

# Precision test of the muon-Higgs coupling at a high-energy muon collider



**ICHEP 2022**  
**XLI**  
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on High Energy Physics  
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6  
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arXiv: 2108.05362 [JHEP 12 (2021) 162]

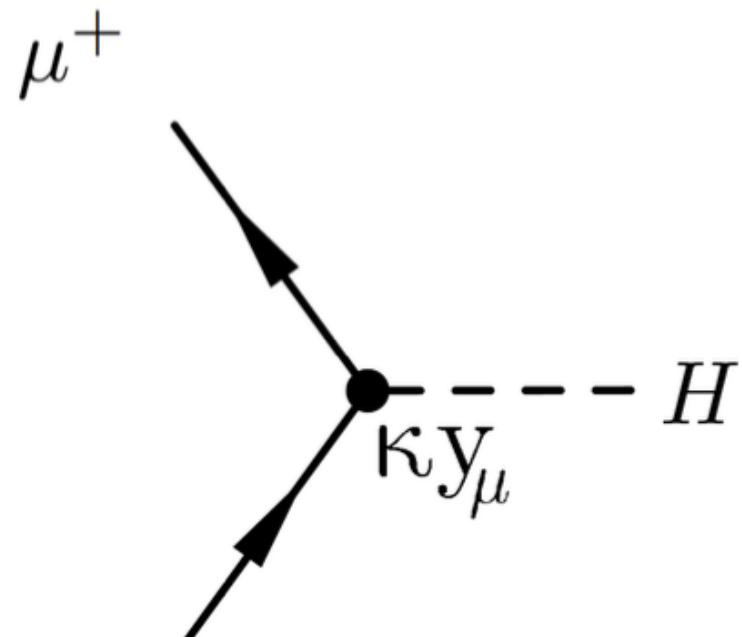
Jürgen R. Reuter



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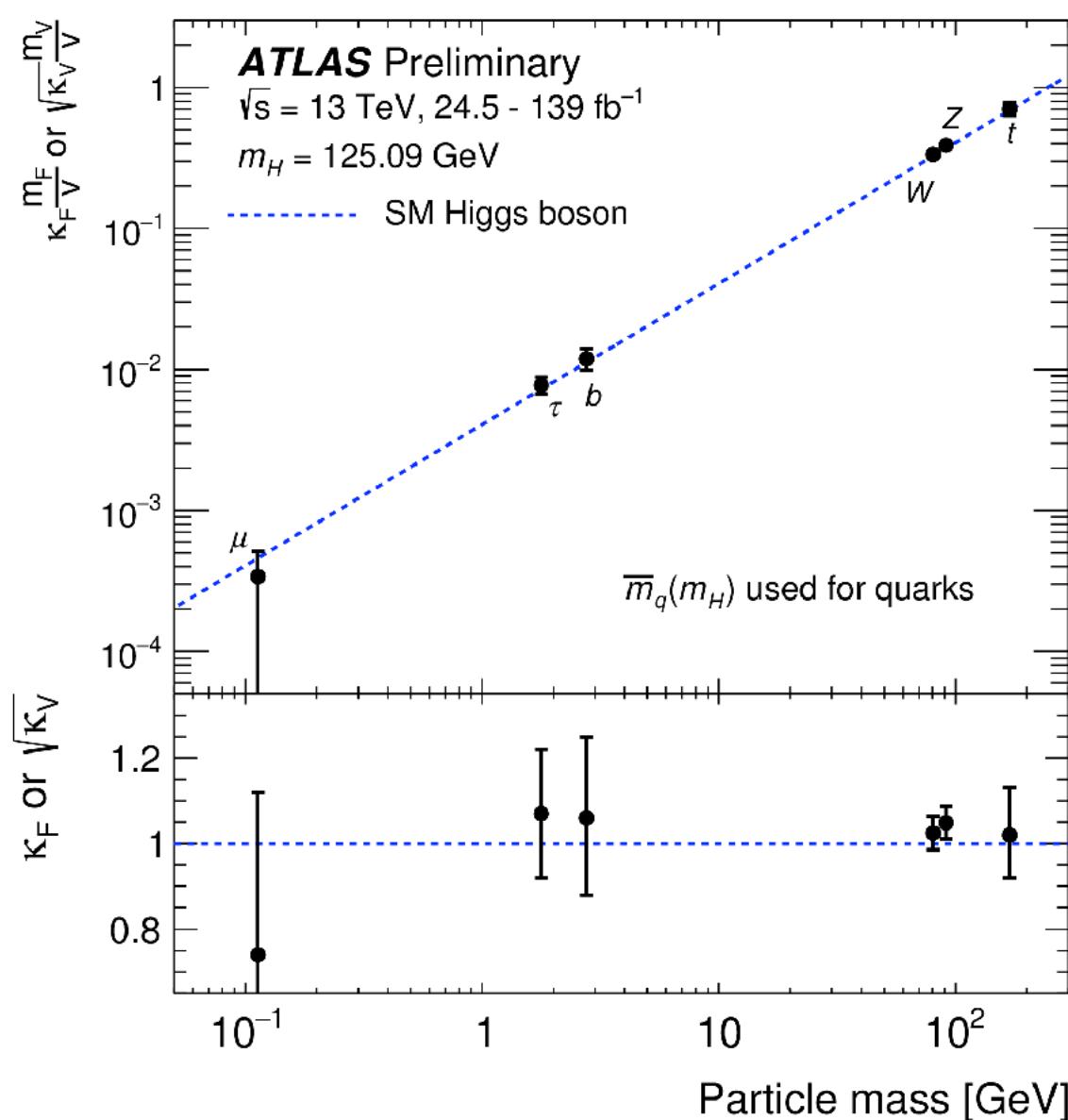
# Higgs Precision Paradigm



