



Higgs boson decay to J/ψ via c -quark fragmentation

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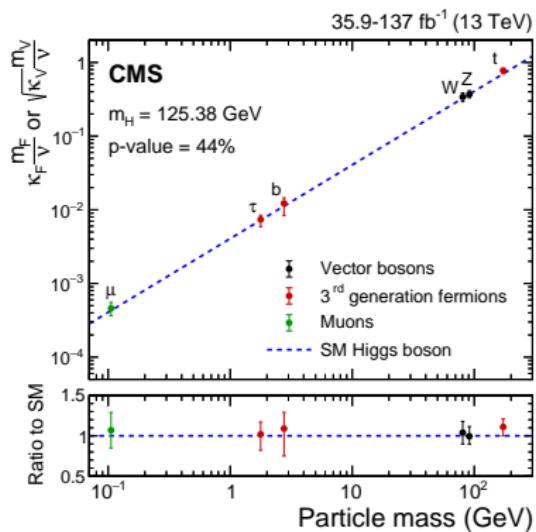
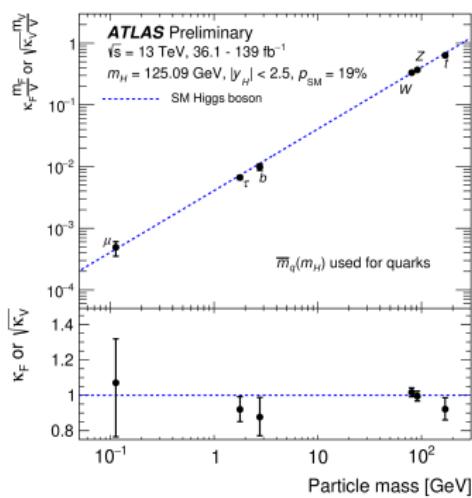
2202.08273, in collaboration with
T. Han, A. Leibovich (PITT), X. Tan (HIT)

Why Charm-Higgs coupling?

Higgs is special

- Higgs provides masses to all other elementary particles.
- Higgs is the only known elementary particle with spin 0.
- A portal to new physics beyond the Standard Model.

Measure the Higgs couplings



[ATLAS-CONF-2021-053]

[2009.04363]

Higgs to light fermion couplings are to be measured

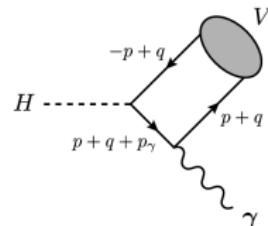
Measuring Charm-Higgs coupling: current status

Measuring $Hc\bar{c}$ coupling is not easy

- Smaller mass \Rightarrow Smaller branching fraction: $\text{BR}(H \rightarrow c\bar{c}) \simeq 2.9\%$
- Large QCD background at hadron colliders \Rightarrow Need c -tagging
- c -tagging is challenging**

Current experimental searching

- κ framework: For $y_c^{\text{SM}} = \sqrt{2}m_c/v$, set $y_c = \kappa_c y_c^{\text{SM}}$
- $pp \rightarrow VH(c\bar{c})$
 - Need c -tagging.
 - LHC Run 2: ATLAS $\kappa_c \leq 8.5$ [ATLAS-CONF-2021-021], CMS $1.1 < |\kappa_c| < 5.5$ [CMS-PAS-HIG-21-008]
 - Future HL-LHC: $\kappa_c \leq 3$. [2001.11428, ATL-PHYS-PUB-2021-039]
- To avoid c -tagging \Rightarrow Higgs decay to J/ψ
 - Clean final states $J/\psi \rightarrow \mu^+ \mu^-$, may avoid c -tagging
 - Use an addition photon as trigger: $H \rightarrow J/\psi + \gamma$
 - The rate is too low: $\text{BR} \sim 10^{-6}$. [1306.5770, 1407.6695]
 - Result is less sensitive: $\kappa_c \leq 100$. [1807.00802, 1810.10056]



Higgs decay to charmonia (I)

The Nonrelativistic QCD framework

- The Higgs decay width in NRQCD factorization

$$\Gamma = \sum_{\mathbb{N}} \hat{\Gamma}_{\mathbb{N}}(H \rightarrow (Q\bar{Q})[\mathbb{N}] + X) \times \langle \mathcal{O}^h[\mathbb{N}] \rangle, \quad d\hat{\Gamma}_{\mathbb{N}} = \frac{1}{2m_H} \frac{|\mathcal{M}|^2}{\langle \mathcal{O}^{Q\bar{Q}} \rangle} d\Phi_3$$

