



# Oscillations of Majorana neutrinos in supernova and CP violation

Artem Popov<sup>1</sup>, Alexander Studenikin<sup>1,2</sup>

1. Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia  
 2. National Centre for Physics and Mathematics, 607328 Satis, Nizhny Novgorod District, Russia  
 Correspondence: ar.popov@physics.msu.ru, studenik@srd.sinp.msu.ru



## 1 Introduction

Following our paper [1], we study the effects of nonzero Dirac and Majorana CP violating phases on neutrino-antineutrino oscillations engendered by magnetic fields of astrophysical environments. We show that in the presence of strong magnetic fields and dense matter, nonzero CP phases can induce new resonances in  $\nu \rightarrow \bar{\nu}$  oscillations, in particular in the channels  $\nu_e \leftrightarrow \bar{\nu}_e$ ,  $\nu_e \leftrightarrow \bar{\nu}_\mu$  and  $\nu_e \leftrightarrow \bar{\nu}_\tau$ . The discovered resonances can lead to appearance of potentially observable phenomena in neutrino oscillations accessible for observation in future experiments, such as JUNO and Hyper-Kamiokande (see [1]). Thus, we conclude that the astrophysical neutrino experiments can provide us a tool for studying leptonic CP violation, as well as for distinguishing between Dirac or Majorana nature of neutrinos.

## 2 Neutrino oscillations in magnetic fields and media

Astrophysical objects, such as supernovae and neutron stars, are characterized by extreme conditions, such as strong magnetic fields and high matter density. Magnetic field during the core collapse can reach magnitudes up to  $10^{18}$  Gauss, while the baryon number density  $n_B$  is  $10^{30}$  cm<sup>-3</sup> and higher. Neutrino interactions with such astrophysical media lead to certain peculiarities in neutrino oscillations phenomena. In this section we briefly present the approach to the calculation of the neutrino oscillations probabilities under extreme conditions described above developed in [1].

The wave functions of the massive neutrino states are described by the following system of Dirac equations

$$(i\gamma^\mu \partial_\mu - m_i - V_{ii}^{(m)} \gamma^0 \gamma_5) \nu_i(t, \vec{x}) - \sum_{k \neq i} (\mu_{ik} \Sigma \mathbf{B} + V_{ik}^{(m)} \gamma^0 \gamma_5) \nu_k(t, \vec{x}) = 0, \quad (1)$$

where  $i, k = 1, 2, 3$  enumerate the neutrino mass states,  $\mu_{ik}$  are the neutrino magnetic moments and  $V^{(m)} = U^\dagger V^{(f)} U$  is the matter potential in the mass basis, where

$$V^{(f)} = \frac{G_F}{\sqrt{2}} \text{diag}(n_n - 2n_e, n_n, n_n) \quad (2)$$

is the Wolfenstein potential and  $U$  is the PMNS matrix. Here we consider electrically neutral, nonmoving, and unpolarised matter composed of electrons, protons and neutrons. Note that the total baryon number density  $n_B$  is a sum of neutron and proton number densities:  $n_B = n_n + n_p$ . Due to electrical neutrality, in our case the proton and electron densities are equal:  $n_p = n_e$ .

Majorana condition  $\nu^c = \nu$  imposes certain constraints on the neutrino magnetic moments matrix. A truly neutral particle as Majorana neutrino cannot possess diagonal magnetic dipole form factors. However nondiagonal entries, i.e. the transition magnetic moments, are possible. The magnetic moments matrix  $\mu_{ik}$  of a Majorana neutrino is antisymmetric and Hermitian, and has only nondiagonal entries that are purely imaginary quantities:  $\mu_{ik} = i|\mu_{ik}| = -\mu_{ki}$  for  $i \neq k$ . The best terrestrial experiment upper bounds on the neutrino magnetic moments, obtained by the GEMMA reactor neutrino experiment and Borexino collaboration by measuring the solar neutrino fluxes, are on the level  $\mu_\nu < 2.8 \div 2.9 \times 10^{-11} \mu_B$ . An order of magnitude more stringent upper bound is provided by the observed properties of the globular cluster stars. For our further analyses we fix the values of the transition magnetic moments accordingly:  $|\mu_{12}| = |\mu_{13}| = |\mu_{23}| = 10^{-12} \mu_B$ . The particular features of the neutrino oscillations described below are generally appropriate for the case of an arbitrary choice of nonzero transition magnetic moments. For a thorough review of neutrino electromagnetic properties see [3] and references therein.

