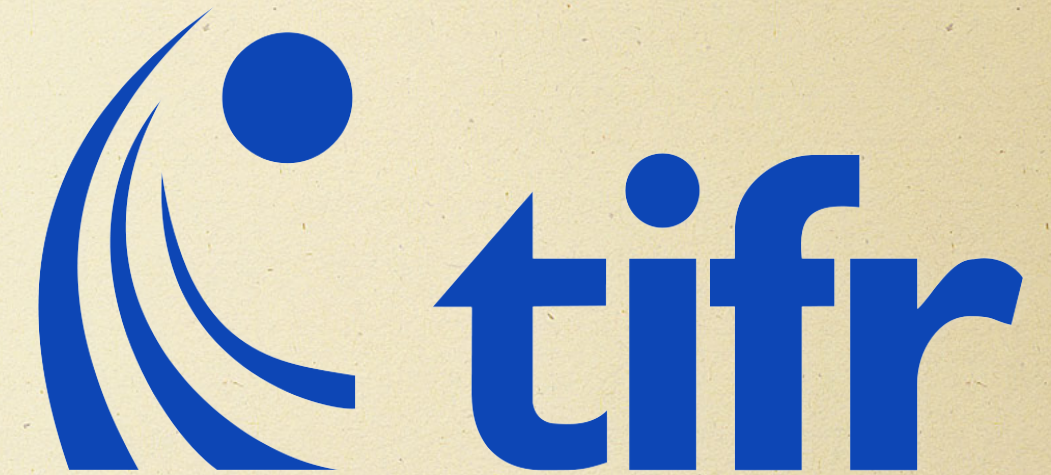


Analytic treatment of neutrino oscillation and decay in matter

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and [arXiv:2204.05803 \[hep-ph\]](#)

In collaboration with Amol Dighe, Kaustav Chakraborty, Srubabati Goswami, SM Lakshmi

Objective

If neutrinos decay, the effective non-Hermitian Hamiltonian needs to be treated carefully due to subtle issues regarding its mass and decay components.

We derive **compact analytic expressions** for 2-flavor 3-flavor neutrino probabilities with:

- Invisible **decay** + **Oscillation** + **Explicit matter effects included.**

Useful for:

1. Long-baseline neutrino experiments
2. Atmospheric neutrino experiments
3. Reactor anti-neutrino experiments

The problem

- The inclusion of decay makes the effective Hamiltonian non-Hermitian

$$\mathcal{H} = H - i\Gamma/2 \qquad \Gamma_{ij} = 2\pi \sum_k \langle \nu_i | \mathcal{H}' | \phi_k \rangle \langle \phi_k | \mathcal{H}' | \nu_j \rangle \delta(E_k - E_\nu)$$

- The decay and the mass eigenstates need not be the same \Rightarrow *Mismatch*

$$[H, \Gamma] \neq 0$$

- Even if there's no mismatch in vacuum, due to matter effects, the components will invariably become non-commuting.

Inevitability of the off-diagonal elements

- In the 2-flavor approximation:

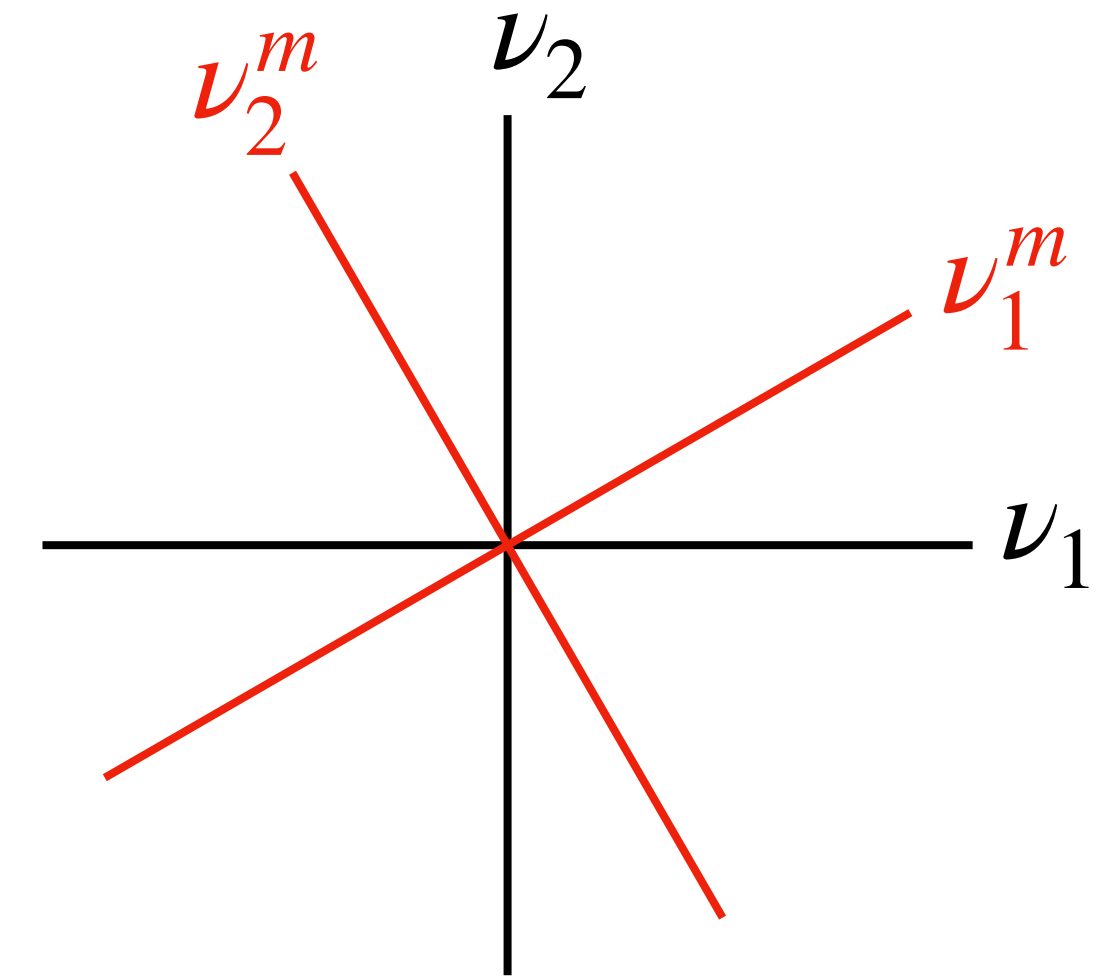
$$\mathcal{H}_m = \begin{pmatrix} a_1 - i b_1 & -\frac{1}{2} i \gamma e^{i\chi} \\ -\frac{1}{2} i \gamma e^{-i\chi} & a_2 - i b_2 \end{pmatrix}$$