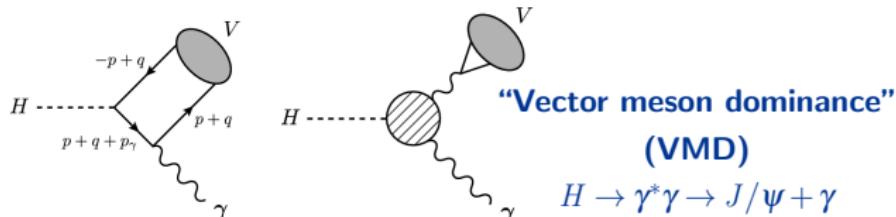
- Long distance matrix element (LDME) are related to the wave function at origin

$$\langle \mathcal{O}^{J/\psi}[{}^3S_1^{[1]}] \rangle = \frac{3N_c}{2\pi} |R(0)|^2, \quad \langle \mathcal{O}^{\eta_c}[{}^1S_0^{[1]}] \rangle = \frac{N_c}{2\pi} |R(0)|^2$$

$$\langle \mathcal{O}^{Q\bar{Q}} \rangle = 6N_c, \text{ for } {}^3S_1^{[1]}, \quad \langle \mathcal{O}^{Q\bar{Q}} \rangle = 2N_c, \text{ for } {}^1S_0^{[1]}$$

Higgs decay to J/ψ and a photon

- $Hc\bar{c}$ diagram is suppressed \Rightarrow Small branching fraction
- The dominant contribution is from $H\gamma\gamma$ diagram \Rightarrow Less sensitive to κ_c
 $\Gamma_{H\gamma\gamma^*} \simeq 1.32 \times 10^{-8} \text{ GeV}, \Gamma_{\text{SM}} \simeq 1.00 \times 10^{-8} \text{ GeV}$ [1306.5770, 1407.6695]



Higgs decay to charmonia (II)

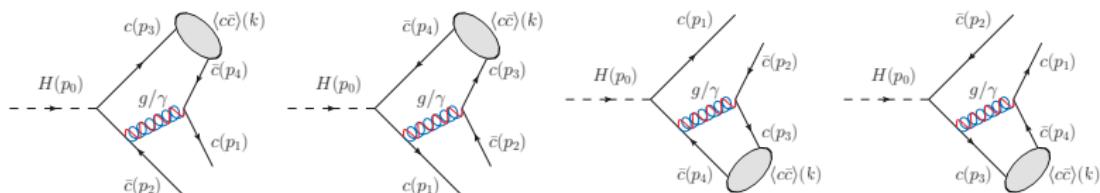
Our idea

- Take advantage of the clean $J/\psi \rightarrow \mu^+ \mu^-$ decay
- Look for a process

$$H \rightarrow c + \bar{c} + J/\psi \text{ (or } \eta_c)$$

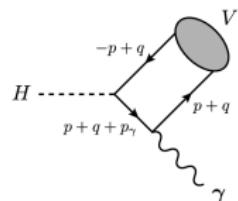
- The rate is larger than that of $H \rightarrow J/\psi + \gamma$
- The $Hc\bar{c}$ channel dominates over possible contaminations

Color-singlet mode: Charm quark fragmentation to ${}^3S_1^{[1]}(J/\psi)$ and ${}^1S_0^{[1]}(\eta_c)$



Compare with $H \rightarrow J/\psi + \gamma$

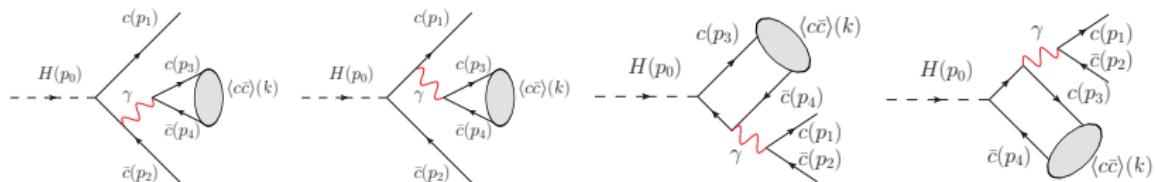
- Enhancement from the quark fragmentation
⇒ Larger rate
- The decay width is more sensitive to κ_c



More corrections from QED and EW sector

Pure QED diagrams: sizable correction to $^3S_1^{[1]}(J/\psi)$ production

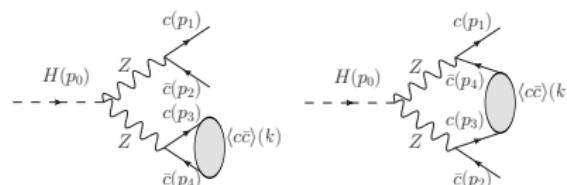
The photon propagator $1/q^2 = 1/m_{J/\psi}^2$



Single photon fragmentation (SPF) \Rightarrow logarithmic enhancement

Electroweak correction from the HZZ diagrams

This may be the contamination for Charm-Higgs coupling determination



One of the Z can be on shell \Rightarrow resonance enhancement

- The resonance peak can be seen in the $J/\psi(\eta_c)$ energy distribution.
- Sizable for $^1S_0^{[1]}(\eta_c)$ due to the larger axial $Zc\bar{c}$ coupling.