Eq. (1) can be rewritten in the Hamiltonian form as

$$i \frac{\partial}{\partial t} \nu(t, \vec{x}) = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{pmatrix} \nu(t, \vec{x}) = H \nu(t, \vec{x}), \quad (3)$$

where  $\nu = (\nu_1, \nu_2, \nu_3)^T$  and the Hamiltonian blocks are defined by

$$H_{ik} = \delta_{ik} \gamma_0 \boldsymbol{\gamma} \mathbf{p} + m_i \delta_{ik} \gamma_0 + \mu_{ik} \gamma_0 \Sigma \mathbf{B} + V_{ik}^{(m)} \gamma_5. \quad (4)$$

Here  $\nu_1, \nu_2$  and  $\nu_3$  are the neutrino mass states wave functions.

Below we use the eigendecomposition of the Hamiltonian from the equation (3) in the following form

$$H = \sum_n E_n |n\rangle \langle n|. \quad (5)$$

In [1] we show that the probabilities of neutrino oscillations as functions of distance  $x$  can be calculated as

$$P(\nu_\alpha^s \rightarrow \nu_\beta^{s'}) = \left| \sum_n \sum_{i,k} (U_{\beta k}^{s'})^* U_{\alpha i}^s C_{nki}^{ss'} e^{-iE_n x} \right|^2, \quad (6)$$

where  $\alpha, \beta = e, \mu, \tau$  are neutrino flavours and  $s, s' = L, R$  are neutrino helicities. Note that Majorana theory of neutrinos implies that  $\nu_\alpha^L$  and  $\nu_\alpha^R$  are neutrino and antineutrino of flavour  $\alpha$  respectively. Thus,  $U^L = U$  and  $U^R = U^*$  are the mixing matrices for neutrinos and antineutrinos.

The coefficients  $C_{nki}^{ss'}$  in Eq. (6) are defined by

$$C_{nki}^{ss'} = \langle \psi_k^s | n \rangle \langle n | \psi_i^{s'} \rangle, \quad (7)$$

where

$$|\psi_1^s\rangle = \begin{pmatrix} |s\rangle \\ 0 \\ 0 \end{pmatrix}, \quad |\psi_2^s\rangle = \begin{pmatrix} 0 \\ |s\rangle \\ 0 \end{pmatrix}, \quad |\psi_3^s\rangle = \begin{pmatrix} 0 \\ 0 \\ |s\rangle \end{pmatrix}, \quad (8)$$

and

$$|L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \quad |R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}. \quad (9)$$

The probabilities (6) can be expressed in the explicit form as

$$P(\nu_\alpha^s \rightarrow \nu_\beta^{s'}; x) = \delta_{\alpha\beta} \delta_{ss'} - 4 \sum_{n>m} \text{Re}(A_{\alpha\beta nm}^{ss'}) \sin^2 \left( \frac{\pi x}{L_{nm}^{osc}} \right) + 2 \sum_{n>m} \text{Im}(A_{\alpha\beta nm}^{ss'}) \sin \left( \frac{2\pi x}{L_{nm}^{osc}} \right), \quad (10)$$

where

$$A_{\alpha\beta nm}^{ss'} = \sum_{i,j,k,l} (U_{\beta k}^{s'})^* U_{\alpha i}^s U_{\beta l}^{s'} (U_{\alpha j}^s)^* (C_{nki}^{ss'})^* C_{mlj}^{ss'} \quad (11)$$

and the oscillation lengths are

$$L_{nm}^{osc} = 2\pi / (E_n - E_m). \quad (12)$$

For the further considerations we derive the expressions for the amplitudes of oscillations as