SM:  $\kappa = 1$   
or  $\Delta\kappa = 0$

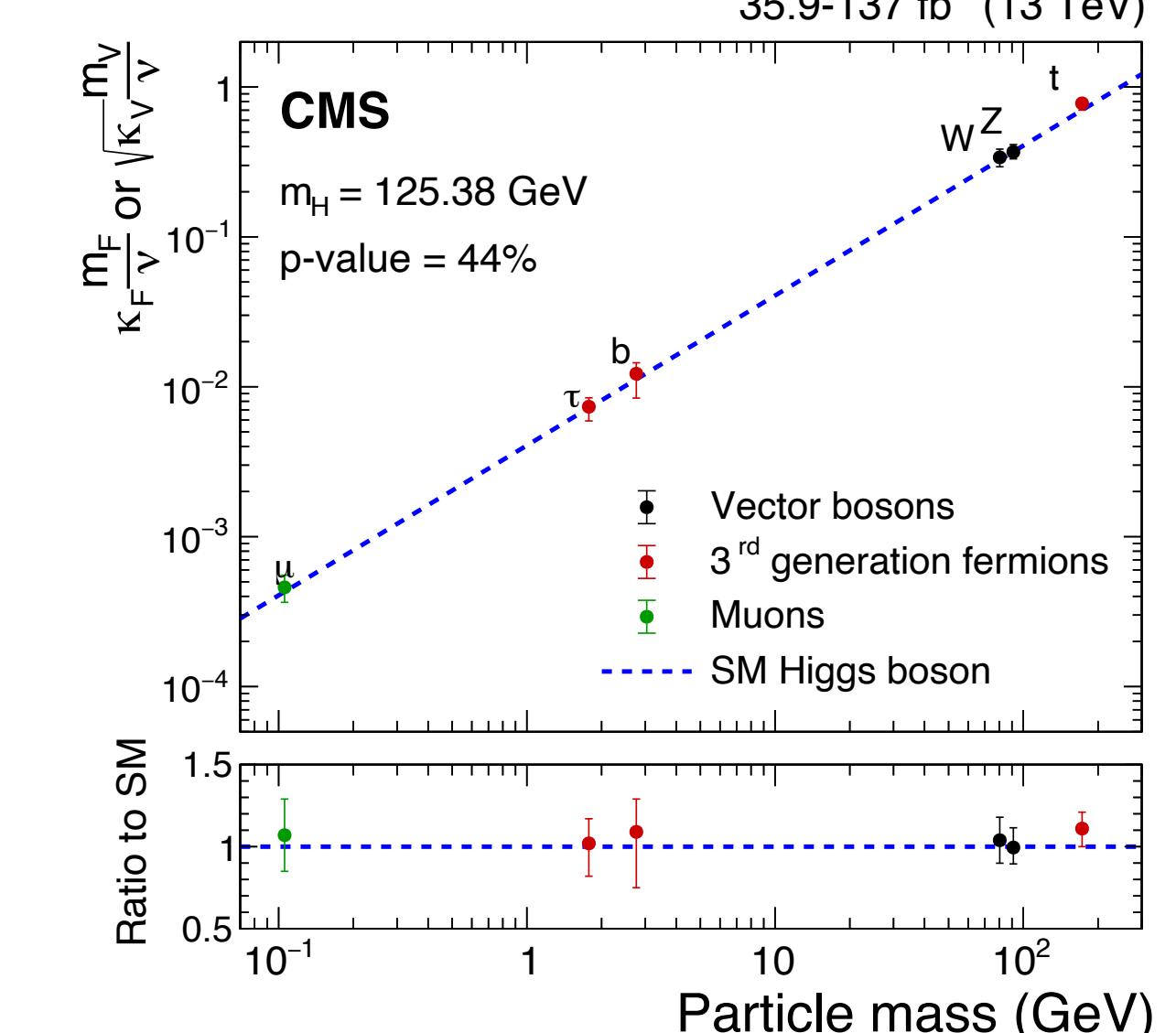
- Higgs properties at high precision utmost priority  $\Rightarrow$  [ESU2020 document](#)
- Higgs potential and Higgs couplings to all SM particles
- Higgs muon Yukawa coupling — connected to muon mass [in the SM!]

- Muon Yukawa coupling established at LHC (not yet  $5\sigma$ )  
[\[ATLAS: 2007.07830 ; CMS: 2009.04363\]](#)
- Projections for the high-luminosity LHC (HL-LHC): sensitivity with precision of (several) 10% [\[ATLAS-PHYS-PUB-2014-016\]](#)

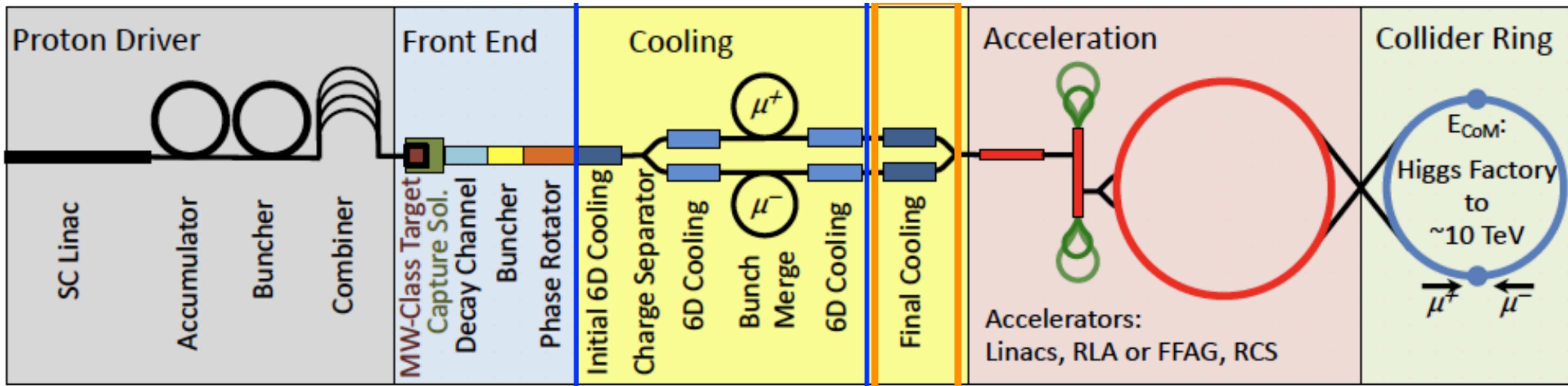


## Challenges / wishlist:

- (very) small coupling needs (very) large luminosity
- Model independence I: Separate production/decay
- Model independence II: sensitivity to many BSM models
- ▶ use high-luminosity lepton (muon) collider