Charmonia production via color-octet states

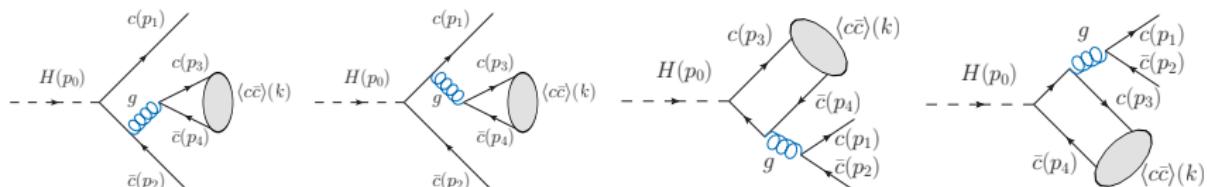
A key property of NRQCD: color-octet states also contribute

- A quarkonium can also be produced through color-octet $Q\bar{Q}$ Fork states
- New states involved: $^3S_1^{[8]}$, $^1S_0^{[8]}$, $^3P_J^{[8]}$, and $^1P_1^{[8]}$
- The LDMEs $\langle \mathcal{O}^h [{}^{2S+1}L_J^{[\text{color}]}] \rangle$ need to be fitted from experimental data

| Reference | $\langle \mathcal{O}^{J/\psi} [{}^1S_0^{[8]}] \rangle$ | $\langle \mathcal{O}^{J/\psi} [{}^3S_1^{[8]}] \rangle$ | $\langle \mathcal{O}^{J/\psi} [{}^3P_0^{[8]}] \rangle / m_c^2$ |
|------------|--|--|--|
| G. Bodwin, | $(9.9 \pm 2.2) \times 10^{-2}$ | $(1.1 \pm 1.0) \times 10^{-2}$ | $(4.89 \pm 4.44) \times 10^{-3}$ |
| K.T. Chao, | $(8.9 \pm 0.98) \times 10^{-2}$ | $(3.0 \pm 1.2) \times 10^{-3}$ | $(5.6 \pm 2.1) \times 10^{-3}$ |
| Y. Feng, | $(5.66 \pm 4.7) \times 10^{-2}$ | $(1.77 \pm 0.58) \times 10^{-3}$ | $(3.42 \pm 1.02) \times 10^{-3}$ |

New diagrams for ${}^3S_1^{[8]}$

Similar to the SPF: The gluon propagator $1/q^2 = 1/m_{J/\psi}^2$



Single gluon fragmentation (SGF) \Rightarrow logarithmic enhancement

Standard Model results (I)

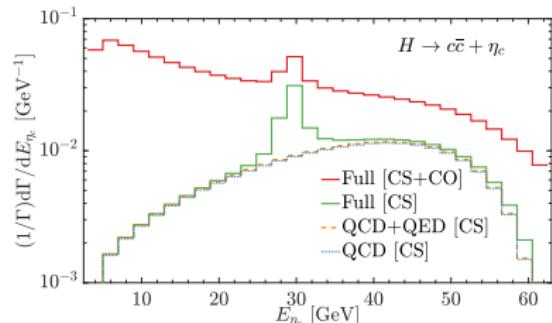
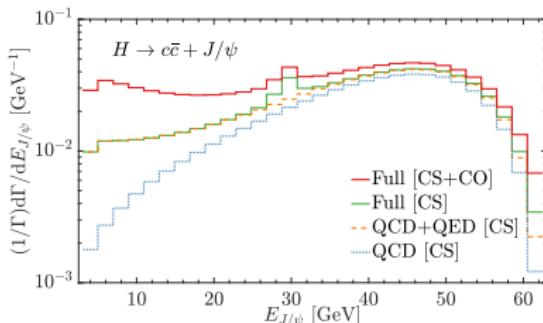
Numerical parameters

$$\alpha = 1/132.5, \quad \alpha_s(2m_c) = 0.235, \quad m_c^{\text{pole}} = 1.5 \text{ GeV}, \quad m_c(m_H) = 0.694 \text{ GeV},$$
$$m_H = 125 \text{ GeV}, \quad m_W = 80.419 \text{ GeV}, \quad m_Z = 91.188 \text{ GeV}, \quad v = 246.22 \text{ GeV}.$$
$$y_c^{\text{SM}} = \frac{\sqrt{2}m_c(m_H)}{v} \approx 3.986 \times 10^{-3},$$

Decay width and branching fraction

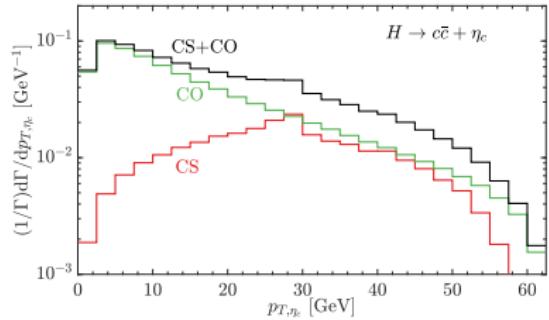
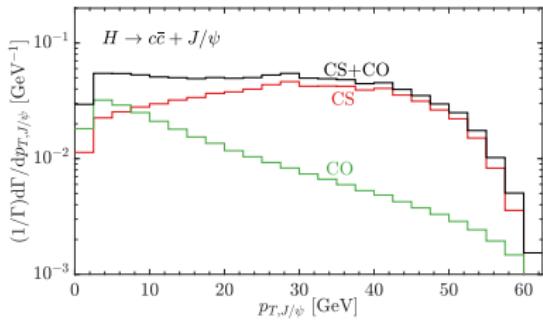
| | QCD [CS] | QCD+QED [CS] | Full [CS] | Full [CO] | Full [CS+CO] |
|---|----------------------|----------------------|----------------------|----------------------|----------------------|
| $\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV) | 4.8×10^{-8} | 5.8×10^{-8} | 6.1×10^{-8} | 2.2×10^{-8} | 8.3×10^{-8} |
| BR($H \rightarrow c\bar{c} + J/\psi$) | 1.2×10^{-5} | 1.4×10^{-5} | 1.5×10^{-5} | 5.3×10^{-6} | 2.0×10^{-5} |
| $\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV) | 4.9×10^{-8} | 5.1×10^{-8} | 6.3×10^{-8} | 1.8×10^{-7} | 2.4×10^{-7} |
| BR($H \rightarrow c\bar{c} + \eta_c$) | 1.2×10^{-5} | 1.2×10^{-5} | 1.5×10^{-5} | 4.5×10^{-5} | 6.0×10^{-5} |

