Bayesian method for waveform analysis with GPU acceleration

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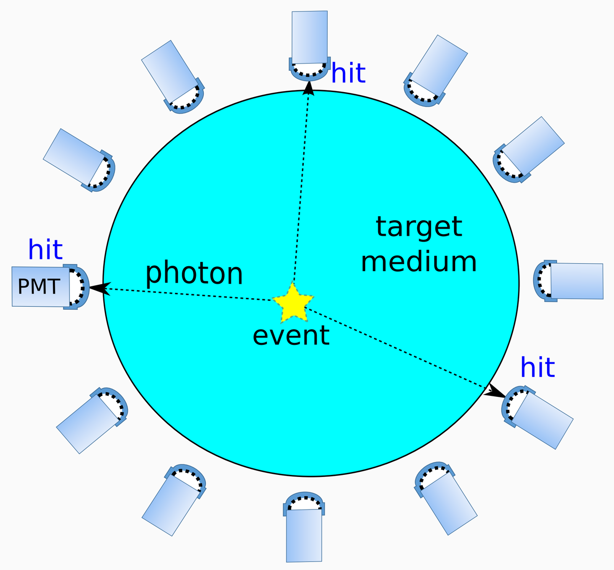
Motivation

We use liquid scintillator detector in neutrino experiments. Our final goal is the neutrino mass ordering(NMO). We need high resolution to achieve that goal.

Therefore, we need much more accurate waveform analysis and reconstruction method, to make use of total information in waveforms, and gain high energy resolution.

accurate waveform analysis → accurate reconstruction → high energy resolution → NMO



Bayesian waveform analysis

We have

$$p(\mu, t_0|\boldsymbol{w}) \propto p(\boldsymbol{w}|\mu, t_0)p(\mu, t_0)$$

where $p(\mu, t_0)$ is a Bayesian prior.

waveform $\boldsymbol{w}$ $\xrightarrow{\text{analysis}}$ [photon intensity μ; time offset t_0; *high resolution needed*] $\xrightarrow{\text{e.g.MLE}}$ [event energy E; event time t; event position $\boldsymbol{r}$]

With maximum likelihood estimate (MLE), to estimate the event $(E, t, \boldsymbol{r})$:

$$(\hat{E}, \hat{t}, \hat{\boldsymbol{r}}) = \underset{E,t,\boldsymbol{r}}{\arg\max}\ p(E, t, \boldsymbol{r}|\mu, t_0, \boldsymbol{w}) = \underset{E,t,\boldsymbol{r}}{\arg\max} \frac{p(\mu, t_0|E, t, \boldsymbol{r})p(E, t, \boldsymbol{r})}{p(\mu, t_0|\boldsymbol{w})}$$

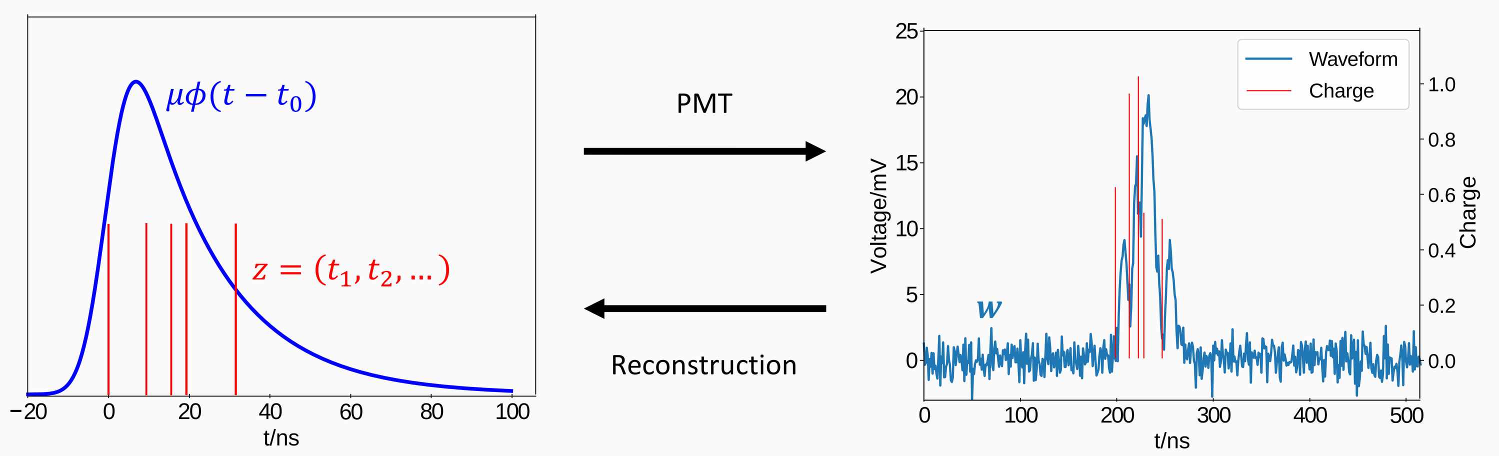
Therefore, it is important to estimate μ and t_0 with high resolution, to guarantee the resolution of latter steps.