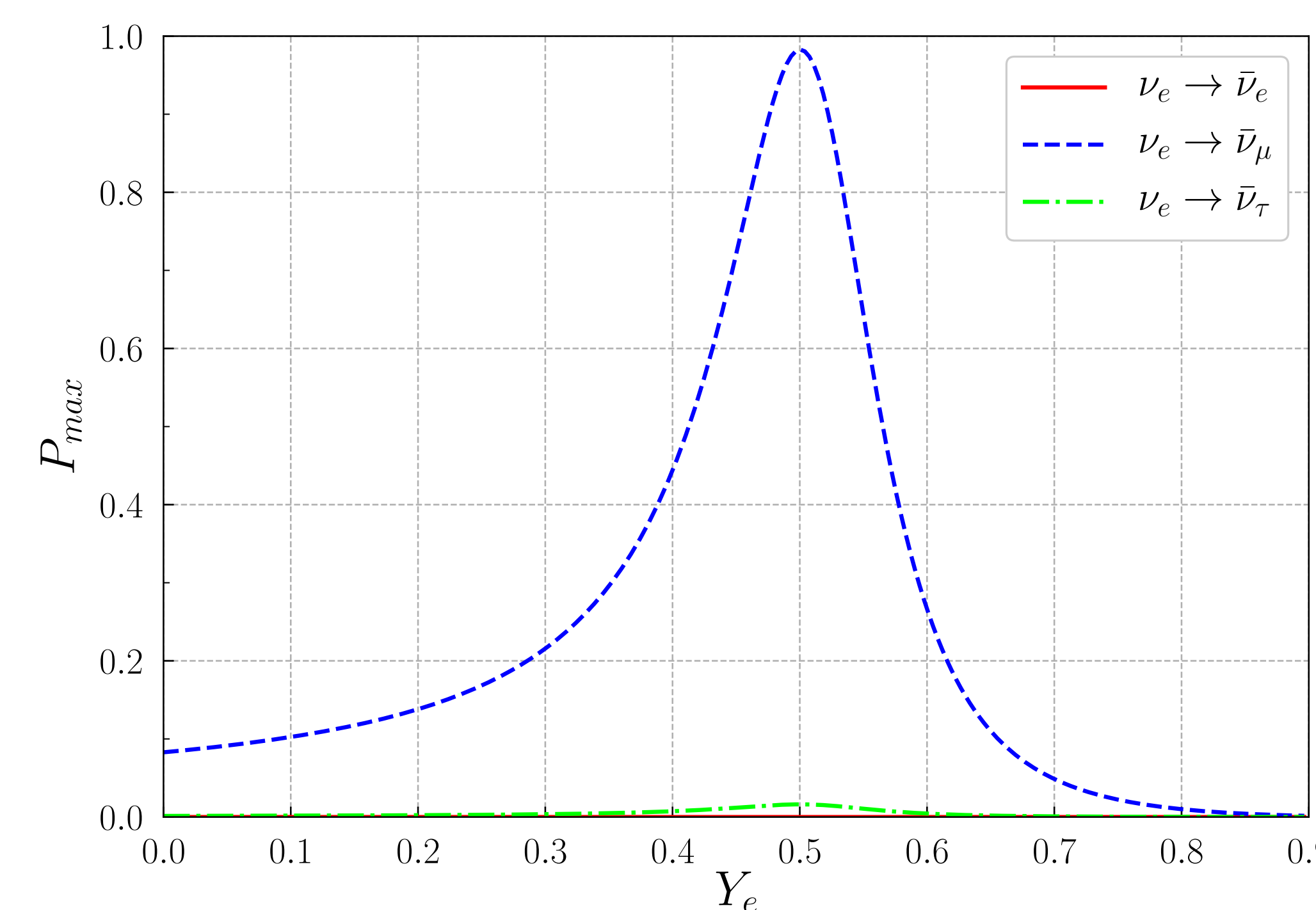
$$P(\nu_\alpha^s \rightarrow \nu_\beta^{s'})_{max} = \left( \sum_n |T_{\alpha\beta}^{ss'}| \right)^2, \quad T_{\alpha\beta}^{ss'} = \sum_{i,k} (U_{\beta k}^{s'})^* U_{\alpha i}^s C_{nki}^{ss'}. \quad (13)$$