# The (high-energy) muon collider



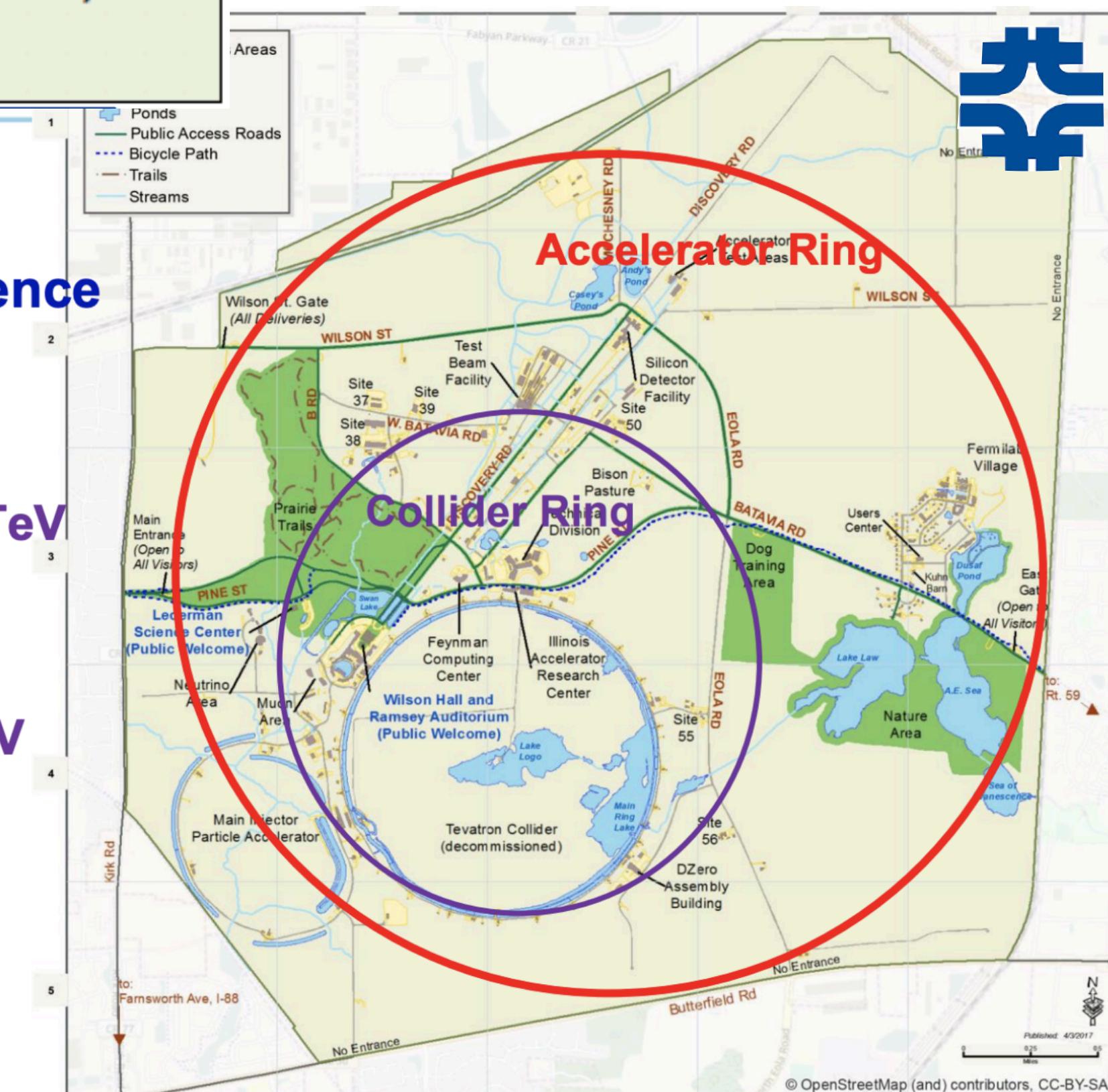
- Muons pointlike objects: cleaner environment than hh
- Much less synchrotron radiation than electrons
- Much smaller beam energy spread:  $\Delta E \approx 0.1 - 0.001\%$
- Complicated production: protons → target →  $\pi \rightarrow \mu$
- Short lifetime: difficult to get high-quality/lumi beams
- Difficult cooling of beams
- Beam-induced bkgds (BIP) from decay @ IP
- Radiation hazard from beam dump (neutrinos)

➤ **Largest Radius is  $\sim 2.65$  km**  
**•  $\sim 16.5$  km Circumference**  
**•  $\sim 2/3$  LHC**

**~RCS accelerator**  
**If  $B_{ave} = 3$  T  $\rightarrow E_\mu = 2.4$  TeV**  
 $(B_{max} = 8$  T,  $B_{pulse} = \pm 2$  T)

**Doubled ?**  
 $B_{ave} = 6.3$  T  $\rightarrow E_\mu = 5$  TeV  
 $(B_{max} = 16$  T,  $B_{pulse} = \pm 4$  T)

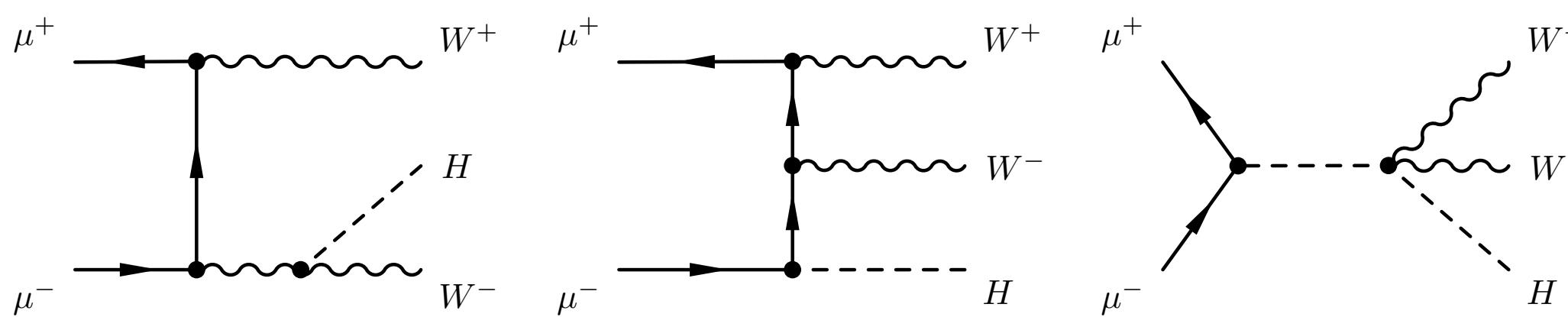
**10 TeV collider**  
**Collider Ring  $\sim 10$  km**  
 $B_{ave} = 10$  T  
 $\tau_{\mu} = 0.104$  s