Charmonium energy distributions

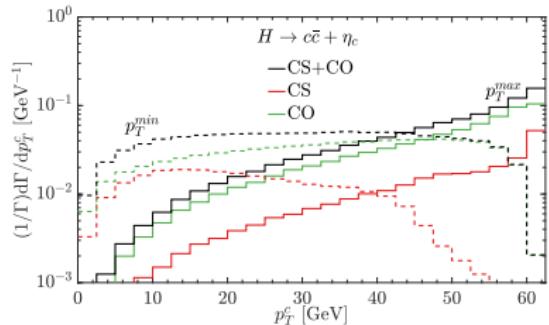
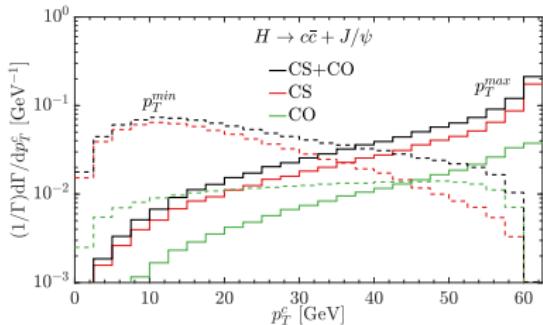


Standard Model results (II)

Charmonium transverse momentum distribution

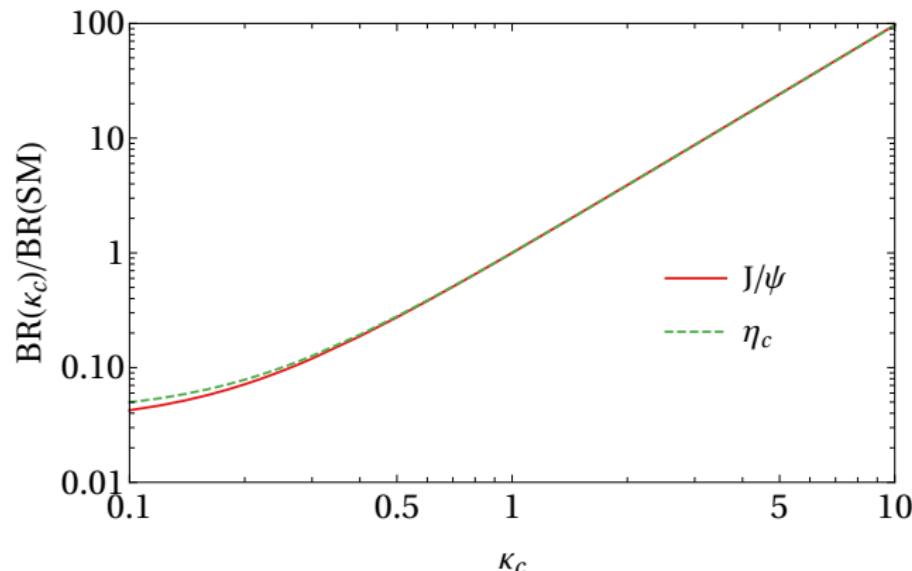


Transverse momentum distribution for the free charm quark



Probe the $Hc\bar{c}$ coupling (I)

Use the κ framework $y_c = \kappa_c y_c^{\text{SM}}$, $\text{BR} \approx \kappa_c^2 \text{BR}^{\text{SM}}$



Note there are small contaminations:

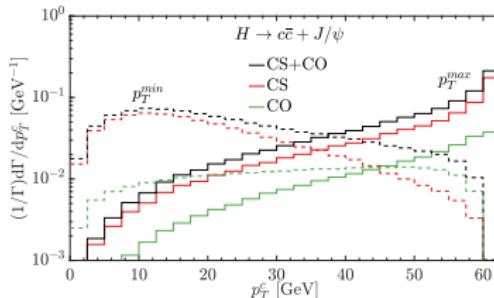
- HZZ diagrams
- The $H \rightarrow g^*g^*/\gamma^*\gamma^* \rightarrow J/\psi + c\bar{c}$ channel

Probe the $Hc\bar{c}$ coupling (II)

Some rough analysis:

- Higgs production cross section at LHC $\sigma_H \sim 50$ pb
- Expect HL-LHC $L \sim 3 \text{ ab}^{-1}$ at ATLAS and CMS and $L \sim 0.3 \text{ ab}^{-1}$ at LHCb
- Detection efficiency ε for the final state $c\bar{c} + \ell^+\ell^-$
- $\text{BR}(J/\psi \rightarrow \ell^+\ell^-) \sim 12\%$, $\text{BR}(H \rightarrow J/\psi + c\bar{c}) \sim 2 \times 10^{-5}$
- Event number $N = L\sigma_H \varepsilon \text{ BR}(H \rightarrow c\bar{c}\ell^+\ell^-) \approx 12 \kappa_c^2 \times \frac{L}{\text{ab}^{-1}} \times \frac{\varepsilon}{10\%}$
- Considering the statistical error only $\delta N \sim \sqrt{N}$ gives

$$\Delta\kappa_c \approx 15\% \times \left(\frac{L}{\text{ab}^{-1}} \times \frac{\varepsilon}{10\%} \right)^{-1/2}$$

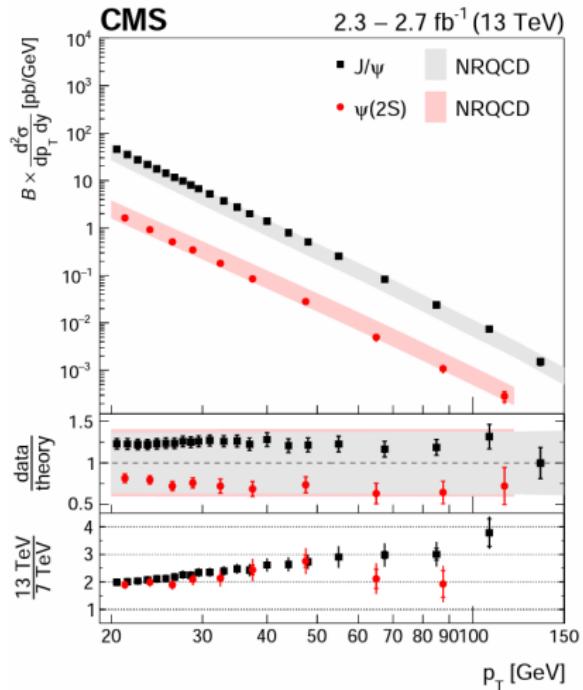


Detection efficiency ε :

- Double charm-tagging $(40\%)^2 \sim 16\%$
- Kinematic acceptance 50%
- Assume $\varepsilon \sim 10\% \Rightarrow \Delta\kappa_c \sim 15\%$

Probe the $Hc\bar{c}$ coupling (III)

Background: $pp \rightarrow J/\psi + X$

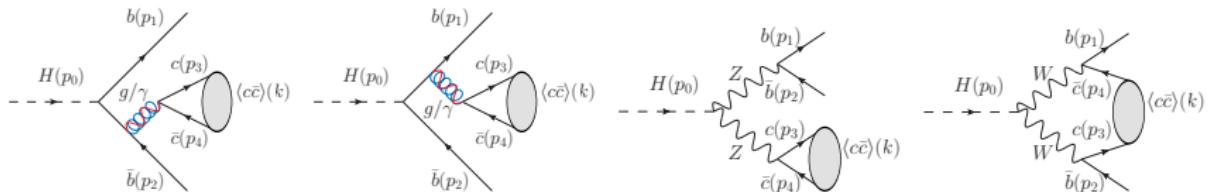


- Prompt J/ψ production $\text{BR}(J/\psi \rightarrow \mu^+ \mu^-) \times \sigma(pp \rightarrow J/\psi) \simeq 860 \text{ pb}$ [1710.11002]
- Estimate 75000 events for $pp \rightarrow J/\psi + c\bar{c} \Rightarrow \sim 25 \text{ fb}$ for a 3 ab^{-1} HL-LHC [2012.14161]
- Charm-tagging is needed. • Some kinematic cuts may help.

Probe the $Hc\bar{c}$ coupling (IV)

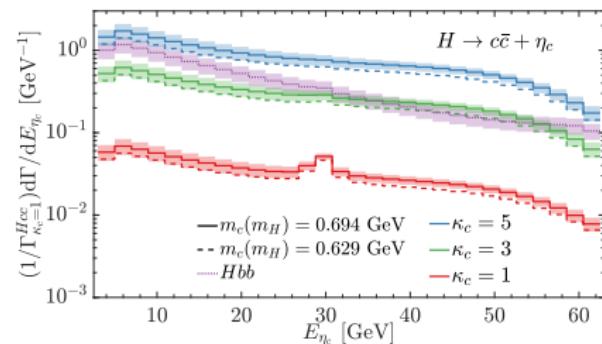
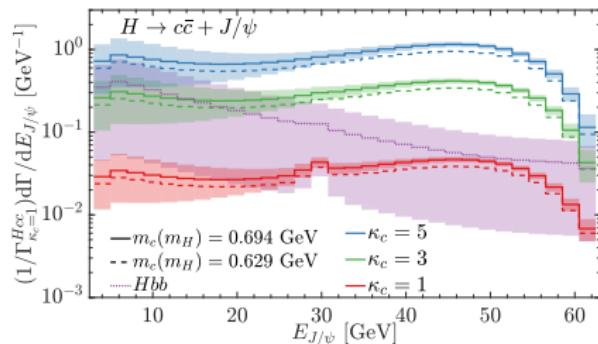
Background: $H \rightarrow J/\psi + b\bar{b}$

