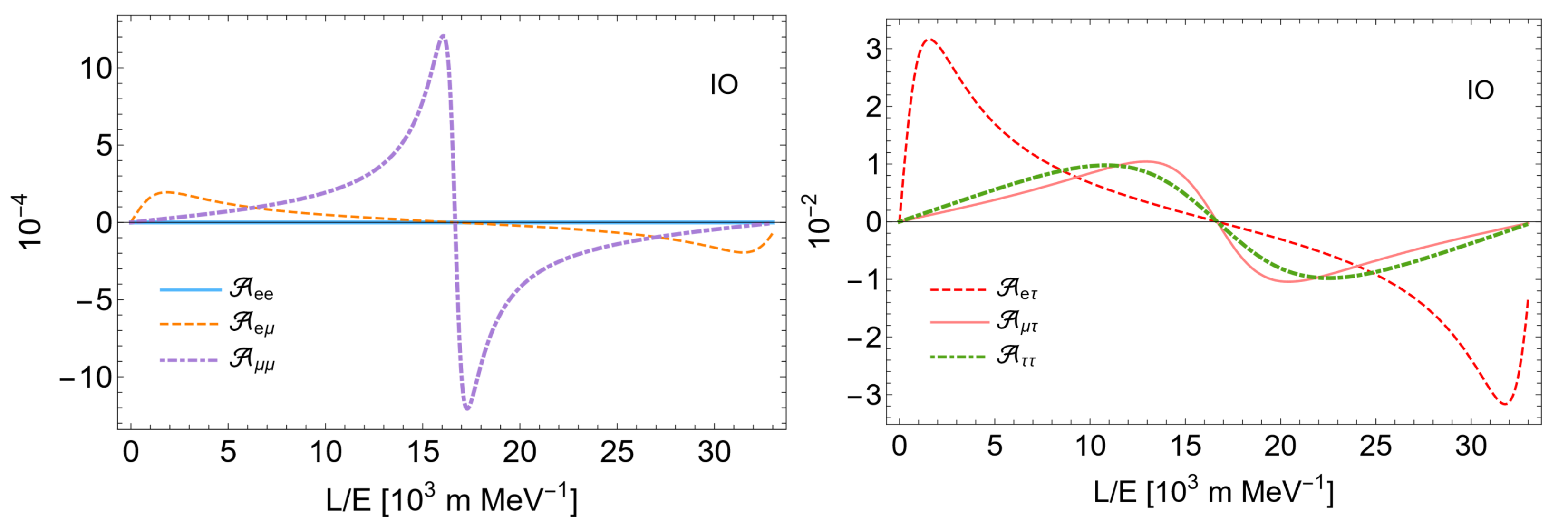
V. Numerical Results

A particularly interesting scenario is to assume all the ordinary Dirac and Majorana CP-violating phases to be vanishing. There are also **non-vanishing** CP asymmetries induced by non-unitary parameters as

$$\begin{aligned}
\mathcal{V}^{12}_{ee} &\approx \mathcal{V}^{13}_{ee} \approx \mathcal{V}^{23}_{ee} \approx \mathcal{V}^{13}_{e\mu} \approx \mathcal{V}^{23}_{e\mu} \approx 0 . \\
\mathcal{V}^{12}_{\mu\mu} &\approx -2\,|\alpha_{21}|\,\alpha^3_{22} s_{12} c_{12} c^3_{23} \sin\phi_{21} , \\
\mathcal{V}^{13}_{\mu\mu} &\approx -\mathcal{V}^{23}_{\mu\mu} \approx -2\,|\alpha_{21}|\,\alpha^3_{22} s_{12} c_{12} s^2_{23} c_{23} \sin\boxed{\phi_{21}} , \\
\mathcal{V}^{12}_{\tau\tau} &\approx +2\,|\alpha_{31}|\,\alpha^3_{33} s_{12} c_{12} s^3_{23} \sin\boxed{\phi_{31}} , \\
\mathcal{V}^{13}_{\tau\tau} &\approx +2\,|\alpha_{31}|\,\alpha^3_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} - 2\,|\alpha_{32}|\,\alpha^3_{33} s^2_{12} s_{23} c_{23} \sin\boxed{\phi_{32}} , \\
\mathcal{V}^{23}_{\tau\tau} &\approx -2\,|\alpha_{31}|\,\alpha^3_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} - 2\,|\alpha_{32}|\,\alpha^3_{33} c^2_{12} s_{23} c_{23} \sin\phi_{32} , \\
\mathcal{V}^{12}_{e\mu} &\approx +\alpha^2_{11}\,|\alpha_{21}|\,\alpha_{22} s_{12} c_{12} c_{23} \sin\phi_{21} , \\
\mathcal{V}^{12}_{e\tau} &\approx -\alpha^2_{11}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} \sin\phi_{31} , \\
\mathcal{V}^{13}_{e\tau} &\approx +\alpha^2_{11} \alpha_{33} c_{12} s_{13} \left(|\alpha_{31}|\, c_{12} c_{23} \sin\phi_{31} - |\alpha_{32}|\, s_{12} \sin\phi_{32}\right) \\
\mathcal{V}^{23}_{e\tau} &\approx +\alpha^2_{11} \alpha_{33} s_{12} s_{13} \left(|\alpha_{31}|\, s_{12} c_{23} \sin\phi_{31} + |\alpha_{32}|\, c_{12} \sin\phi_{32}\right) \\
\mathcal{V}^{12}_{\mu\tau} &\approx +\alpha^2_{22}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} , \\
\mathcal{V}^{13}_{\mu\tau} &\approx -\alpha^2_{22}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} + \alpha^2_{22}\,|\alpha_{32}|\,\alpha_{33} s^2_{12} s_{23} c_{23} \sin\phi_{32} , \\
\mathcal{V}^{23}_{\mu\tau} &\approx +\alpha^2_{22}\,|\alpha_{31}|\,\alpha_{33} s_{12} c_{12} s_{23} c^2_{23} \sin\phi_{31} + \alpha^2_{22}\,|\alpha_{32}|\,\alpha_{33} c^2_{12} s_{23} c_{23} \sin\phi_{32} .
\end{aligned}$$

Non-unitary phases

In the inverted-ordering of neutrino masses, the CP asymmetries $\mathcal{A}_{\alpha\beta}$ with trivial Dirac and Majorana CP-violating phases varying with L/E are shown as



VI. Summary

- In order to understand the Majorana nature of neutrinos, it is necessary to observe the Majorana phases. 0νββ cannot determine the two phases simultaneously. Therefore, we consider the neutrino-antineutrino oscillations.
- Non-unitary flavor mixing is actually a natural prediction in the type-I seesaw model.
- We have examined the CP asymmetries in the neutrino-antineutrino oscillations in the presence of a non-unitary flavor mixing matrix.
- By using the QR factorization, we establish the relation between the Hermitian parametrization and the triangular parametrization of a non-unitary mixing matrix.
- Even with trivial values of ordinary CP-violating phases, one can obtain nonzero CP asymmetries due to the extra non-unitary CP phases.
- In addition, the probabilities of heavy neutrino-antineutrino oscillations may not be suppressed with the enhanced mass scale and the induced CP asymmetries could also be resonantly enhanced.