- Even if only ν_2 in vacuum decays, with $\alpha_2 = m_2/\tau_2$, in matter, we get:

$$a_{1,2} = \frac{\tilde{m}_{1,2}^2}{2E} \quad , \quad b_{1,2} = \frac{\alpha_2}{4E} [1 \mp \cos[2(\theta - \theta_m)] \quad ,$$

$$\chi = 0 \quad , \quad \gamma = \frac{\alpha_2}{2E} \sin[2(\theta - \theta_m)] \quad .$$

- The **off-diagonal term** γ is **generated**, even though it was absent in vacuum.
- Inevitable “mismatch”** in matter.
- We develop techniques using Zassenhaus (inverse BCH) expansion and Cayley-Hamilton theorem.



Decay of ν_3 only

- Strong constraints from solar neutrino data on ν_1 and ν_2 decay.
- Therefore, the special case where only **ν_3 mass eigenstate in vacuum decays**:

$$\mathcal{H}_f^{(\gamma_3)} = \frac{1}{2E_\nu} U \left[\Delta m_{31}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} - i \Delta m_{31}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \gamma_3 \end{pmatrix} \right] U^\dagger + \begin{pmatrix} V_{cc} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Here, γ_i is defined such that $\gamma_i \Delta m_{31}^2 = m_i / \tau_i$.

$$\alpha \equiv \Delta m_{21}^2 / \Delta m_{31}^2$$

- Current long-baseline constraints[†]: $\tau_3 / m_3 > 1.5 \times 10^{-12}$ s/eV (3σ)

[†]arXiv:1805.01848

The general decay matrix Γ

- The solar neutrino constraint on decay is for neutrinos propagating in vacuum.
- For matter induced decay, the solar neutrino constraint may be relaxed.
- For the **general decay matrix Γ** we have:

$$\mathcal{H}_f^{(\Gamma)} = U \left[\frac{\Delta m_{31}^2}{2E_\nu} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} - \frac{i}{2} \Gamma \right] U^\dagger + \begin{pmatrix} V_{cc} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\Gamma = \frac{\Delta m_{31}^2}{E_\nu} \begin{pmatrix} \gamma_1 & \frac{1}{2}\gamma_{12}e^{i\chi_{12}} & \frac{1}{2}\gamma_{13}e^{i\chi_{13}} \\ \frac{1}{2}\gamma_{12}e^{-i\chi_{12}} & \gamma_2 & \frac{1}{2}\gamma_{23}e^{i\chi_{23}} \\ \frac{1}{2}\gamma_{13}e^{-i\chi_{13}} & \frac{1}{2}\gamma_{23}e^{-i\chi_{23}} & \gamma_3 \end{pmatrix}.$$

Formalism and scales



$$\alpha \approx 0.03 \simeq O(\lambda^2) , \quad s_{13} \equiv \sin \theta_{13} \simeq 0.14 \simeq O(\lambda) .$$

- Decay has not been observed yet over the timescale of oscillations.
- Decay must be subleading to oscillation, i.e. decay length must be larger.
- Therefore, $\gamma_3 < O(1)$ and $\gamma_1, \gamma_2 < O(\alpha)$.

$$\gamma_3 \sim O(\lambda) , \quad \gamma_1, \gamma_2 \sim O(\lambda^3) .$$

- Decay matrix should be positive definite.