Color-octet contribution dominates



Charmonium energy distributions

Take the ${}^3S_1^{[8]}$ LDME uncertainty for error estimation



- Need to determine charm from bottom \Rightarrow Charm-tagging is needed.
- Large uncertainty from LDME \Rightarrow More work on LDMEs fitting is needed.

Probe the $Hc\bar{c}$ coupling (V)

- If there were no background: $\Delta\kappa_c \sim 15\%$
- However, there is background in the real world:
 - Assume 10,000 background events after the selection cuts at the HL-LHC
 - Assume the detection efficiency $\epsilon \sim 10\%$
 - The signal event number is given by

$$N = L\sigma_H \epsilon \text{BR}(H \rightarrow c\bar{c}\ell^+\ell^-) \approx 12 \kappa_c^2 \times \frac{L}{ab^{-1}} \times \frac{\epsilon}{10\%}$$

- Sensitivity $S \simeq N_{\text{signal}} / \sqrt{N_{\text{Background}}}$
⇒ It is possible to reach 2σ for $\kappa_c \approx 2.4$.
- systematic effect $N_{\text{signal}}/N_{\text{Background}} = 2\%$ for $\kappa_c \approx 2.4$.

Conclusion

- **Higgs is special and important**

- The Higgs sector is the portal to new physics beyond SM.
- Testing the SM mass generation mechanism helps BSM physics searches.
- The Yukawa couplings of the 3rd generation fermions are precisely measured
⇒ Charm quark is the next target.

- **For the current determination of the Charm-Higgs coupling**

- $p p \rightarrow V H(c\bar{c})$, *c*-tagging is challenging

ATLAS: $\kappa_c < 8.5$, CMS: $1.1 < |\kappa_c| < 5.5$, Future 3 ab⁻¹ HL-LHC: $\kappa_c < 3$

- $H \rightarrow J/\psi + \gamma$, no need for *c*-tagging but insensitive to κ_c ATLAS: $\kappa_c < 100$

- **Another possible approach:** $H \rightarrow J/\psi + c\bar{c}$

- The rate is larger due to the fragmentation enhancements
- There are both color-singlet and color-octet contributions
- The QED and EW corrections can be sizable, so need to be included
- The SM prediction gives $BR \sim 2 \times 10^{-5}$
- For a possible 3 ab⁻¹ HL-LHC, with a 10% final state detection efficiency
⇒ $\Delta \kappa_c \sim 10\%$

- Assume there are 10,000 background events ⇒ 2σ for $\kappa_c \simeq 2.4$

- **More work in progress:**

- Background analysis, detector/systematic effects
- Better LDMEs fittings, higher order calculations/resummation ...

Worry about VMD ?

$$H \rightarrow J/\psi + c\bar{c}$$

- Larger decay rate

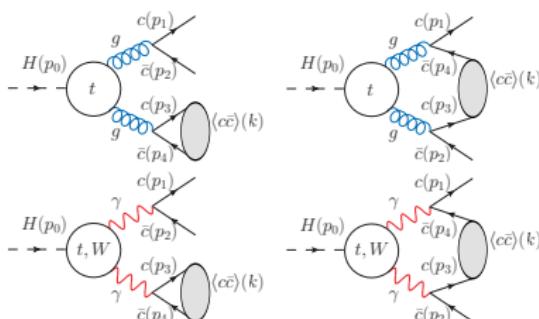
$$\text{BR}(H \rightarrow J/\psi + c\bar{c}) \simeq 2 \times 10^{-5}$$

- Sensitive to $Hc\bar{c}$ coupling

QCD and QED dominates

- Other diagrams

$$H \rightarrow g^*g^*/\gamma^*\gamma^* \rightarrow J/\psi + c\bar{c}$$



$$\text{BR}(g^*g^*) \sim 2.5 \times 10^{-6}, \text{ BR}(\gamma^*\gamma^*) < 2 \times 10^{-7}$$

- No need to worry about VMD

$$H \rightarrow J/\psi + \gamma$$

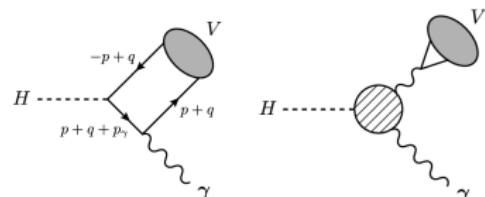
- Small decay rate

$$\text{BR}(H \rightarrow J/\psi + \gamma) \simeq 2.8 \times 10^{-6}$$

- Insensitive to $Hc\bar{c}$ coupling

$$\Rightarrow \kappa_c \leq 100$$

VMD dominates



- $\gamma^* \rightarrow J/\psi$ dominates over $Hc\bar{c}$
Two orders of magnitude larger.

