Elastic neutrino scattering on nuclear systems as a probe of neutrino electromagnetic interactions.

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1 Introduction

There are a large number of experiments investigating both neutrino oscillations and their interactions. In both cases, it is important to theoretically investigate neutrino scattering on various targets [1,2], since scattering processes are either a tool for detecting neutrino fluxes: the processes of neutrino scattering on a nucleon or a nucleus studied in this work contribute to the signals of such experiments as MiniBooNE [3], COHERENT [4,5] and registration of supernova neutrinos in JUNO [6]; or a tool for studying fundamental interactions of neutrinos: in this work, the contribution of the electromagnetic properties of neutrinos is studied. The latter neutrino properties emerge in different extensions of the Standard Model, and they include [7]: millicharges and charge radii, electric, magnetic and anapole moments.

2 Cross sections of elastic neutrino scattering on nucleons and nuclei

We consider the process where an ultrarelativistic neutrino with energy E_ν originates from a source (reactor, accelerator, the Sun, etc.) and elastically scatters on a nucleon (nucleus) in a detector at energy-momentum transfer $q = (T, \mathbf{q})$. If the neutrino is born in the source in the flavor state $|\nu_\ell\rangle$, then its state in the detector is $|\nu_\ell(L)\rangle = \sum_{k=1}^3 U^*_{\ell k}\exp(-i\frac{m_k^2}{2E_\nu}\mathcal{L})|\nu_k\rangle$, where $\mathcal{L}$ is the source-detector distance. We assume the target nucleon (nucleus) to be free and at rest in the lab frame. The matrix element of the transition $\nu_\ell(L)+X \to \nu_j+X$, where X is either a proton, a neutron or a nucleus, due to weak interaction is given by

$$\mathcal{M}_j^{(w)} = \frac{G_F}{\sqrt{2}}U^*_{\ell j}e^{-i\frac{m_j^2}{2E_\nu}\mathcal{L}}\bar{u}^{(\nu)}_{j,\lambda'}(k')\gamma^\mu(1-\gamma^5)u^{(\nu)}_{j,\lambda}(k)J_\mu^{(NC)}, \tag{1}$$

here $J_\lambda^{(NC)}$ is a weak neutral current of a nucleon (nucleus). $\bar{u}^{(\nu)}_{j,\lambda'}(k') = u^{(\nu)\dagger}_{j,\lambda'}(k')\gamma^0$, where $u^{(\nu)}_{j,\lambda}(k)$ is the bispinor amplitude of the massive neutrino state $|\nu_j\rangle$ with 4-momentum k and spin state λ. The matrix element due to electromagnetic interaction is

$$\mathcal{M}_j^{(\gamma)} = -\frac{4\pi\alpha}{q^2}\sum_{k=1}^3 U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}\bar{u}^{(\nu)}_{j,\lambda'}(k')\Lambda_{jk}^{(\mathrm{EM};\nu)\mu}(q)u^{(\nu)}_{k,\lambda}(k)J_\mu^{(EM)}, \tag{2}$$

where $J_\mu^{(EM)}$ is the electromagnetic current of the nucleon (nucleus). Assuming the target to be a free nucleon, these currents can be expanded as follows:

$$\begin{aligned} J_\lambda^{(NC)}(q) &= \bar{u}_{s'}^{(N)}(p')\Lambda_\lambda^{(\mathrm{NC};N)}(-q)u_s^{(N)}(p),\\ J_\lambda^{(EM)}(q) &= \bar{u}_{s'}^{(N)}(p')\Lambda_\lambda^{(\mathrm{EM};N)}(-q)u_s^{(N)}(p), \end{aligned} \tag{3}$$

where $\Lambda_\lambda^{(\mathrm{NC};N)}(q)$ and $\Lambda_\lambda^{(\mathrm{EM};N)}(-q)$ are nucleon neutral weak and electromagnetic vertexes, respectively. $\Lambda_{jk}^{(\mathrm{EM};\nu)\mu}(q)$ is the electromagnetic neutrino vertex. We consider the following vertexes:

$$\begin{aligned} \Lambda_\mu^{(\mathrm{EM};\nu)fi}(q) &= (\gamma_\mu - q_\mu \not{q}/q^2)[f_Q^{fi}(q^2) + f_A^{fi}(q^2)q^2\gamma_5] - i\sigma_{\mu\nu}q^\nu[f_M^{fi}(q^2) + if_E^{fi}(q^2)\gamma_5],\\ \Lambda_\mu^{(\mathrm{EM};N)}(q) &= \gamma_\mu F_Q^N(q^2) - \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_M^N(q^2)\\ &\quad + \frac{1}{2m_N}\sigma_{\mu\nu}q^\nu\gamma_5 F_E^N(q^2) - \left(q^2\gamma_\mu - q_\mu\not{q}\right)\gamma_5\frac{F_A^N(q^2)}{(2m_N)^2},\\ \Lambda_\mu^{(\mathrm{NC};N)}(q) &= \gamma_\mu F_1^N(q^2) - \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2^N(q^2) - \gamma_\mu\gamma_5 G_A^N(q^2) + \frac{1}{m_N}G_P^N(q^2)q_\mu\gamma_5. \end{aligned} \tag{4}$$

Assuming the target to be a free nucleus, the nuclear currents can be expanded as follows:

$$J_\lambda^{(NC)}(q) = 2M[\delta_0^\mu\mathcal{F}_1(\vec{q}) - \delta_i^\mu\mathcal{G}_A^i(\vec{q})], \qquad J_\lambda^{(EM)}(q) = 2M\delta_0^\mu\mathcal{F}_Q(\vec{q}), \tag{5}$$

where M is the nuclear mass, and

$$\begin{aligned} \mathcal{F}_1(\vec{q}) &= \frac{1}{(2\pi)^3}\int d^3r e^{i\vec{q}\cdot\vec{r}}\langle n, J, M_J'|\sum_{k=0}^{Z+N} g_V^{N_{(k)}}\delta^3(\vec{r}-\vec{r}_k)|n, J, M_J\rangle,\\ \mathcal{G}_A^i(\vec{q}) &= \frac{1}{(2\pi)^3}\int d^3r e^{i\vec{q}\cdot\vec{r}}\langle n, J, M_J'|\sum_{k=0}^{Z+N} g_A^{N_{(k)}}\sigma^i_{(k)}\delta^3(\vec{r}-\vec{r}_k)|n, J, M_J\rangle,\\ \mathcal{F}_Q(\vec{q}) &= \frac{1}{(2\pi)^3}\int d^3r e^{i\vec{q}\cdot\vec{r}}\langle n, J, M_J'|\sum_{k=0}^{Z}\delta^3(\vec{r}-\vec{r}_k)|n, J, M_J\rangle. \end{aligned} \tag{6}$$

When evaluating the cross section, we neglect the neutrino masses. Since the final massive state of the neutrino is not resolved in the detector, the differential cross section measured in the scattering experiment is given by

$$\frac{d\sigma}{dT} = \frac{|\mathcal{M}|^2}{32\pi E_\nu^2 m_N}, \tag{7}$$

with the following absolute matrix element squared:

$$|\mathcal{M}|^2 = \sum_{j=1}^3\left|\mathcal{M}_j^{(w)} + \mathcal{M}_j^{(\gamma)}\right|^2, \tag{8}$$

where averaging over initial and summing over final spin polarizations is assumed. The differential cross section can be presented as a sum of the helicity-preserving (hp) and helicity-flipping (hf) parts: $\frac{d\sigma}{dT} = \frac{d\sigma_{\mathrm{hp}}}{dT} + \frac{d\sigma_{\mathrm{hf}}}{dT}$. The cross section for neutrino-nucleon scattering is