$$\gamma_{12} \sim O(\lambda^3) , \quad \gamma_{13}, \gamma_{23} \sim O(\lambda^2) .$$

$$\lambda \equiv 0.2$$

Book-keeping
parameter

Probabilities expanded in s_{13} , α and γ_3

$$\begin{aligned}
 P_{\mu\mu}^{(0)} &= 1 - \sin^2 2\theta_{23} \sin^2 \Delta - \frac{2}{A-1} s_{13}^2 \sin^2 2\theta_{23} \\
 &\times \left(\sin \Delta \cos A\Delta \frac{\sin[(A-1)\Delta]}{A-1} - \frac{A}{2} \Delta \sin 2\Delta \right) \\
 &- 4s_{13}^2 s_{23}^2 \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} \\
 &+ \alpha c_{12}^2 \sin^2 2\theta_{23} \Delta \sin 2\Delta + O(\lambda^3), \\
 P_{\mu\mu}^{(\gamma_3)} &= -\gamma_3 \Delta (\sin^2 2\theta_{23} \cos 2\Delta + 4s_{23}^4) \\
 &+ \gamma_3^2 \Delta^2 (\sin^2 2\theta_{23} \cos 2\Delta + 8s_{23}^4) + O(\lambda^3), \\
 P_{\mu\mu}^{(\Gamma)} &= \sin 2\theta_{23} (\gamma_{13} s_{12} \cos \chi_{13} - \gamma_{23} c_{12} \cos \chi_{23}) \\
 &\times \sin 2\Delta + O(\lambda^3).
 \end{aligned}$$

with $P_{\mu e} = P_{e\mu}(\delta_{\text{CP}} \rightarrow -\delta_{\text{CP}}, \chi_{ij} \rightarrow -\chi_{ij})$.

$$A = \frac{2E_\nu V_{cc}}{\Delta m_{31}^2}, \quad \Delta = \frac{\Delta m_{31}^2 L}{4E_\nu}$$

$$P_{\alpha\beta} = P_{\alpha\beta}^{(0)} + P_{\alpha\beta}^{(\gamma_3)} + P_{\alpha\beta}^{(\Gamma)}$$

$$\begin{aligned}
 P_{e\mu}^{(0)} &= 4s_{13}^2 s_{23}^2 \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} \\
 &+ 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta - \delta_{\text{CP}}) \\
 &\times \frac{\sin[(A-1)\Delta]}{A-1} \frac{\sin A\Delta}{A} + O(\lambda^4), \\
 P_{e\mu}^{(\gamma_3)} &= -8\gamma_3 s_{13}^2 s_{23}^2 \Delta \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} + O(\lambda^4), \\
 P_{e\mu}^{(\Gamma)} &= -4s_{13} s_{23}^2 (\gamma_{23} s_{12} \sin[\delta_{\text{CP}} + \chi_{23}] \\
 &+ \gamma_{13} c_{12} \sin[\delta_{\text{CP}} + \chi_{13}]) \frac{\sin^2[(A-1)\Delta]}{(A-1)^2} \\
 &+ O(\lambda^4),
 \end{aligned}$$

Key observations

- **The leading effect of ν_3 decay** at the muon neutrino **survival** channel.
- No matter effects in the muon neutrino survival channel.
- In the **conversion** channel, the effect of **off-diagonal decay terms are as important** as effects of γ_3 .
- Matter dependence in the conversion channel decay terms.

Probabilities expanded in s_{13} and α , exact in γ_3

$$P_{\mu\mu} = \left| c_{23}^2 + s_{23}^2 e^{-2i(1-i\gamma_3)\Delta} - 2i\alpha c_{12}^2 c_{23}^2 \Delta + s_{13}^2 s_{23}^2 \left(e^{-2iA\Delta} \frac{(1-i\gamma_3)^2}{[A-(1-i\gamma_3)]^2} + e^{-2i(1-i\gamma_3)\Delta} \left[2iA\Delta [A-(1-i\gamma_3)] - (1-i\gamma_3) \right] \frac{1-i\gamma_3}{[A-(1-i\gamma_3)]^2} \right) \right|^2 + O(\lambda^3) .$$

