Abstract

- The uniqueness of this work is that it employs the Linear Seesaw mechanism in SUSY to create a model that explains neutrino oscillation data under $A'$ modular symmetry.
- The Standard Model (SM) is supplemented by three RH neutrinos ($N_R$) and three Sterile neutrinos ($S_L$), as well as one weight that eliminates unnecessary components in the superpotential and maintains the model consistent.

Introduction

- **$A'$ Modular Symmetry:** The utilisation of flavour fields is minimised with $A'$ modular symmetry, and the modular group is the group of linear fractional transformations occurring on a complex variable, $τ$, used in expressing Dedekind eta-function $η(τ)$.

\[
η(τ) = e^{\frac{2πiτ}{c+d}} \prod_{n=1}^{∞} (1 - e^{2πinτ}) \tag{1}
\]

- The dimension of $Γ'$ is $5k + 1$ hence their are six basis vectors used in expressing lowest weight Yukawa coupling $Y^{(1)}$.

\[
Y^{(i)} = \begin{pmatrix} 1 + 3 + 4 + 5 + 6 + 7 + \cdots \end{pmatrix}
\]

- Linear Seesaw: The light neutrino mass matrix under the linear seesaw in the flavor basis of ($ν_L$, $N_R$, $S_L$) is expressed as

\[
M = \begin{pmatrix} 0 & M_D & M_{D_4} \\ M_{D_4}^T & M_{L_4} & 0 \\ M_{L_4}^T & 0 & M_{N_4} \end{pmatrix}
\]

The resulting mass formula

\[
m_{ν} = M_{ν} M^{-1}_{N_4} M_{L_4}^{T} \text{ transpose} \tag{3}
\]

Model Framework

- The complete superpotential is given by

\[
W = A_M \left[ \left( L_{L_{i}} L_{R} \right) Y^{(1)}_{D} \right] H_d + \mu H_u H_d + G \left[ \left( L_{L} N_{R} \right) s Y^{(2)}_{S} \right] H_u + G_{D} \left[ \left( L_{L} S_{L} \right) H_u \sum_{i=1}^{2} Y^{(6)}_{i} \right] \zeta'. \tag{4}
\]

- **Fields**

<table>
<thead>
<tr>
<th>SU(2)_{L}</th>
<th>T_{R}</th>
<th>B_{R}</th>
<th>Y_{R}</th>
<th>N_{R}</th>
<th>S_{L}</th>
<th>H_{d}</th>
<th>H_{u}</th>
<th>Z_{C}</th>
<th>Z_{C}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>


Table 1: The particle spectrum and their charges under the symmetry groups SU(2)_L x U(1)_Y x U(1)_{B-L} x A'_4 where k_1 represents the modular weight.

Mass matrices expression

\[
M_D = \frac{v_\mu G_\mu}{\sqrt{30}} \begin{pmatrix} ( \sqrt{3} v^{(2)}_{L} )_{1} & ( v^{(2)}_{L} )_{2} & ( v^{(2)}_{L} )_{3} \\ ( v^{(2)}_{R} )_{1} & -\sqrt{3} ( v^{(2)}_{R} )_{2} & ( v^{(2)}_{R} )_{3} \\ ( v^{(2)}_{S} )_{1} & -2 ( v^{(2)}_{S} )_{2} & - ( v^{(2)}_{S} )_{3} \end{pmatrix}, M_{R,S} = \frac{v_\mu G_\mu}{\sqrt{60}} \begin{pmatrix} ( \sqrt{2} v^{(6)}_{L} )_{1} & -\sqrt{3} ( v^{(6)}_{L} )_{2} & - ( v^{(6)}_{L} )_{3} \\ ( v^{(6)}_{R} )_{1} & ( v^{(6)}_{R} )_{2} & ( v^{(6)}_{R} )_{3} \\ ( v^{(6)}_{S} )_{1} & ( v^{(6)}_{S} )_{2} & ( v^{(6)}_{S} )_{3} \end{pmatrix} \tag{5}
\]

Leptogenesis

- To account for leptogenesis, a higher dimensional mass term for the Majorana fermion ($N_R$) is introduced as in eq.(6), resulting in a tiny mass splitting between the heavy fermions, where $G_{R}$ is the coupling.

\[
W_{MR} = -G_{R} \left[ \sum_{i=1}^{2} \left( Y^{(4)}_{L_{i}} N_{R} N_{R} \right) \right] \frac{1}{\lambda^2} \tag{6}
\]

- **Boltzmann equations**

\[
\frac{dY_{N}}{dz} = -\frac{z}{sH(M_{N})} \left[ \left( Y_{N} \right)_{Y_{N}} - 1 \right] \gamma_D + \left( Y_{N} \right)_{Y_{N}}^2 - 1 \gamma_S \tag{7}
\]

Here, $s$ denotes the entropy density, $z = M_{N}^2/T$ and the equilibrium number densities are given by

\[
Y_{N}^{eq} = \frac{135 \sqrt{5}}{16 \pi^4 g_{*}} z^2 K_{2}(z), \quad Y_{T}^{eq} = \frac{135 \sqrt{5}}{8 \pi^4 g_{*}} \tag{8}
\]

Results

Figure 1: Extreme left plot represents $\sin^2 \theta_{12}$, ($\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$) (middle) versus $\Sigma m_{i}[\text{eV}]$, right middle and extreme right plot shows evolution of $Y_{B-L}$ (dashed) as a function of $z = M_{N}^2/T$ for one flavor approximation and favoured case respectively.

Conclusion

- We investigated the linear seesaw mechanism under modular symmetry and the local $U(1)_{B-L}$, which aids in the prohibition of needful terms and the constraint structures of crucial Yukawa interactions.
- In addition, our model is good enough to handle Leptogenesis.

References


Acknowledgements

- I want to acknowledge DST-INSPIRE for its financial support.
- I want to acknowledge IOE-UOH and SERB, INDIA for financial support.
- I want to acknowledge ICHEP-2022 for its financial support.