$$\begin{aligned} \frac{d\sigma_{hp}}{dT} &= \frac{G_F^2 m_N}{2\pi}\Bigg[(C_V - 2\mathrm{Re}\,C_{V\&A} + C_A) + (C_V + 2\mathrm{Re}\,C_{V\&A} + C_A)\left(1-\frac{T}{E_\nu}\right)^2\\ &\quad + (C_A - C_V)\frac{m_N T}{E_\nu^2} + C_M\frac{T}{2m_N}\left(2 + \frac{m_N T}{E_\nu^2} - \frac{2T}{E_\nu}\right) - C_E\frac{T}{2m_N}\left(2 - \frac{m_N T}{E_\nu^2} - \frac{2T}{E_\nu}\right)\\ &\quad + 2\frac{T}{E_\nu}\mathrm{Re}\,C_{A\&M}\left(2-\frac{T}{E_\nu}\right) - 2\mathrm{Re}\,C_{V\&M}\frac{T^2}{E_\nu^2}\Bigg],\\ \frac{d\sigma_{hf}}{dT} &= \frac{\pi\alpha^2}{m_e^2}|\mu_\nu(\mathcal{L}, E_\nu)|^2\Bigg[\left(\frac{1}{T} - \frac{1}{E_\nu}\right)F_Q^2 + \left(\frac{1}{T} - \frac{1}{E_\nu} - \frac{m_N}{2E_\nu^2}\right)\frac{T^2}{4m_N^2}F_A^2\\ &\quad - \frac{T}{2E_\nu^2}F_Q F_M + \frac{\left(2-\frac{T}{E_\nu}\right)^2 - \frac{2m_N T}{E_\nu^2}}{8m_N}F_M^2 - \frac{\left(2-\frac{T}{E_\nu}\right)^2}{8m_N}F_E^2 + \frac{(2-\frac{T}{E_\nu})T}{4E_\nu m_N}F_A(F_M - F_Q)\Bigg], \end{aligned} \tag{9}$$

where [1]

$$\begin{aligned} C_V &= \sum_j\left|\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}(\delta_{jk}F_1 - F_Q Q_{jk})\right|^2, \quad Q_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F q^2}\left(f_{jk}^Q - q^2 f_{jk}^A\right),\\ C_{V\&A} &= \sum_j\left(\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}(\delta_{jk}F_1 - F_Q QL_{jk})\right)\left(\sum_n U_{\ell n}e^{i\frac{m_n^2}{2E_\nu}\mathcal{L}}(-\delta_{jn}G_A + \frac{q^2 F_A Q_{jn}}{4m_N^2})\right),\\ C_A &= \sum_j\left|\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}(G_A\delta_{jk} - \frac{q^2 F_A Q_{jk}}{4m_N^2})\right|^2,\\ C_M &= \sum_j\left|\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}(F_2\delta_{jk} - F_M Q_{jk})\right|^2,\\ C_{A\&M} &= \sum_j\left(\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}(\delta_{jk}G_A - \frac{q^2 F_A Q_{jk}}{4m_N^2})\right)\left(\sum_n U_{\ell n}e^{i\frac{m_n^2}{2E_\nu}\mathcal{L}}(F_2\delta_{jn} - F_M Q_{jn})\right),\\ C_{V\&M} &= \sum_j\left(\sum_k U^*_{\ell k}e^{-\frac{m_k^2}{2E_\nu}\mathcal{L}}(-\delta_{jk}F_1 + F_Q Q_{jk})\right)\left(\sum_n U_{\ell n}e^{i\frac{m_n^2}{2E_\nu}\mathcal{L}}(F_2\delta_{jn} - F_M Q_{jn})\right),\\ C_E &= \sum_j\left|\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}F_E Q_{jk}\right|^2, \; |\mu(\mathcal{L}, E_\nu)|^2 = \sum_j\left|\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}2m_e(f_{jk}^M - if_{jk}^E)\right|^2. \end{aligned} \tag{10}$$

The cross section for neutrino-nucleus scattering is

$$\begin{aligned} \frac{d\sigma}{dT} &= \frac{G_F^2 M}{2\pi}\left[\mathfrak{C}_V\left(2 - \frac{MT}{E_\nu^2}\right) + \frac{1}{2J+1}\sum_{M_J M_{J'}}|\mathcal{G}_A^z|^2\left(2 + \frac{MT}{E_\nu^2}\right)\right] + \frac{\pi\alpha}{m_e^2}\frac{|\mathcal{F}_Q|^2}{T}|\mu(\mathcal{L}, E_\nu)|^2,\\ \mathfrak{C}_V &= \sum_j\left|\sum_k U^*_{\ell k}e^{-i\frac{m_k^2}{2E_\nu}\mathcal{L}}(\delta_{jk}\mathcal{F}_1 - \mathcal{F}_Q Q_{jk})\right|^2, \quad Q_{jk} = \frac{2\sqrt{2}\pi\alpha}{G_F q^2}\left(f_{jk}^Q - q^2 f_{jk}^A\right). \end{aligned} \tag{11}$$