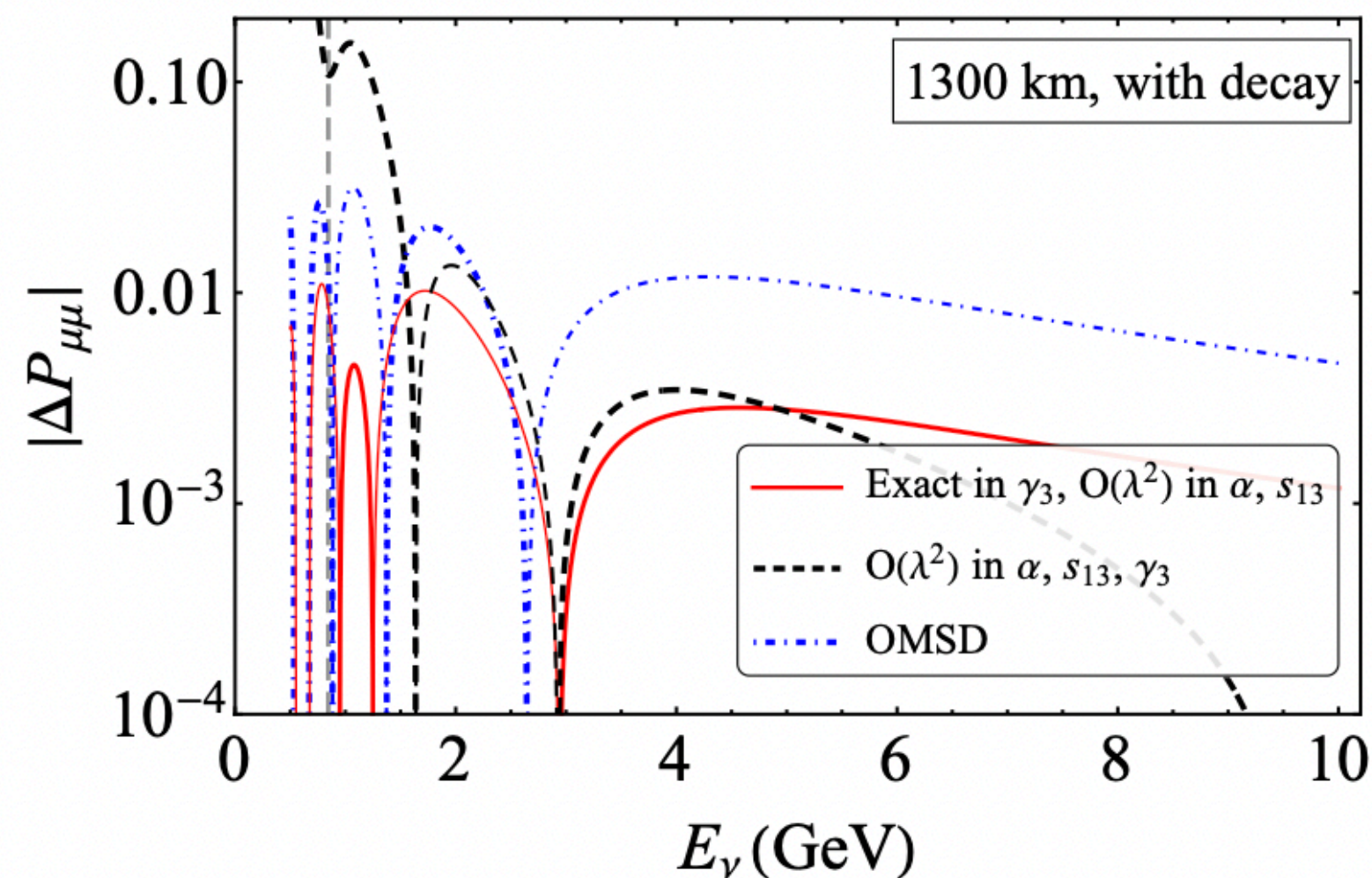
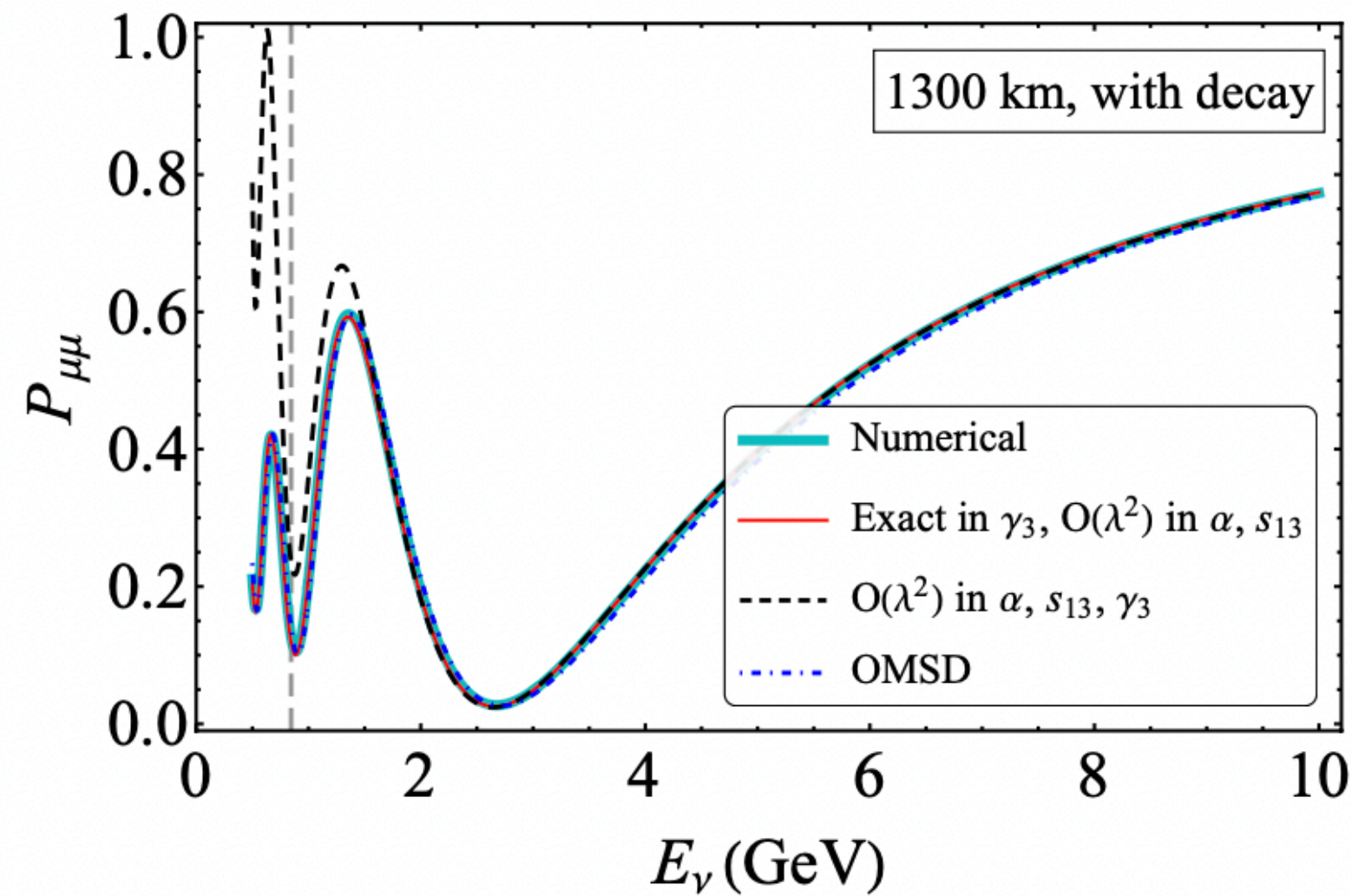
$$\begin{aligned} P_{\mu\mu}^{\text{leading}} &= c_{23}^4 + s_{23}^4 e^{-4\gamma_3\Delta} + 2s_{23}^2 c_{23}^2 \cos(2\Delta) e^{-2\gamma_3\Delta} \\ &= 1 - \sin^2 2\theta_{23} \sin^2 \Delta - s_{23}^4 (1 - e^{-4\gamma_3\Delta}) - 2s_{23}^2 c_{23}^2 \cos(2\Delta) (1 - e^{-2\gamma_3\Delta}) . \end{aligned}$$