Below we study the amplitudes of neutrino-antineutrino oscillations in astrophysical media using Eq. (13).

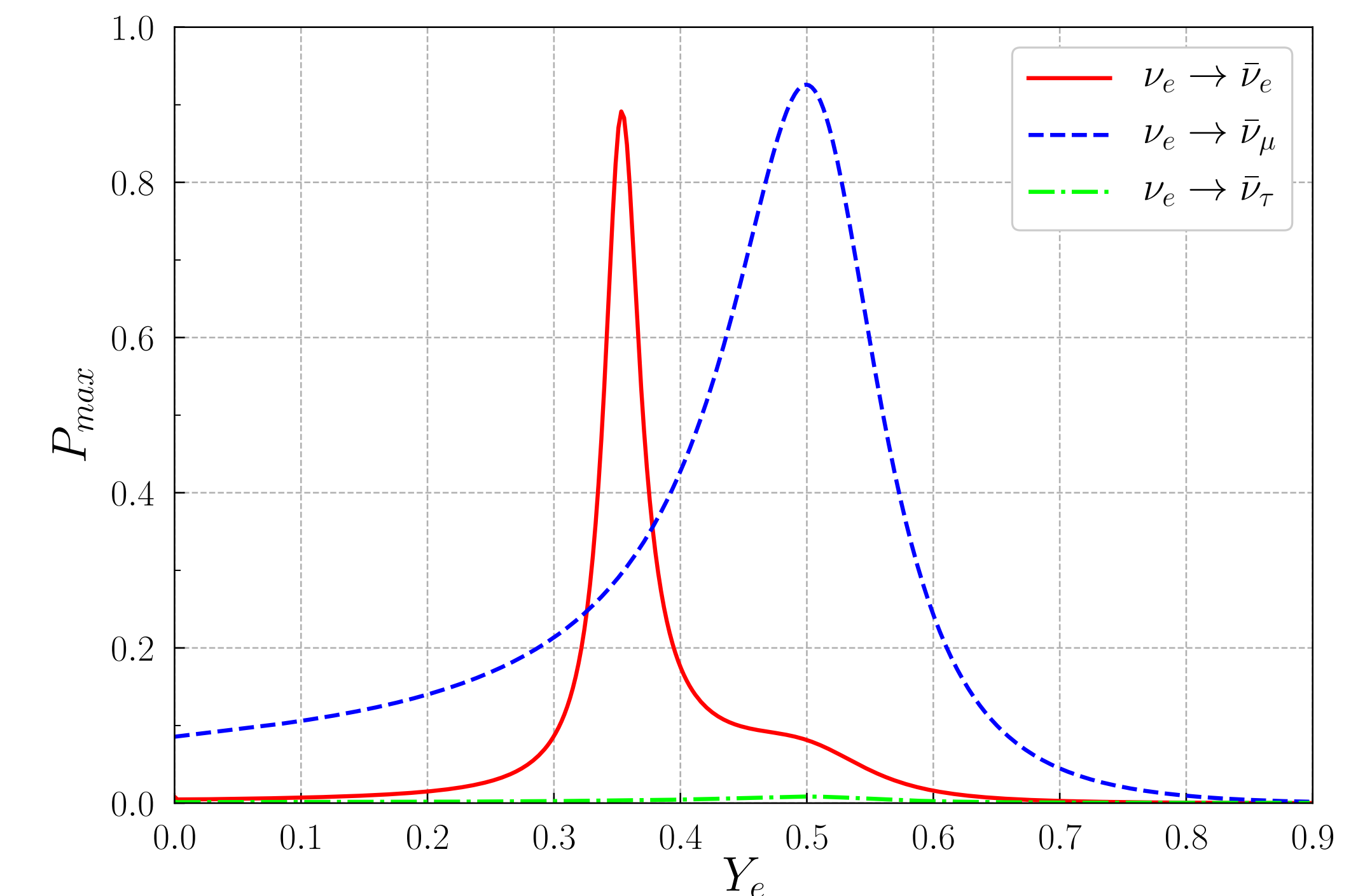
## 3 Results

In this section we present our numerical results on resonances in neutrino-antineutrino oscillations in astrophysical environments. For our analysis, we use relatively conservative magnitudes for magnetic field strength and baryon number density:  $B = 10^{13}$  Gauss and  $n_B = 10^{33}$  cm<sup>-3</sup>.