3 Numerical results

Here we present numerical calculations for neutrino-nucleon and neutrino-nucleus scattering. We restrict ourselves to the case of charge and magnetic electromagnetic form factors of a nucleon, accounting for the relation between the nucleon neutral weak and electromagnetic form factors

$$\begin{aligned} F_{1,2}^p(q^2) &= (\frac{1}{2} - 2\sin^2\theta_W)F_{Q,M}^p(q^2) - 1\frac{1}{2}F_{Q,M}^n(q^2) - \frac{1}{2}F_{1,2}^S(q^2),\\ F_{1,2}^n(q^2) &= (\frac{1}{2} - 2\sin^2\theta_W)F_{Q,M}^n(q^2) - 1\frac{1}{2}F_{Q,M}^p(q^2) - \frac{1}{2}F_{1,2}^S(q^2),\\ G_{A,P}^N(q^2) &= \frac{\tau_3}{2}G_{A,P}^{\mathrm{a}}(q^2) - \frac{1}{2}G_{A,P}^S(q^2), \end{aligned} \tag{12}$$

where $F^S_{1,2}$, $G^S_{A,P}$ are strange form factors of the nucleon. We use the parameterization that can be found in [8] (and references therein). We present the numerical results for a zero source-detector distance, with and without strange form-factor contribution, with and without neutrino charge radii (Figs. 1 and 2), with and without diagonal neutrino magnetic moments (Fig. 3).

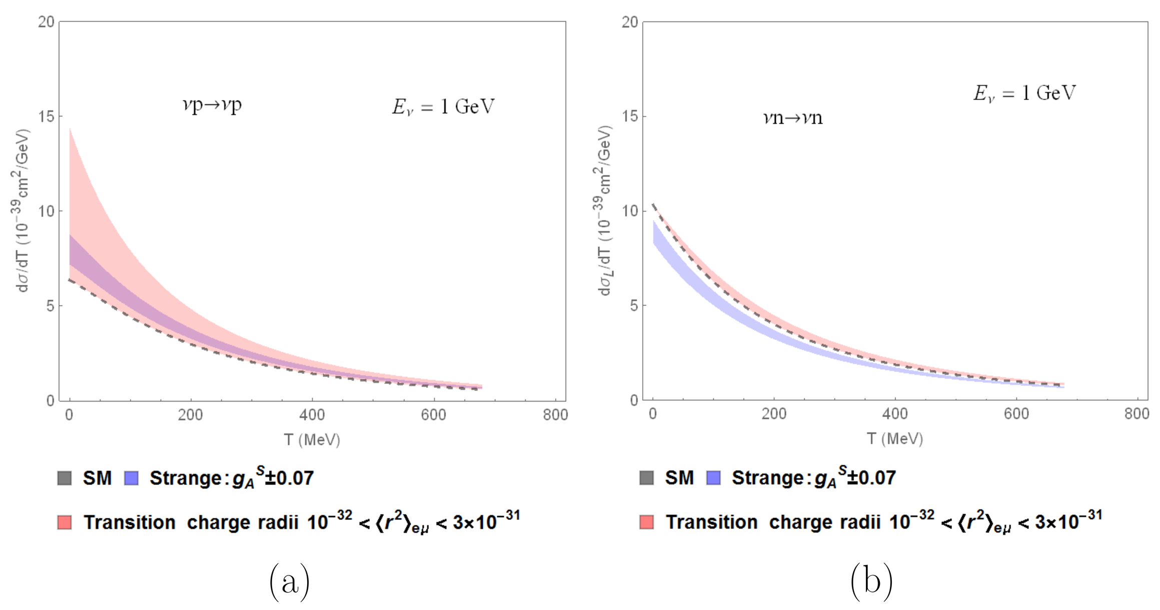

Figure 1: The differential cross sections of neutrino scattering on (a) a proton and (b) a neutron at E_ν = 1 GeV.