- Exact dependence on γ_3 .
- The region of validity increases to $\alpha\Delta \lesssim 1$ from $\gamma_3\Delta \lesssim 1$.
- Valid at lower energies.

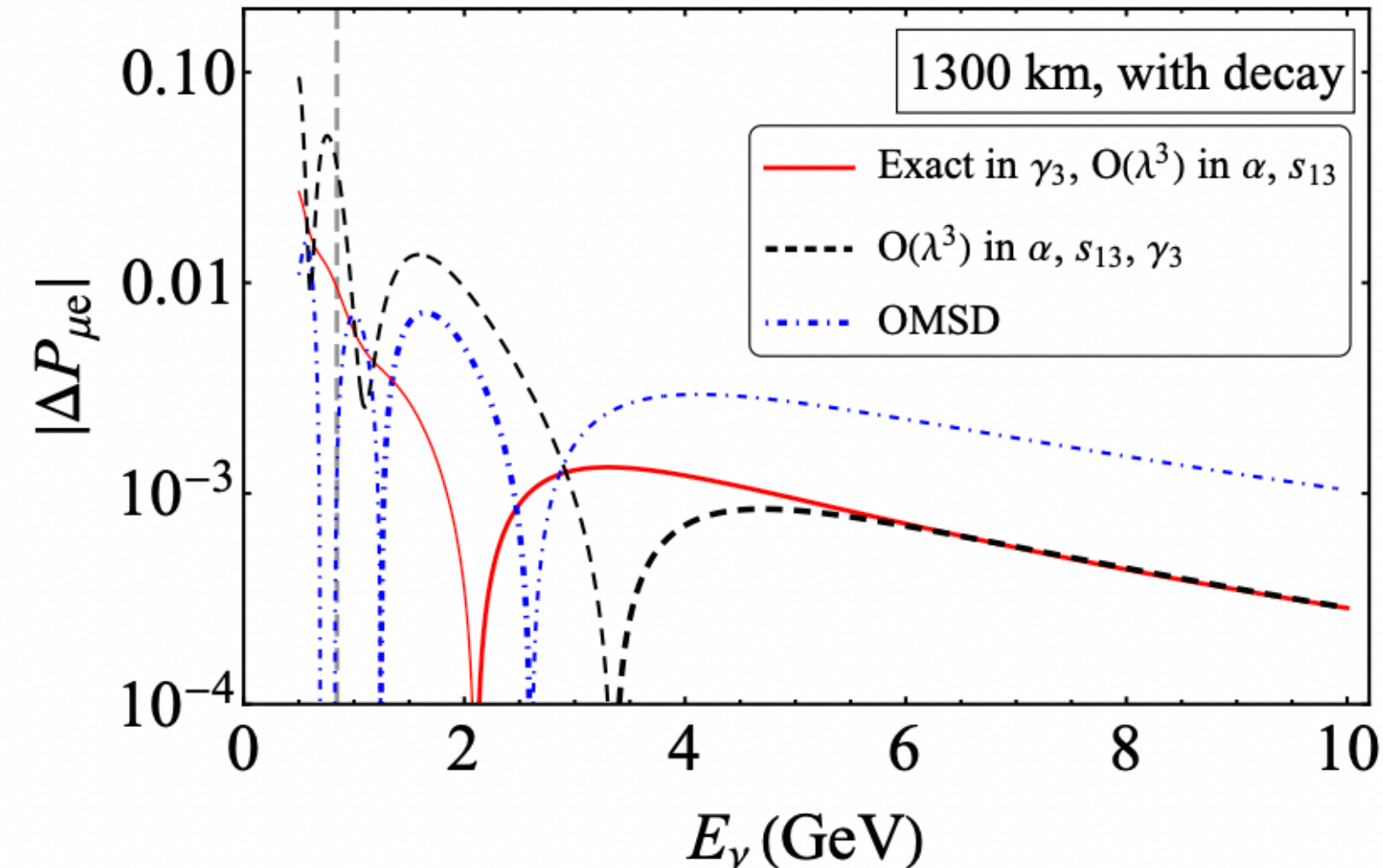
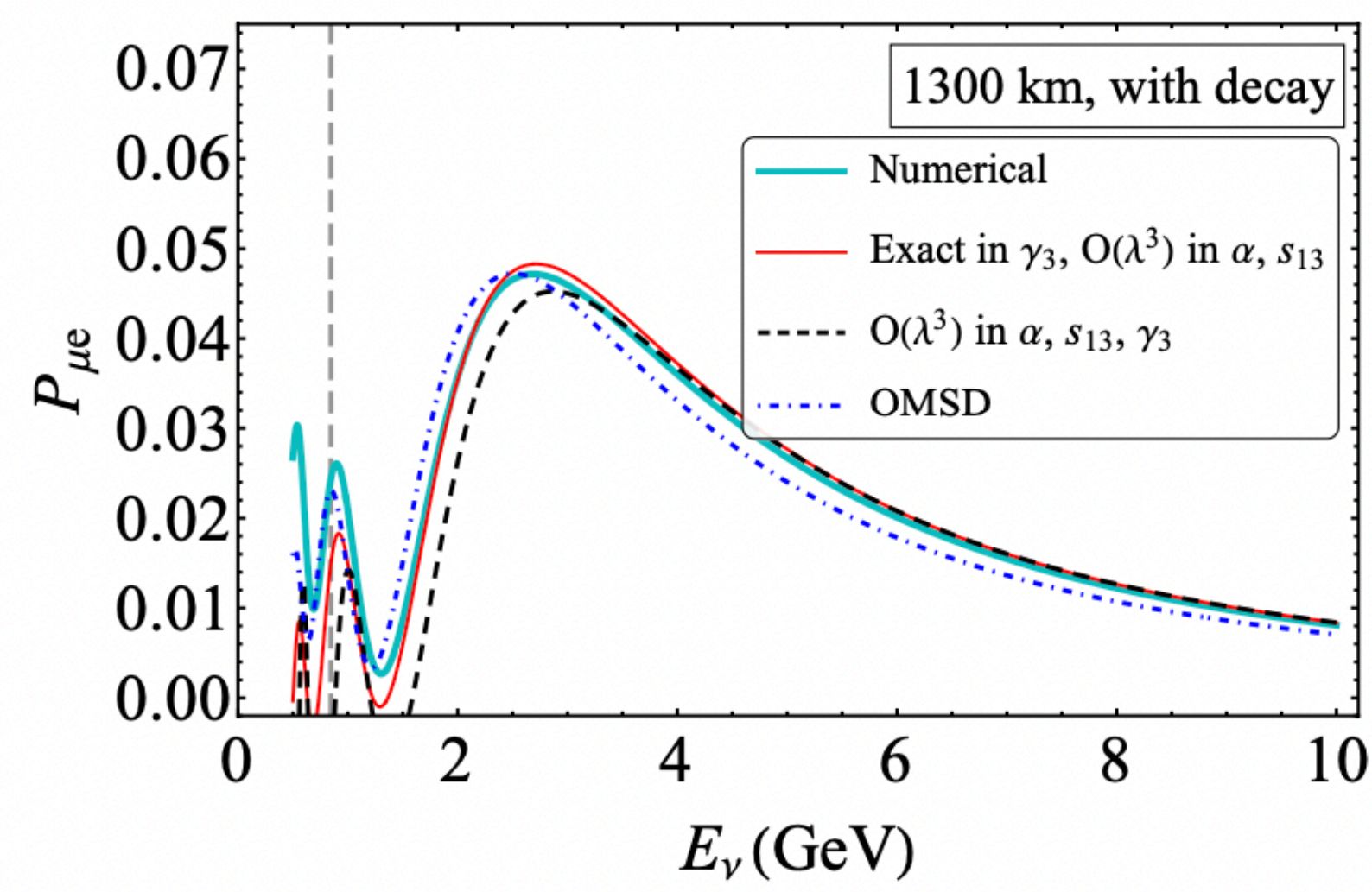
$$\begin{aligned} P_{e\mu} &= s_{13}^2 s_{23}^2 \left(1 + e^{-4\gamma_3\Delta} - 2e^{-2\gamma_3\Delta} \cos[2(A-1)\Delta] \right) \frac{\gamma_3^2 + 1}{(A-1)^2 + \gamma_3^2} \\ &\quad + \alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \frac{\sin A\Delta}{A} \\ &\quad \times \left[\left(\sin[(A-2)\Delta + \delta_{\text{CP}}] e^{-2\gamma_3\Delta} + \sin[A\Delta - \delta_{\text{CP}}] \right) \frac{(A-1) - \gamma_3^2}{(A-1)^2 + \gamma_3^2} \right. \\ &\quad \left. + \gamma_3 \left(\cos[A\Delta - \delta_{\text{CP}}] - \cos[(A-2)\Delta + \delta_{\text{CP}}] e^{-2\gamma_3\Delta} \right) \frac{A}{(A-1)^2 + \gamma_3^2} \right] + O(\lambda^4) . \end{aligned}$$