It is a Poisson process from μ, t_0 to PE sequence $\boldsymbol{z}$, the expectation of this process is $\mu\phi(t - t_0)$, the curve on the left figure. ϕ is a normalized shape function. $\boldsymbol{z} = (t_1, t_2, \dots)$ represents the times of PEs.

$$p(\boldsymbol{w}|\mu, t_0) = \widetilde{\sum_{\boldsymbol{z}}} p(\boldsymbol{w}|\boldsymbol{z})p(\boldsymbol{z}|\mu, t_0)$$

BOOM

The coefficient space of $\boldsymbol{z}$ explodes because there are any number of possible $\boldsymbol{z}$. We need MCMC to sample the most possible ones.

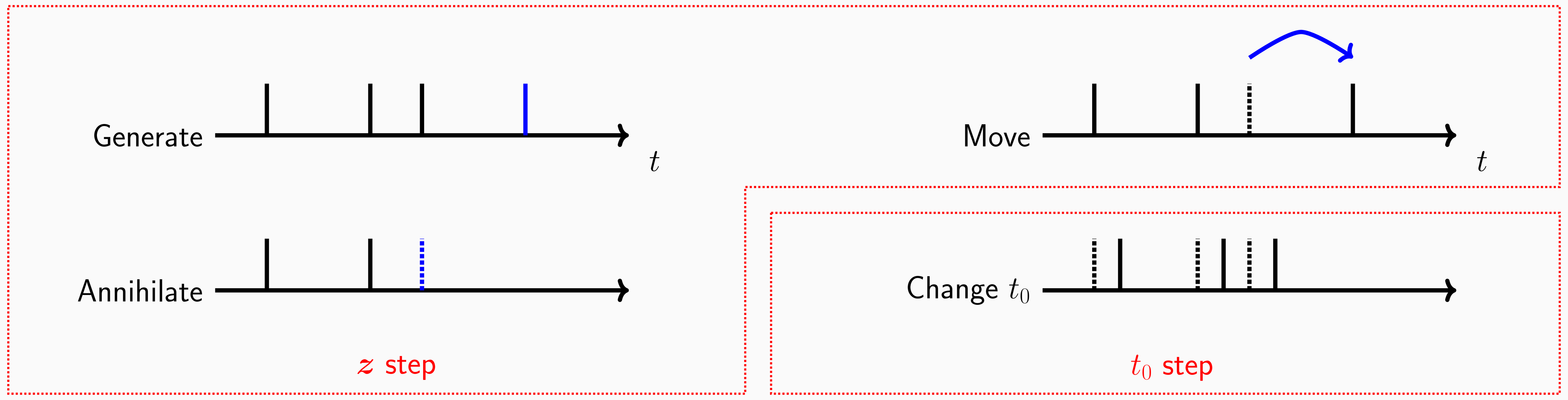


Sample from Poisson process

Convolution with single PE waveform

The MCMC steps in FSMP

We sample $t_0, \boldsymbol{z}$ with Monte-Carlo Markov chain (MCMC).[1, 2] μ is estimated with MLE from posterior distribution. This algorithm is called fast stochastic matching pursuit (FSMP). Fast Bayesian methods are used in calculating.[3]



Bias and resolution

FSMP deals with total information from waveforms, and gives us better accuracy in both time and intensity measurements. It opens the opportunity to boost energy resolution $(\times 1.07)$ and particle identification in PMT-based neutrino experiments.