Color-singlet VS color-octet

Recall the NRQCD factorization formalism

$$\Gamma = \sum_{\mathbb{N}} \hat{\Gamma}_{\mathbb{N}}(H \rightarrow (Q\bar{Q})[\mathbb{N}] + X) \times \langle \mathcal{O}^h[\mathbb{N}] \rangle$$

Long distance: the color-octet LDMEs are suppressed

They are in higher orders of v than the color-singlet one

$$\frac{\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle}{\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle} \sim \mathcal{O}(v^3), \quad \frac{\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle}{\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle} \sim \mathcal{O}(v^4), \quad \frac{\langle \mathcal{O}^{J/\psi}(^3P_J^{[8]}) \rangle}{\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle} \sim \mathcal{O}(v^4),$$
$$\frac{\langle \mathcal{O}^{\eta_c}(^3S_1^{[8]}) \rangle}{\langle \mathcal{O}^{\eta_c}(^1S_0^{[1]}) \rangle} \sim \mathcal{O}(v^3), \quad \frac{\langle \mathcal{O}^{\eta_c}(^1P_1^{[8]}) \rangle}{\langle \mathcal{O}^{\eta_c}(^1S_0^{[1]}) \rangle} \sim \mathcal{O}(v^4)$$

Short distance coefficient (SDC)

- The color factors are different for color-singlet and color-octet states

| | Charm fragmentation | | | SPF | SGF |
|----|---------------------|-----|---------|-----|-----|
| | QCD | QED | QCD×QED | QED | QCD |
| CS | 16/9 | 1 | 4/3 | 9 | - |
| CO | 2/9 | 8 | -4/3 | - | 2 |

- There may appear new diagrams for color-octet state production

The SGF diagrams result in large ${}^3S_1^{[8]}$ SDC

⇒ Sizable color-octet contribution (mainly from ${}^3S_1^{[8]}$)

Standard Model results: some details

Color-octet contributions

| | $^3S_1^{[8]}$ | $^1S_0^{[8]}$ | $^1P_1^{[8]}$ | $^3P_J^{[8]}$ | Total |
|---|----------------------|-----------------------|-----------------------|-----------------------|----------------------|
| $\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV) | 2.0×10^{-8} | 9.8×10^{-10} | - | 2.2×10^{-10} | 2.2×10^{-8} |
| $\text{BR}(H \rightarrow c\bar{c} + J/\psi)$ | 5.0×10^{-6} | 2.4×10^{-7} | - | 5.3×10^{-8} | 5.3×10^{-6} |
| $\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV) | 1.8×10^{-7} | 3.6×10^{-11} | 1.0×10^{-10} | - | 1.8×10^{-7} |
| $\text{BR}(H \rightarrow c\bar{c} + \eta_c)$ | 4.5×10^{-5} | 8.9×10^{-9} | 2.5×10^{-8} | - | 4.5×10^{-5} |

Contributions with respect to QCD

| $\hat{\Gamma}_N / \hat{\Gamma}_N^{\text{QCD}}$ | $^1S_0^{[1]}$ | $^3S_1^{[1]}$ | $^1S_0^{[8]}$ | $^3S_1^{[8]}$ | $^1P_1^{[8]}$ | $^3P_0^{[8]}$ | $^3P_1^{[8]}$ | $^3P_2^{[8]}$ |
|--|----------------------|---------------|---------------|----------------------|---------------|---------------|---------------|---------------|
| QCD | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 |
| QED | 1.1×10^{-4} | 0.077 | 0.0073 | 1.1×10^{-5} | 0.0068 | 0.0073 | 0.0073 | 0.0073 |
| $\text{QCD} \times \text{QED}$ | 0.021 | 0.14 | -0.17 | 0.0012 | -0.15 | -0.17 | -0.17 | -0.17 |
| EW | 0.24 | 0.051 | 0.28 | 2.6×10^{-4} | 1.4 | 0.29 | 0.33 | 1.5 |

Some observations

- QCD is dominant in most of the Fock states
- SPF brings sizable QED correction to $^3S_1^{[1]}$, but it is forbidden for $^1S_0^{[1]}$
- SGF makes $^3S_1^{[8]}$ super large
- For $^1S_0^{[8]}$ and $^3P_J^{[8]}$, charm-quark fragmentation is the only production channel, so that QED and QCD differ by a universal factor
- EW correction is large since Z is closed to its mass shell