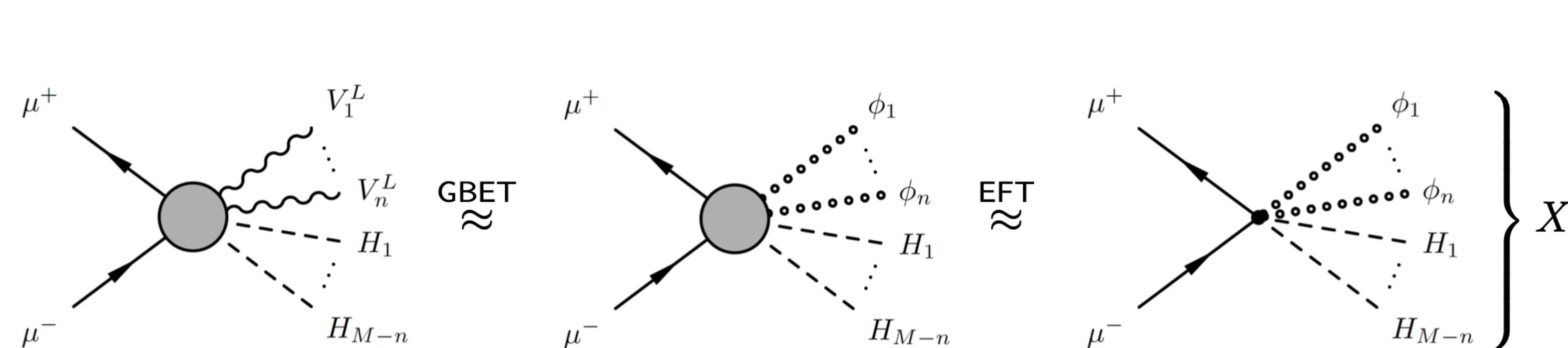
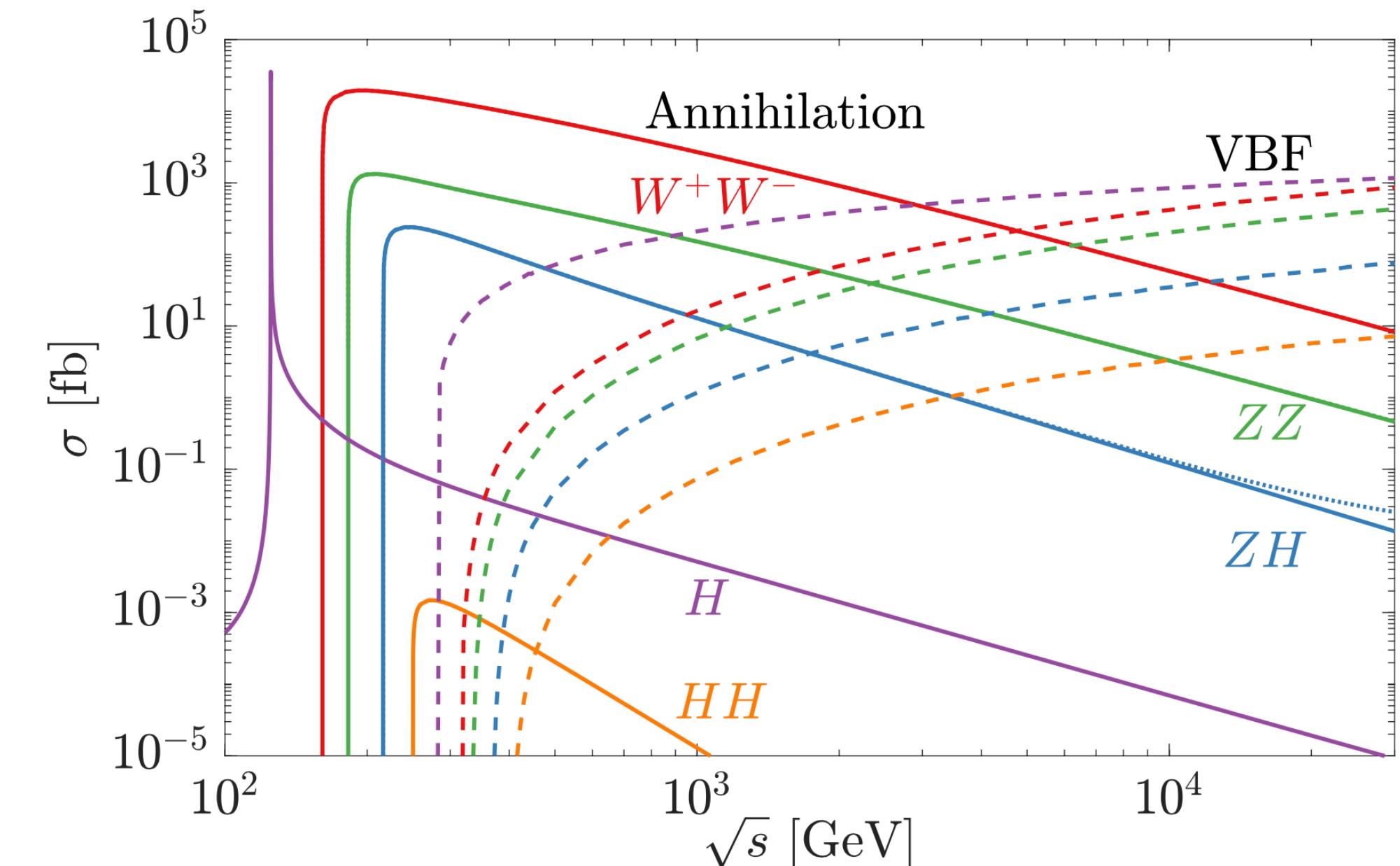
# Multi-boson final states

- Subtle cancellation between Yukawa coupling and multi-boson final states

[hep-ph/0106271]



- (Multi-) boson final states: **longitudinal polarizations** dominate high energies
- Analytic calculations can be approximated by Goldstone-boson Equivalence Theorem (GBET) [NPB261(1985) 379; PRD34(1986) 379]
- New physics parameterized by EFT operator insertions (Wilson coeff.  $C_X$ )



$$\sigma_X \approx \frac{1}{4} \left( \frac{\pi}{2(2\pi)^4} \right)^{M-1} \frac{s^{M-2}}{\Gamma(M)\Gamma(M-1)} |C_X|^2 \left( \prod_{j \in J_X} \frac{1}{n_j!} \right)$$

