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Neutrino propagation in moving and polarized matter

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Introduction

Analytical description of neutrino behavior in external matter and fields and specifically of its spin dynamics gives not only the understanding of the basics of neutrino spin oscillations in astrophysical media but also enables to discover and describe another new phenomena. Examples include effects of neutrino Spin Light [1], and a neutrino beam self-polarization in matter [2]. All these effects arise due to helicity dependence of the neutrino energy in non-moving matter. In this study we go further in developing the neutrino theory in matter and consider its motion. In this case the helicity is no longer the exact quantum number and for consistent treatment one has to find the spin integral of motion.

Objectives:

- ability to classify neutrino states according to full set of observables
- neutrino dispersion relation with spin quantum number that has clear physical meaning
- new features of neutrino oscillations in astrophysical media

Need in a conserved spin quantity

Neutrino dynamics

Modified Dirac equation in matter [1]

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2}\gamma_\mu(1+\gamma^5) f^\mu - m \right\} \Psi(x) = 0$$

Within the minimally extended Standard Model in **moving homogeneous unpolarized medium consisting of electrons** with density n_0

$$\frac{1}{2} f^\mu = \tilde{n}_0 v^\mu, \quad \tilde{n}_0 = \frac{1}{2\sqrt{2}} G_F (1 + 4\sin^2\theta_W) n_0,$$

Neutrino Hamiltonian in moving matter (the structure also suits the case of polarized matter)

$$\mathrm{H} = (\boldsymbol{\alpha}\mathbf{p}) - \tilde{n}(\boldsymbol{\alpha}\mathbf{v}) + \tilde{n}(\boldsymbol{\Sigma}\mathbf{v}) + \gamma^5\tilde{n} + \tilde{n} + \gamma^0 m.$$

$$\Sigma_i = \gamma^0\gamma^5\gamma^i, \tilde{n} = \gamma\tilde{n}_0$$

$$\alpha_i = \gamma^0\gamma^i$$

Polarization operators

Helicity operator $\boldsymbol{\Sigma}\mathbf{p}/\mathrm{p}$

$$[\boldsymbol{\Sigma}\mathbf{p}/\mathrm{p}, \mathrm{H}] \neq 0$$

4-vector spin polarization operator $\mathrm{T}^\mu = \gamma^5\gamma^\mu - \gamma^5 p^\mu/m$

$$[\mathrm{T}^\mu, \mathrm{H}] \neq 0, \quad [\mathrm{T}^\mu v_\mu, \mathrm{H}] \neq 0$$

When using the substitution $p^\mu \to \tilde{p}^\mu \equiv p^\mu - \tilde{n}_0 v^\mu$:

$$\mathrm{T}^\mu \to \tilde{\mathrm{T}}^\mu = \gamma^5\gamma^\mu - \gamma^5\tilde{p}^\mu/m$$

$$[\tilde{\mathrm{T}}^\mu v_\mu, \mathrm{H}] = 0$$

We define the spin operator as

$$S = m(\tilde{\mathrm{T}}^\mu v_\mu)$$

Eigenvalues and dispersion relation

Let us introduce notations:

$$P = (\tilde{p}^\mu v_\mu)$$

$$\Lambda = \sqrt{P^2 - m^2}$$

Eigenvalues in the problem:

$$S\Psi_{E,s} = s\Lambda\Psi_{E,s}, s = \pm 1$$

$$\mathrm{H}\Psi_{E,s} = E\Psi_{E,s}$$

Dispersion relation ($p^2 = E^2 - \mathbf{p}^2$):

$$p^2 - m^2 = 2\tilde{n}_0(P - s\Lambda)$$

In the expanded form (known earlier [3]):

$$(p^2 - m^2)^2 - 4\tilde{n}_0(pv)(p^2 - m^2) + 4\tilde{n}_0^2 p^2 = 0$$

Applications

In non-moving matter: $E = \varepsilon\sqrt{(\mathrm{p} - s\tilde{n}_0)^2 + m^2} + \tilde{n}_0, \ \varepsilon = \pm 1$

Longitudinal matter motion $(\mathbf{v} \| \mathbf{p})$: $E = \varepsilon\sqrt{(\mathrm{p} \mp \mathrm{v}\tilde{n} - s\tilde{n})^2 + m^2} \pm s\mathrm{v}\tilde{n} + \tilde{n}$

Transversal matter motion, $(\mathbf{pv}) = 0$:

$$\begin{cases} E_{s=+1} = \sqrt{(\mathrm{p} - \tilde{n})^2 + m^2} + \tilde{n}, \\ E_{s=-1} = \sqrt{\mathrm{p}^2 + 4\tilde{n}^2\mathrm{v}^2 + m^2} + 2\tilde{n}. \end{cases}$$

Standard MSW potential: $A = \sqrt{2}G_F n$

MSW potential in relativistic transversal matter current: $A' = \sqrt{2}G_F n + 8\dfrac{G_F^2 n^2 \mathrm{v}^2}{\mathrm{p}} \sin^2\theta_W$

Shift of the resonance condition:

$$\frac{\Delta}{2\mathrm{p}}\cos 2\theta = A + 8\frac{G_F^2 n^2 \mathrm{v}^2}{\mathrm{p}}\sin^2\theta_W$$

References

1. A. Studenikin, A. Ternov, Phys. Lett. B 608, 107 (2005)
2. A. Lobanov, A. Studenikin, Phys. Lett. B 601, 171 (2004)
3. I. Pivovarov, A. Studenikin, PoS HEP 2005, 191 (2006)

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