Analytic vs Numerical Comparison

$\nu_\mu \rightarrow \nu_\mu$



$\nu_\mu \rightarrow \nu_e$



$$\Delta P_{\alpha\beta} = P_{\alpha\beta}(\text{analytic}) - P_{\alpha\beta}(\text{numerical}) .$$

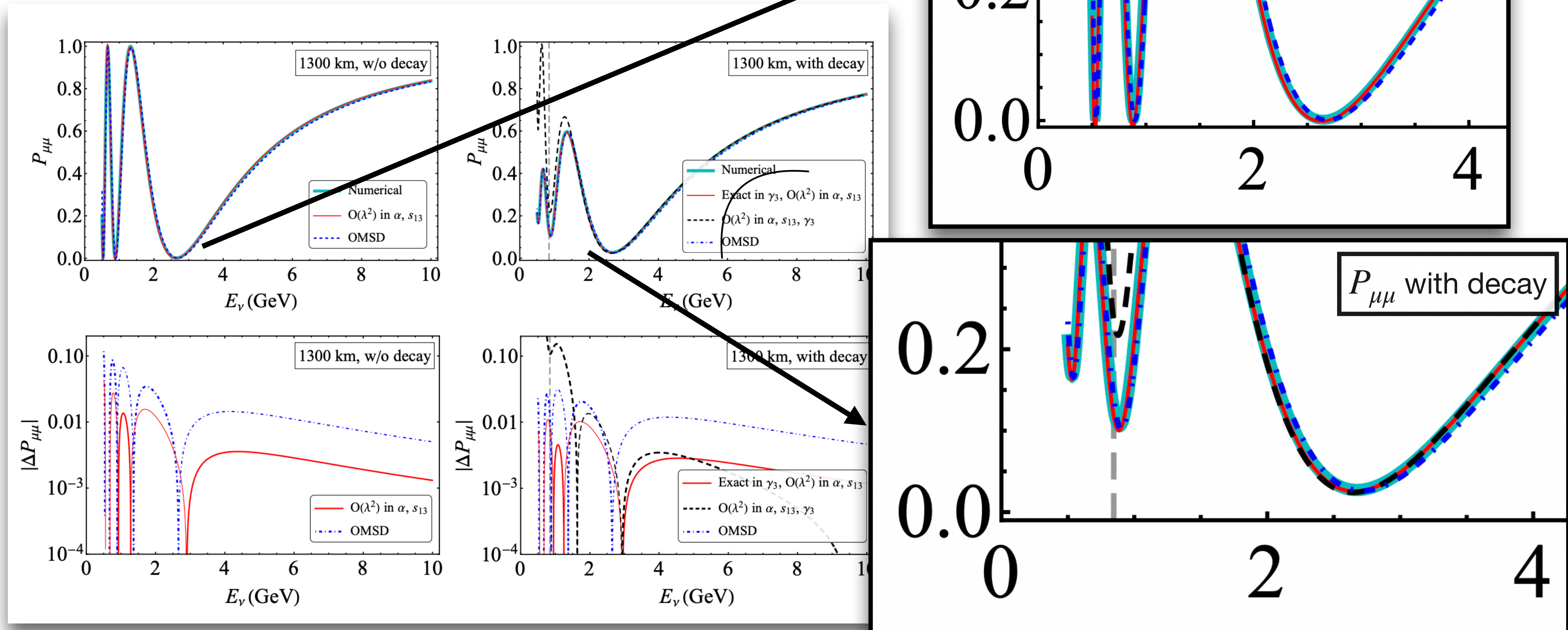
$$\Delta m_{21}^2 = 7.37 \times 10^{-5} \text{ eV}^2, \Delta m_{31}^2 = 2.56 \times 10^{-3} \text{ eV}^2,$$

$$\theta_{12} = 33^\circ, \theta_{23} \simeq 45^\circ, \theta_{13} \simeq 8.5^\circ, \delta_{\text{CP}} = 0^\circ, \gamma_3 = 0.1$$