Cross section ratios:

$$R = \frac{\sigma_X}{\sigma_Y} \approx \frac{|C_X|^2 \left( \prod_{j \in J_X} \frac{1}{n_j!} \right)}{|C_Y|^2 \left( \prod_{j \in J_Y} \frac{1}{n_j!} \right)}$$

$$F_U(H) = 1 + \sum_{n \geq 1} f_{U,n} \left( \frac{H}{v} \right)^n$$

# EFT modelling of SM deviations

## Non-linear representation (HEFT)

Scalar  $H$  NGB  $U = e^{i\phi^a \tau_a/v}$   $\phi^a \tau_a = \sqrt{2} \begin{pmatrix} \frac{\phi^0}{\sqrt{2}} & \phi^+ \\ \phi^- & -\frac{\phi^0}{\sqrt{2}} \end{pmatrix}$

Generalized ( $\mu$ ) Yukawa sector

$$\mathcal{L}_{UH} = \frac{v^2}{4} \text{tr}[D_\mu U^\dagger D^\mu U] F_U(H) + \frac{1}{2} \partial_\mu H \partial^\mu H - V(H) - \frac{v}{2\sqrt{2}} \left[ \sum_{n \geq 0} y_n \left( \frac{H}{v} \right)^n (\bar{\nu}_L, \bar{\mu}_L) U (1 - \tau_3) \begin{pmatrix} \nu_R \\ \mu_R \end{pmatrix} + \text{h.c.} \right]$$

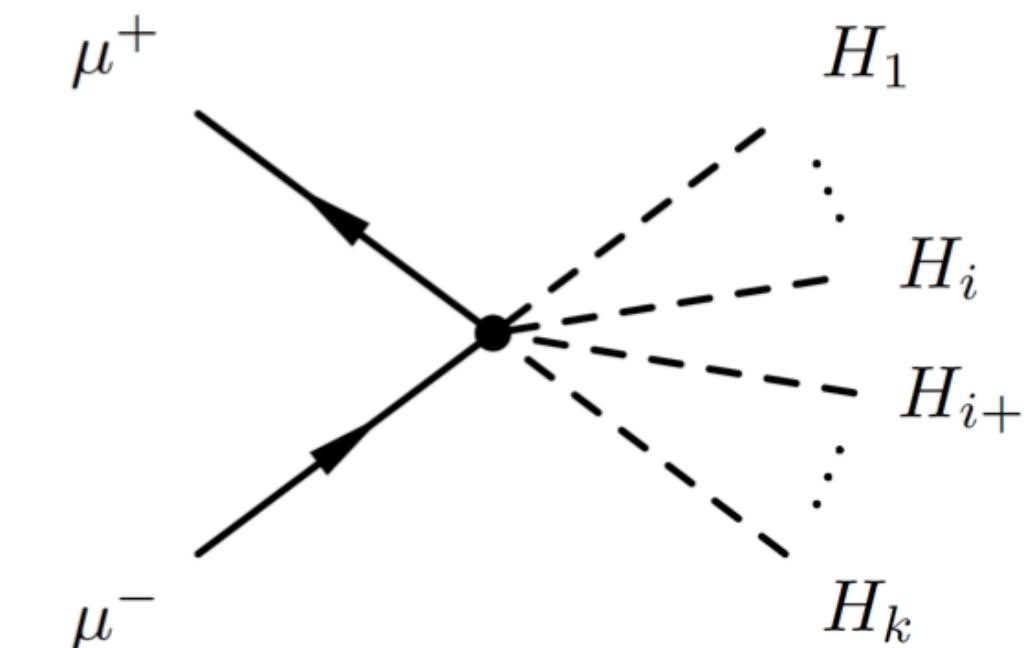
$$m_\mu = \frac{v}{\sqrt{2}} y_0$$

$$\kappa = \frac{v}{\sqrt{2} m_\mu} y_1$$

Parameterization of  $\mu$  mass and Yukawa modifier

**Extreme case:** vanishing  $\mu$  Yukawa: no pure Higgs final states at tree-level !

$$-i \frac{k!}{\sqrt{2}} \left[ Y_\ell \delta_{k,1} - \sum_{n=n_k}^{M-1} \frac{C_{\mu\varphi}^{(n)}}{\Lambda^{2n}} \binom{2n+1}{k} \frac{v^{2n+1-k}}{2^n} \right] = 0 =$$



Benchmark scenario: "matched" case

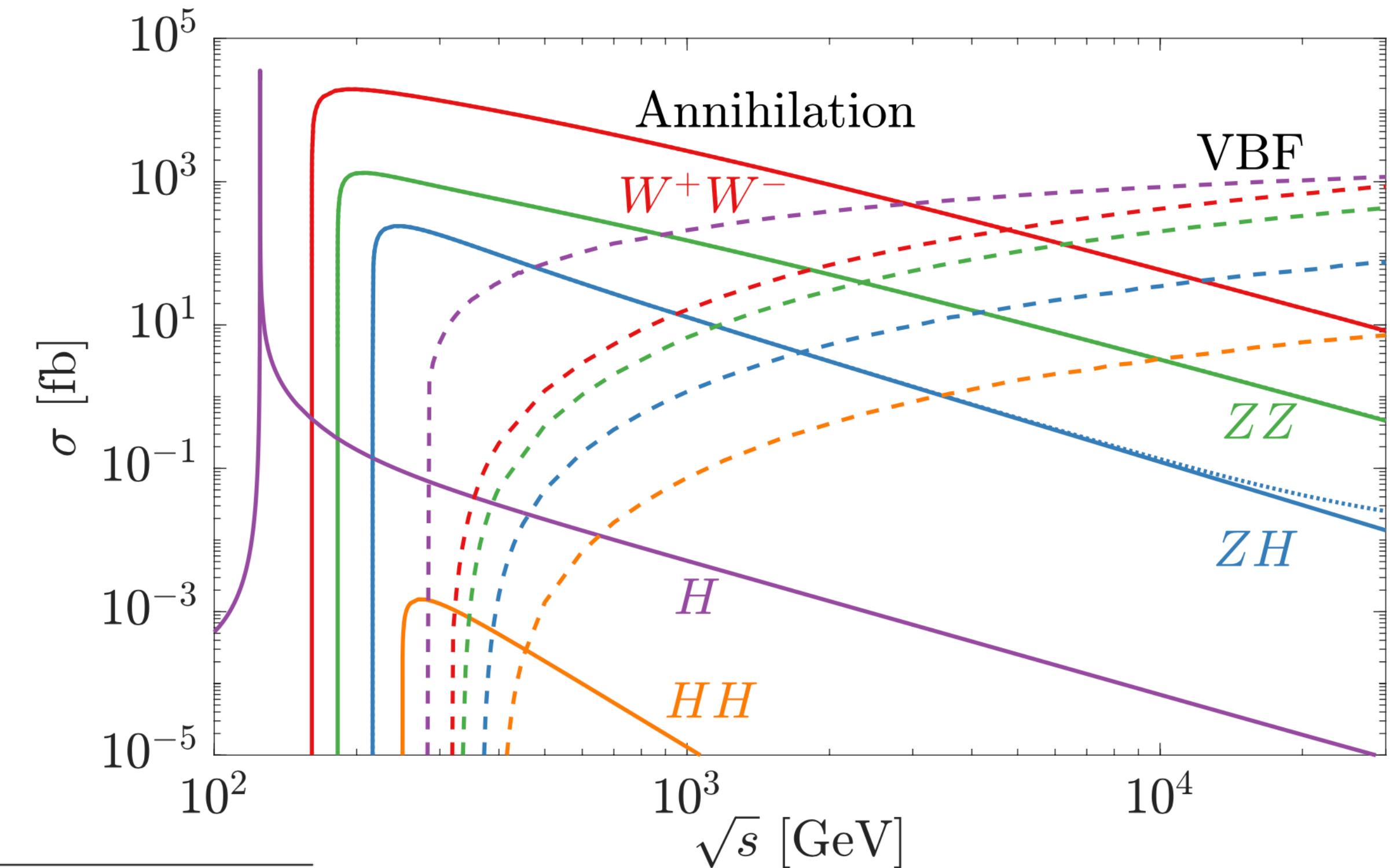
# Simulation, Consistency, Unitarity & Cross Sections