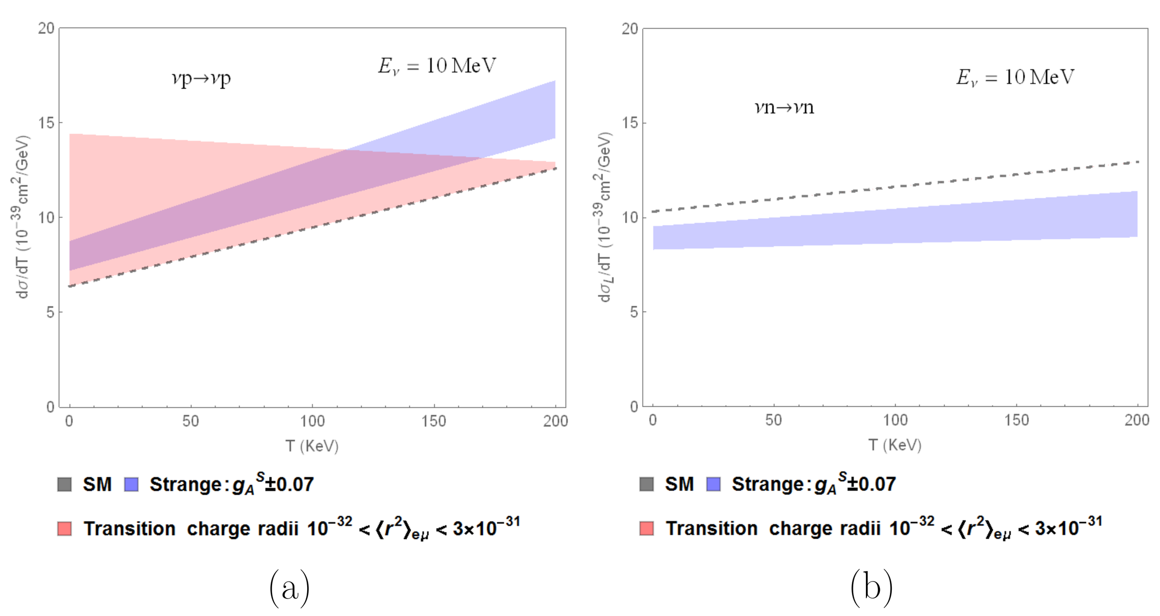

Figure 2: The differential cross sections of neutrino scattering on (a) a proton and (b) a neutron at E_ν = 10 MeV.

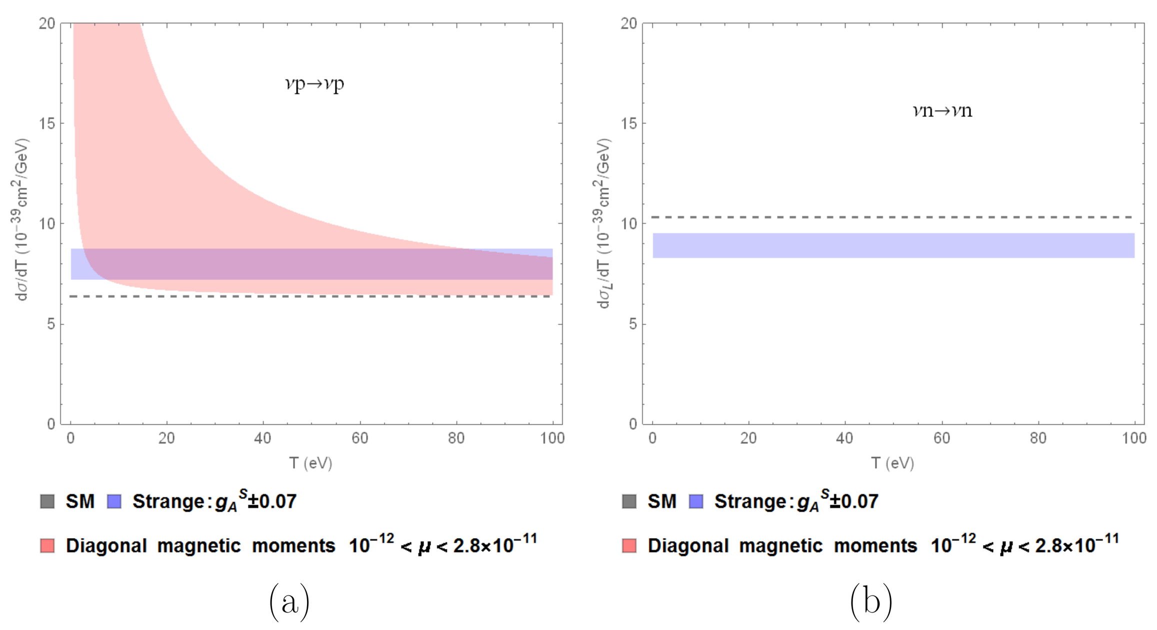

Figure 3: The differential cross sections of neutrino scattering on (a) a proton and (b) a neutron at small energy transfer values, accounting for diagonal neutrino magnetic moments.