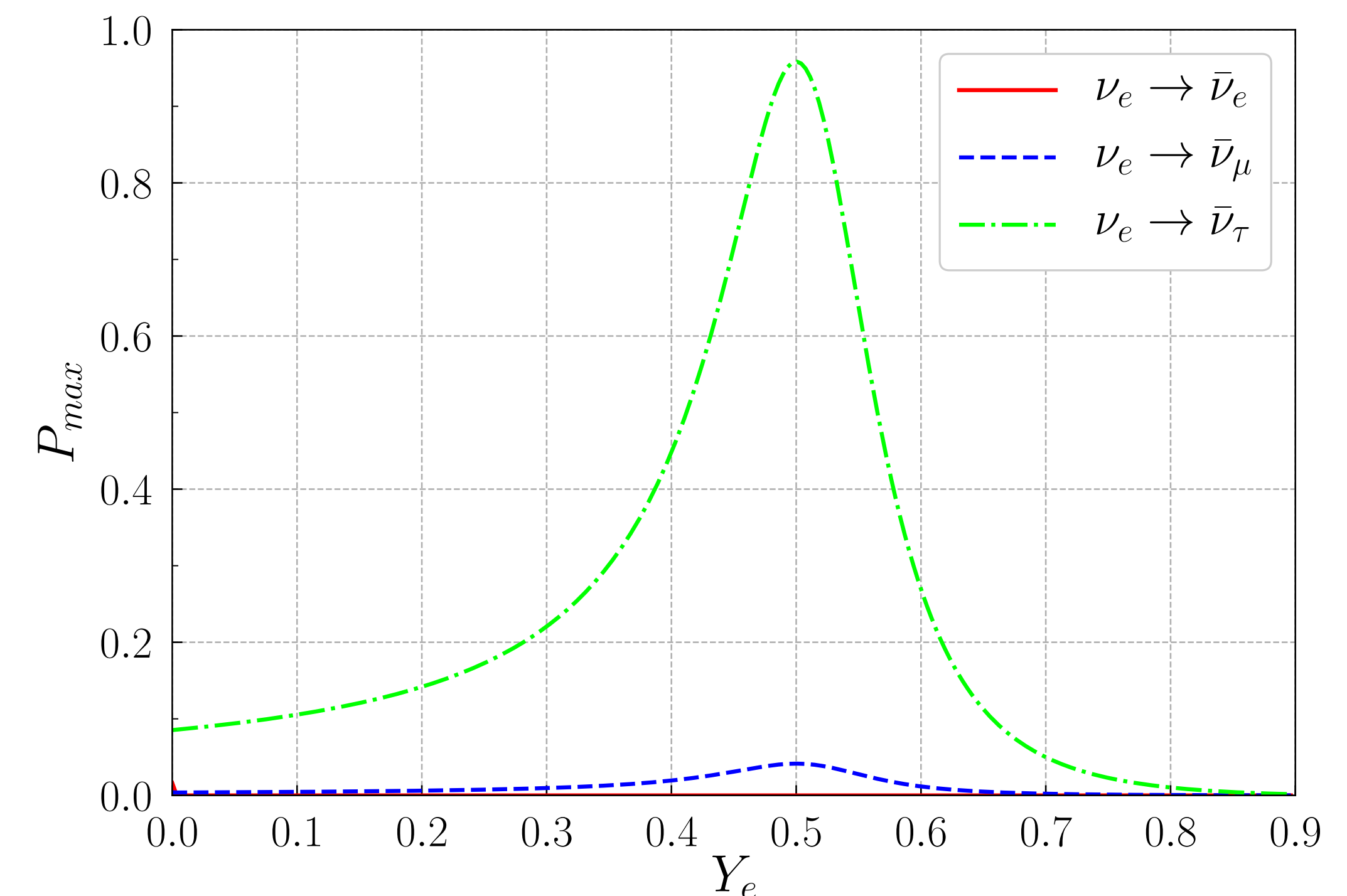
Figure 1 shows the amplitudes (13) of the neutrino-antineutrino oscillations as functions of the electron fraction  $Y_e = n_e/n_B$  for the CP violating phases given by  $\delta = 0$ ,  $\alpha_1 = 0$ ,  $\alpha_2 = 0$ . The resonant curve in Figure 1 reproduces the well-known resonant behavior of the spin-flavor conversion studied in [4,5], with the resonance in the  $\nu_e \rightarrow \bar{\nu}_\mu$  channel for  $Y_e \approx 0.5$ , described by [4,5]