- Analytical calculations checked independently by 3 groups
- Validation of analytic calculation with 2 different MCs
- Final simulation: using UFO files in WHIZARD

## States with multiplicity 2

- ⌚ Different cases: dim 6 alone, dim 8 alone, dim 6+8 combined
- ⌚ Matched case: combination such that Yukawa coupling is zero
- ⌚ HEFT contains in principle all orders: matched is zero Yukawa

	$\Delta\sigma^X/\Delta\sigma^{W^+W^-}$					
	SMEFT			HEFT		
$X$	dim <sub>6</sub>	dim <sub>8</sub>	dim <sub>6,8</sub>	dim <sub>6,8</sub> <sup>matched</sup>	dim <sub><math>\infty</math></sub>	dim <sub><math>\infty</math></sub> <sup>matched</sup>
$W^+W^-$	1	1	1	1	1	1
$ZZ$	1/2	1/2	1/2	1/2	1/2	1/2
$ZH$	1	1/2	1	1	$R_{(2),1}^{\text{HEFT}}$	1
$HH$	9/2	25/2	$R_{(2),1}^{\text{SMEFT}}/2$	0	$2 R_{(2),2}^{\text{HEFT}}$	0



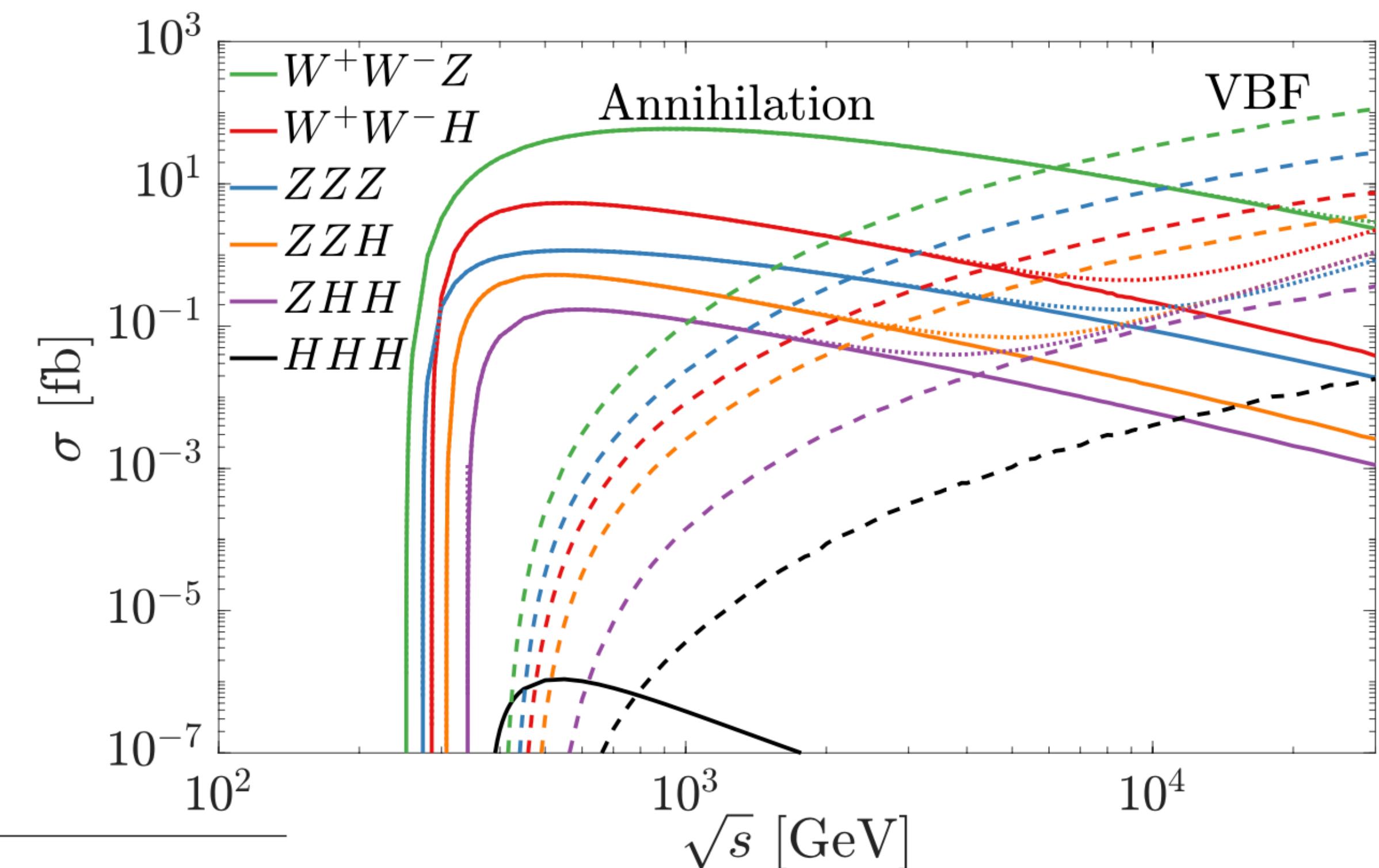
# Simulation, Consistency, Unitarity & Cross Sections