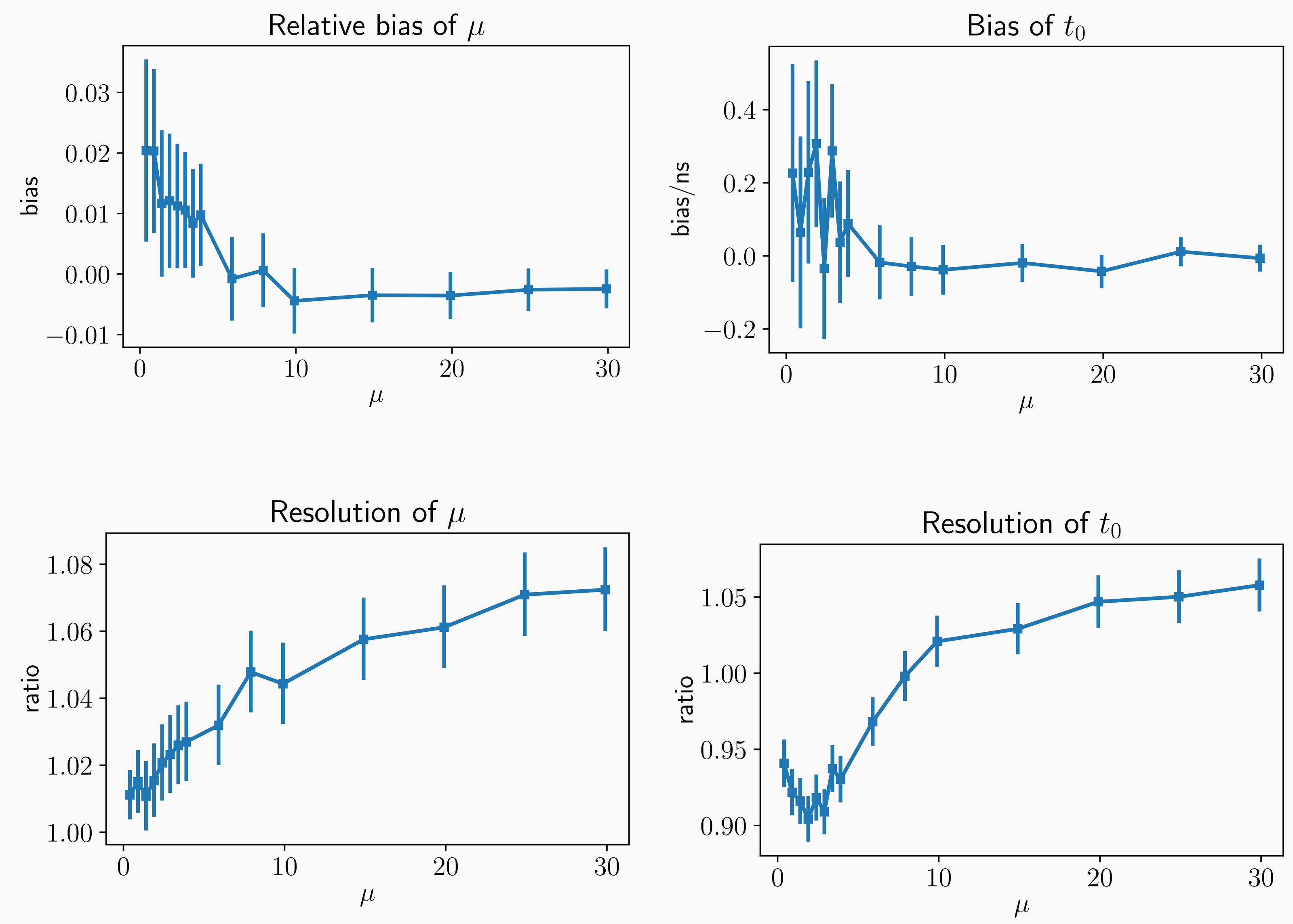
The resolution of μ is defined as

$$\frac{\sqrt{\mathrm{Var}[\hat{\mu}]}/\,\mathrm{E}[\hat{\mu}]}{\sqrt{\mathrm{Var}[N_{\mathrm{PE}}]}/\,\mathrm{E}[N_{\mathrm{PE}}]}$$

N_{PE} is number of PEs; and the resolution of t_0 is defined as

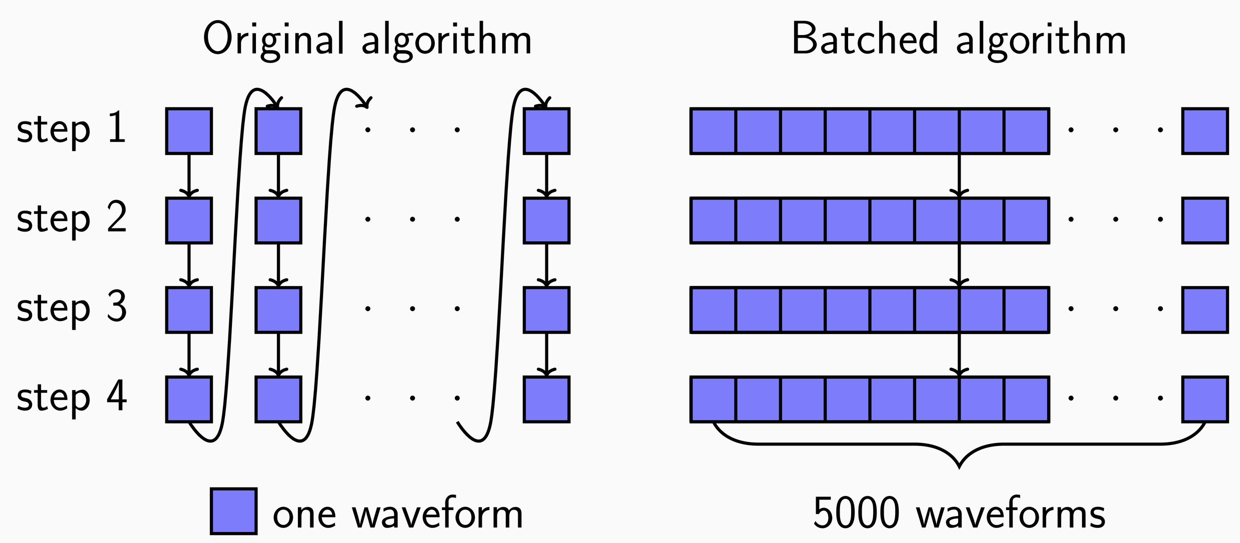
$$\sqrt{\frac{\mathrm{Var}[\hat{t} - t_0]}{\mathrm{Var}[\hat{t}_{\mathrm{ALL}} - t_0]}}$$