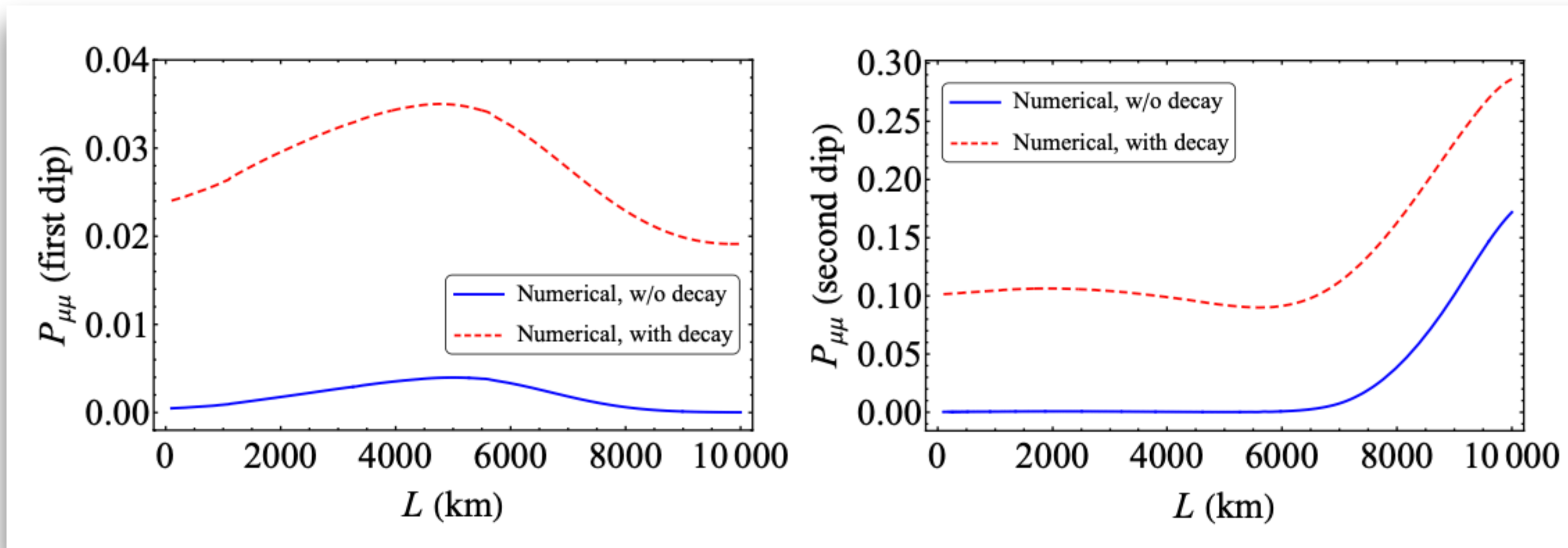
**Accuracy
Checked!**

Increase of probability due to decay!

- $P_{\mu\mu}$ at the oscillation dips



The first two oscillation dips in $P_{\mu\mu}$



$$P_{\mu\mu}^{\text{leading}}(\text{dip}) = 1 - \sin^2 2\theta_{23} - s_{23}^4 (1 - e^{-4\gamma_3\Delta}) + 2s_{23}^2 c_{23}^2 (1 - e^{-2\gamma_3\Delta})$$

$$P_{\mu\mu}(\text{first dip}) \simeq P_{\mu\mu}^{\text{leading}}(\Delta \simeq \pi/2) = \frac{1}{4} (1 - e^{-\pi\gamma_3})^2 \geq 0$$

$$P_{\mu\mu}(\text{second dip}) \simeq P_{\mu\mu}^{\text{leading}}(\Delta \simeq 3\pi/2) \simeq \frac{1}{4} (1 - e^{-3\pi\gamma_3})^2 \geq 0$$

- **Increase in probability** at first and second osc. dips due to ν_3 decay.
- For $\gamma_3 = 0.1$, increase of ~ 0.02 at first and ~ 0.1 at second oscillation dip.
- **Explained by our analytic expressions (like a damped oscillator).**
- The second osc. dip at: $E_\nu \simeq 0.69 (L/1000 \text{ km})$ GeV. Hence possible to observe at DUNE.

Take Home Message

- If neutrinos decay, **mismatch** between mass and decay eigenstates is **inevitable**.
- We have presented the modifications to the neutrino probabilities due to possible **invisible decay in matter**, in a **compact analytic form**.
- Analytic expressions can explain many features of the probabilities: for example, $P_{\mu\mu}$ at oscillation dips increases due to ν_3 decay.

Thank you for your attention

1. D. S. Chattopadhyay, K. Chakraborty, A. Dighe, S. Goswami and S. M. Lakshmi, “*Neutrino Propagation When Mass Eigenstates and Decay Eigenstates Mismatch*”, **Phys. Rev. Lett. 129, no.1, 011802 (2022)** (arXiv:2111.13128 [hep-ph])
2. Dibya S. Chattopadhyay, Kaustav Chakraborty, Amol Dighe, Srubabati Goswami, “*Analytic treatment of 3-flavor neutrino oscillation and decay in matter*”, **arXiv:2204.05803 [hep-ph]**



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