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## States with multiplicity 3

- ⌚ Different cases: dim 6 alone, dim 8 alone, dim 6+8 combined
- ⌚ Matched case: combination such that Yukawa coupling is zero
- ⌚ HEFT contains in principle all orders: matched is zero Yukawa

	$\Delta\sigma^X/\Delta\sigma^{W^+W^-H}$					
	SMEFT				HEFT	
$\mu^+\mu^- \rightarrow X$	dim <sub>6</sub>	dim <sub>8</sub>	dim <sub>6,8</sub>	dim <sub>6,8</sub> <sup>matched</sup>	dim <sub><math>\infty</math></sub>	dim <sub><math>\infty</math></sub> <sup>matched</sup>
WWZ	1	1/9	$R_{(3),1}^{\text{SMEFT}}$	1/4	$R_{(3),1}^{\text{HEFT}}/9$	1/4
ZZZ	3/2	1/6	$3 R_{(3),1}^{\text{SMEFT}}/2$	3/8	$R_{(3),1}^{\text{HEFT}}/6$	3/8
WWH	1	1	1	1	1	1
ZZH	1/2	1/2	1/2	1/2	1/2	1/2
ZHH	1/2	1/2	1/2	1/2	$2 R_{(3),2}^{\text{HEFT}}$	1/2
HHH	3/2	25/6	$3 R_{(3),2}^{\text{SMEFT}}/2$	75/8	$6 R_{(3),3}^{\text{HEFT}}$	0

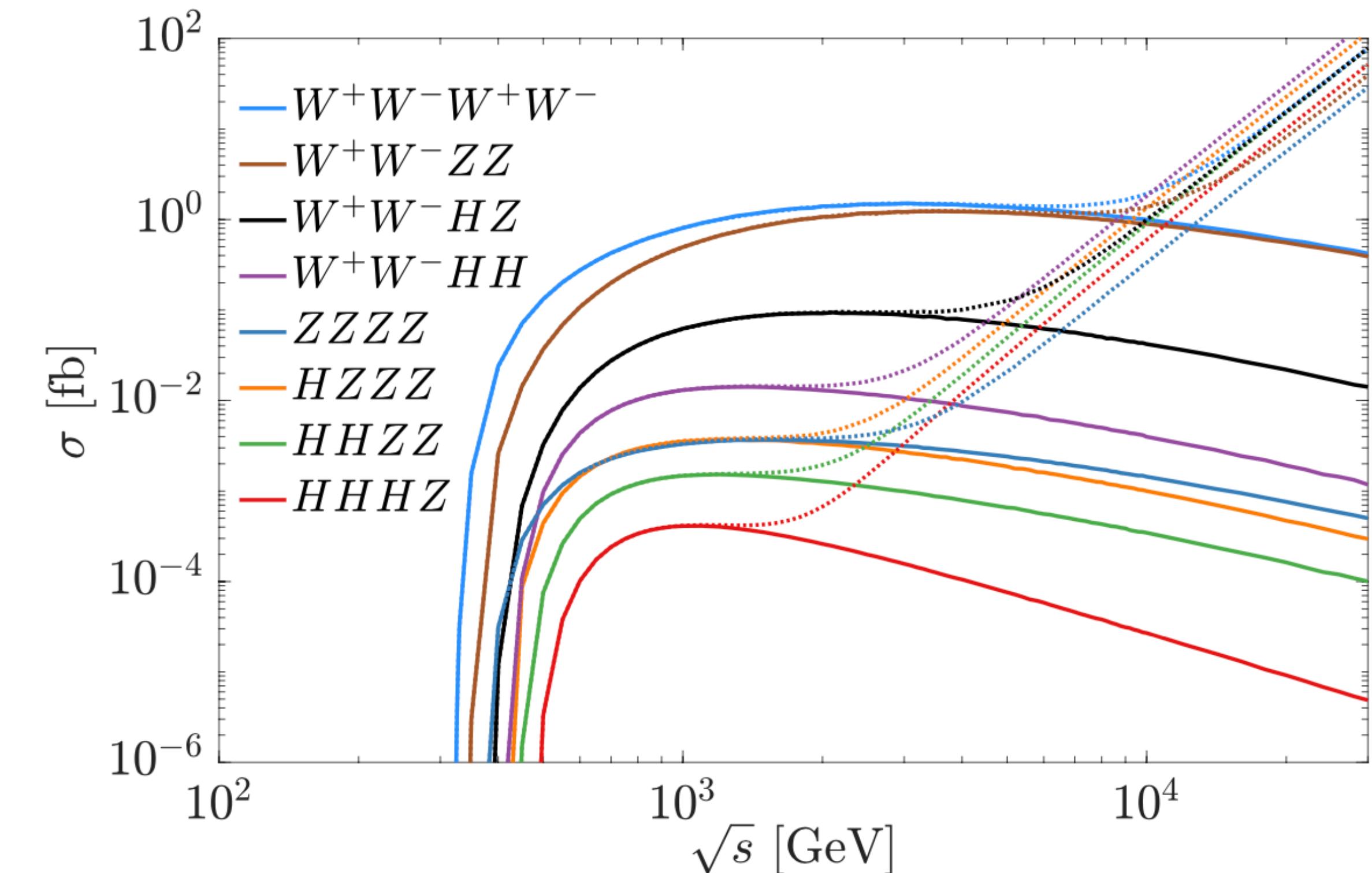


# Simulation, Consistency, Unitarity & Cross Sections

- Analytical calculations checked independently by 3 groups
- Validation of analytic calculation with 2 different MCs
- Final simulation: using UFO files in WHIZARD

## States with multiplicity 4

- ⌚ Different cases: dim 6 alone, dim 8 alone, dim 6+8 combined
- ⌚ Matched case: combination such that Yukawa coupling is zero
- ⌚ HEFT contains in principle all orders: matched is zero Yukawa