The amplitudes of the neutrino-antineutrino oscillations for the case of nonzero Dirac CP violating phase are shown in Figure 2. The resonance in the channel  $\nu_e \rightarrow \bar{\nu}_\mu$  is persistent for all values of  $\delta$ . There is also a new resonance in the channel  $\nu_e \rightarrow \bar{\nu}_e$  that appears at  $Y_e \approx 0.35$ . The location of the resonance does not depend either on the magnetic field strength  $B$  or the baryon density  $n_B$  and the neutrino energy  $p$ . This resonance occurs even for values of the Dirac CP violating phase which are only slightly different from the CP conserving values, i.e.  $\delta = 0$  or  $\pi$ . Thus we can expect significant  $\nu_e \rightarrow \bar{\nu}_e$  conversions at a certain point of a supernova if neutrinos are Majorana particles, Dirac CP violating phase  $\delta$  is nonzero and the interaction with the stellar magnetic field is strong enough ( $B \sim 10^{12} \div 10^{13}$  G).



The amplitudes (13) of neutrino-antineutrino oscillations for the case  $\delta = 0$ ,  $\alpha_1$  and  $\alpha_2$  are shown in Fig. 3. A particular feature is that now the resonant peak in the  $\nu_e \leftrightarrow \bar{\nu}_e$  oscillations can appear at  $Y_e \approx 0.5$  for certain magnitudes of the Majorana CP violating phases in addition to the resonances in the  $\nu_e \leftrightarrow \bar{\nu}_e$  and  $\nu_e \leftrightarrow \bar{\nu}_\mu$  conversions.



Thus, we have shown that the new resonances in neutrino-antineutrino oscillations in a magnetic field appear at  $Y_e \approx 0.35$  and  $Y_e \approx 0.5$ , given that the Dirac or Majorana CP violating phases are nonzero. In our paper [1] we show that appearance of these resonances can lead to observable phenomena, in particular to disproportion between  $\nu_e$  and  $\bar{\nu}_e$  fluxes from supernova.

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