Color-octet uncertainties from the LDMEs

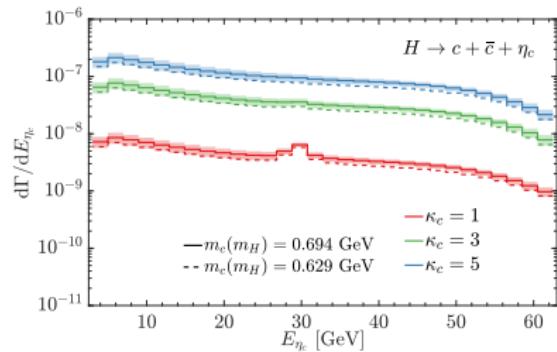
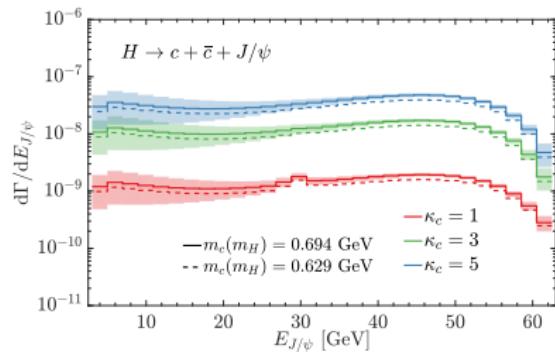
Color-octet contributions: $^3S_1^{[8]}$ dominates

| | $^3S_1^{[8]}$ | $^1S_0^{[8]}$ | $^1P_1^{[8]}$ | $^3P_J^{[8]}$ | Total |
|---|----------------------|-----------------------|-----------------------|-----------------------|----------------------|
| $\Gamma(H \rightarrow c\bar{c} + J/\psi)$ (GeV) | 2.0×10^{-8} | 9.8×10^{-10} | - | 2.2×10^{-10} | 2.2×10^{-8} |
| BR($H \rightarrow c\bar{c} + J/\psi$) | 5.0×10^{-6} | 2.4×10^{-7} | - | 5.3×10^{-8} | 5.3×10^{-6} |
| $\Gamma(H \rightarrow c\bar{c} + \eta_c)$ (GeV) | 1.8×10^{-7} | 3.6×10^{-11} | 1.0×10^{-10} | - | 1.8×10^{-7} |
| BR($H \rightarrow c\bar{c} + \eta_c$) | 4.5×10^{-5} | 8.9×10^{-9} | 2.5×10^{-8} | - | 4.5×10^{-5} |

Take the $^3S_1^{[8]}$ LDME for the uncertainty estimation

$$\text{BR}(H \rightarrow c\bar{c} + J/\psi) = (2.0 \pm 0.5) \times 10^{-5},$$

$$\text{BR}(H \rightarrow c\bar{c} + \eta_c) = (6.0 \pm 1.0) \times 10^{-5}.$$



When is y_c not related to the charm mass?

Higgs Effective Field Theory (HEFT)

$SU(2)$ doublets of the global $SU(2)_{L,R}$ symmetries:

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad Q_R = \begin{pmatrix} U_R \\ D_R \end{pmatrix}, \quad L_L = \begin{pmatrix} v_L \\ E_L \end{pmatrix}, \quad L_R = \begin{pmatrix} 0 \\ E_R \end{pmatrix}.$$

Define

$$U(x) \equiv \exp(i\sigma_a \pi^a(x)/v)$$

so that the Lagrangian contains

$$\mathcal{L} \supset -\frac{v}{\sqrt{2}} \bar{Q}_L U y_Q(h) Q_R - \frac{v}{\sqrt{2}} \bar{L}_L U y_L(h) L_R + h.c.$$

The functions $y_Q(h)$ and $y_L(h)$ control the Yukawa couplings

$$y_Q(h) \equiv \text{diag} \left(\sum_n y_U^{(n)} \frac{h^n}{v^n}, \sum_n y_D^{(n)} \frac{h^n}{v^n} \right)$$

$$y_L(h) \equiv \text{diag} \left(0, \sum_n y_\ell^{(n)} \frac{h^n}{v^n} \right) L$$

$n=0$ is for mass term, $n=1$ is for Yukawa coupling.

Fragmentation formalism

The decay width is written as a convolution

Define $z \equiv 2E_\psi/m_H$

$$\frac{d\Gamma}{dz}(H \rightarrow \psi(z)q\bar{q}) = 2C_q \otimes D_q + C_g \otimes D_g, \quad C \otimes D \equiv \int_z^1 C(y)D(z/y) \frac{dy}{y}$$

Hard coefficient

$$C_q(\mu^2, z) = \Gamma(H \rightarrow q\bar{q})\delta(1-z)$$

$$C_g(\mu^2, z) = \frac{4\alpha_s}{3\pi} \Gamma(H \rightarrow q\bar{q}) \left[\frac{(z-1)^2+1}{z} \log \left(\frac{(1-z)z^2 m_H^2}{\mu^2} \right) - z \right]$$

Fragmentation functions

$$D_{c \rightarrow J/\psi}^{(1)}(\mu^2, z) = \frac{128\alpha_s^2}{243m_{J/\psi}^3} \frac{z(1-z^2)}{(2-z)^6} (16 - 32z + 72z^2 - 32z^3 + 5z^4) \langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle$$

$$D_{q \rightarrow \psi}^{(8)}(\mu^2, z) = \frac{2\alpha_s^2}{9m_\psi^3} \left[\frac{(z-1)^2+1}{z} \log \left(\frac{\mu^2}{m_\psi^2(1-z)} \right) - z \right] \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$$

$$D_{g \rightarrow \psi}(\mu^2, z) = \frac{\pi\alpha_s}{3m_\psi^3} \delta(1-z) \langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$$