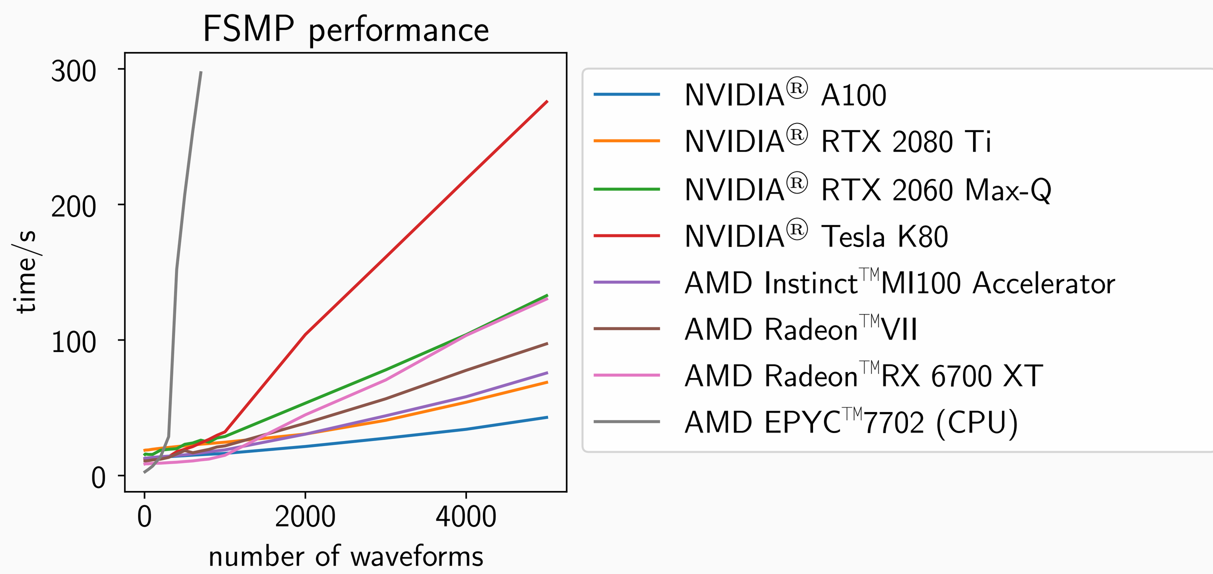
$\hat{t}_{\mathrm{ALL}}$ is ideal estimator for t_0 by truth PE times.



GPU acceleration

The FSMP algorithm is accelerated with batched method on GPU. The left 2 figures show what is batched method: a lot of waveforms are operated together, instead of analyzing them one by one.





The batched method performs 0.01 s/waveform with batched size 5000 on NVIDIA® A100, and it is faster than original algorithm on CPU by 2 orders of magnitude.

Summary

- FSMP method could make use of total information in waveforms.
- FSMP performs fast on consumer GPUs, with high precision results.
- FSMP proves the practicability of the Bayesian method, and we are willing to extend it to event reconstruction.

Reference

[1] W. K. Hastings. Monte Carlo sampling methods using Markov chains and their applications. *Biometrika*, 57(1):97–109, April 1970.

[2] Luke Tierney. Markov chains for exploring posterior distributions. *The Annals of Statistics*, 22(4):1701–1728, 1994.

[3] P. Schniter, L. C. Potter, and J. Ziniel. Fast Bayesian matching pursuit. In *2008 Information Theory and Applications Workshop*, pages 326–333, January 2008.