For the neutrino-nucleus case we chose ^{40}Ar as a target [9] with parametrization of nuclear form factors that can be found in [10] (and references therein). We present our results for a zero source-detector distance, with and without transition neutrino charge radii (Fig. 4), with and without diagonal neutrino magnetic moments (Fig. 5)

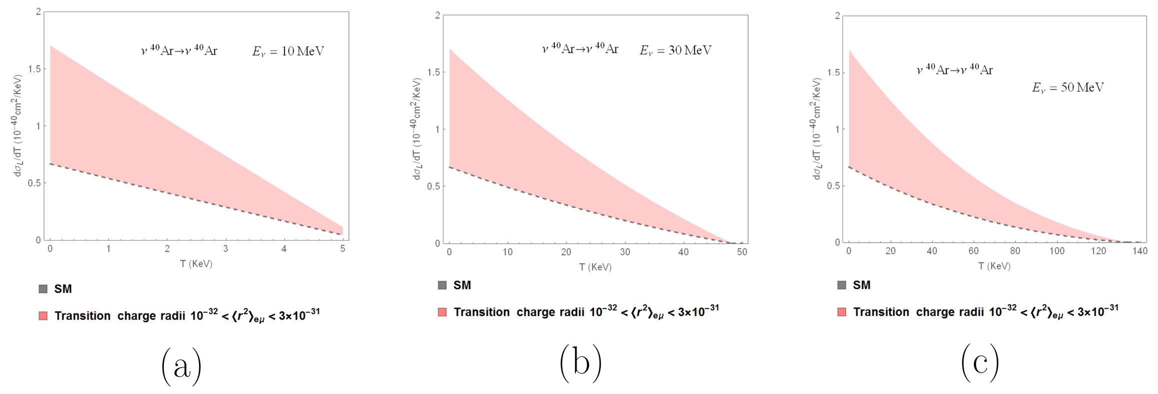

Figure 4: The differential cross sections of neutrino scattering on ^{40}Ar, accounting for transition neutrino charge radii, at E_ν values of (a) 10 MeV, (b) 30 MeV, and (c) 50 MeV.

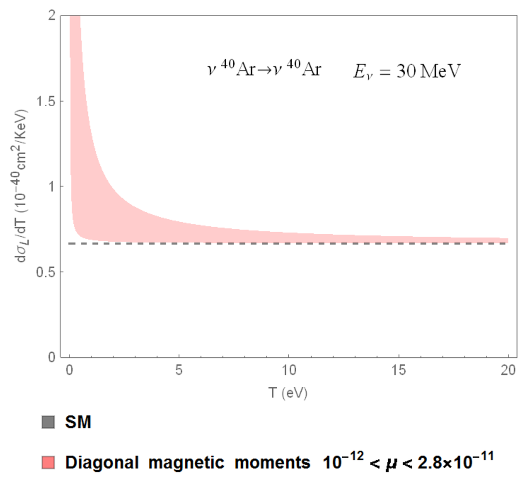

Figure 5: The differential cross section of neutrino scattering on ^{40}Ar with account for diagonal neutrino magnetic moments.

4 Summary

Elastic scattering of neutrinos by a nucleon and a nucleus has been considered theoretically, taking into account the electromagnetic interactions of massive neutrinos. Thus, the processes under consideration have two channels: through the exchange of a Z boson and a photon. In both cases, the nucleon and nuclear form factors are taken into account. In addition, neutrino oscillations on the source-detector base are taken into account.

The formulas obtained contain information about both neutrino electromagnetic form factors and nucleon and nuclear form factors. This feature allows the formulas to be used in various studies. Among them are neutrino experiments with short and long baselines, the study of neutrino interactions and oscillations in matter, registration of neutrinos from supernova explosions using elastic neutrino-proton scattering [6], the study of the anapole moment of the nucleon, the search for the electric dipole moment of the neutron, the search for the electromagnetic characteristics of neutrinos. We have performed numerical calculations for neutrino energies relevant for the MiniBooNE (Fig.1,3) and COHERENT (Fig.4,5) experiments.

The results of this work contribute to the development of a systematic approach to studying the properties of neutrinos in their elastic scattering on complex targets (nuclei, atoms, condensed matter).

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