$\mu^+ \mu^- \rightarrow X$	SMEFT				HEFT	
	dim <sub>6,8</sub>	dim <sub>10</sub>	dim <sub>6,8,10</sub>	dim <sub>6,8,10</sub> <sup>matched</sup>	dim <sub>∞</sub>	dim <sub>∞</sub> <sup>matched</sup>
WWWW	2/9	2/25	$2 R_{(4),1}^{\text{SMEFT}}/9$	1/2	$R_{(4),1}^{\text{HEFT}}/18$	1/2
WWZZ	1/9	1/25	$R_{(4),1}^{\text{SMEFT}}/9$	1/4	$R_{(4),1}^{\text{HEFT}}/36$	1/4
ZZZZ	1/12	3/100	$R_{(4),1}^{\text{SMEFT}}/12$	3/16	$R_{(4),1}^{\text{HEFT}}/48$	3/16
WWZH	2/9	2/25	$2 R_{(4),1}^{\text{SMEFT}}/9$	1/2	$R_{(4),2}^{\text{HEFT}}/8$	1/2
WWHH	1	1	1	1	1	1
ZZZH	1/3	3/25	$R_{(4),1}^{\text{SMEFT}}/3$	3/4	$R_{(4),2}^{\text{HEFT}}/12$	3/4
ZZHH	1/2	1/2	1/2	1/2	1/2	1/2
ZHHH	1/3	1/3	1/3	1/3	$3 R_{(4),3}^{\text{HEFT}}$	1/3
HHHH	25/12	49/12	$25 R_{(4),2}^{\text{SMEFT}}/12$	1225/48	$12 R_{(4),4}^{\text{HEFT}}$	0

# Simulation, Consistency, Unitarity & Cross Sections

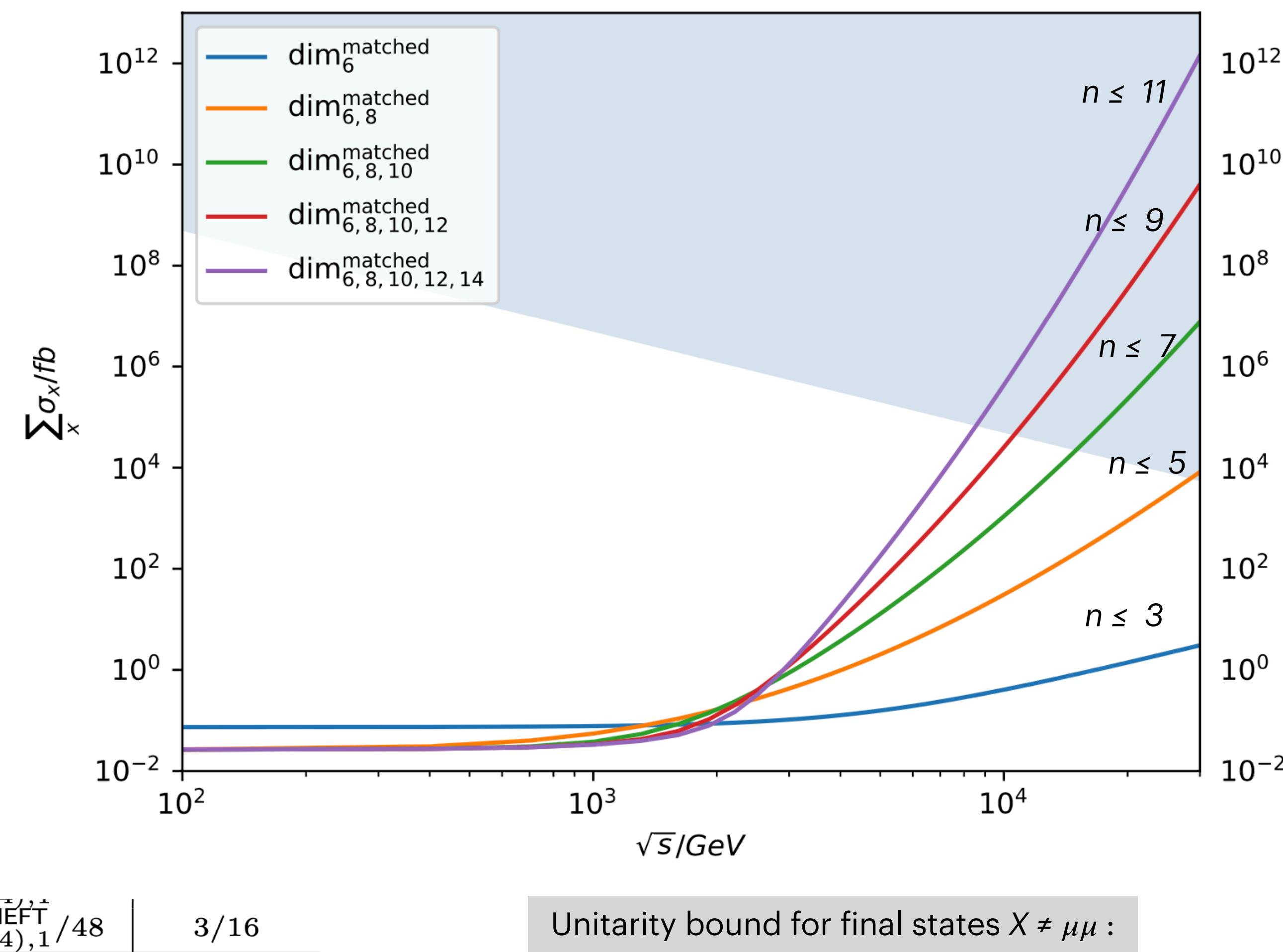
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## States with multiplicity 4

- ⌚ Different cases: dim 6 alone, dim 8 alone, dim 6+8 combined
- ⌚ Matched case: combination such that Yukawa coupling is zero
- ⌚ HEFT contains in principle all orders: matched is zero Yukawa

SMEFT						
$\mu^+ \mu^- \rightarrow X$	dim <sub>6,8</sub>	dim <sub>10</sub>	dim <sub>6,8,10</sub>	dim <sub>6,8,10</sub> <sup>matched</sup>	R	
WWWW	2/9	2/25	$2 R_{(4),1}^{\text{SMEFT}}/9$	1/2	$R_{(4),1}/48$	3/16
WWZZ	1/9	1/25	$R_{(4),1}^{\text{SMEFT}}/9$	1/4	$R_{(4),2}^{\text{HEFT}}/8$	1/2
ZZZZ	1/12	3/100	$R_{(4),1}^{\text{SMEFT}}/12$	3/16	$R_{(4),2}^{\text{HEFT}}/12$	3/4
WWZH	2/9	2/25	$2 R_{(4),1}^{\text{SMEFT}}/9$	1/2	$R_{(4),3}^{\text{HEFT}}/12$	1/2
WWHH	1	1	1	1	1	1
ZZZH	1/3	3/25	$R_{(4),1}^{\text{SMEFT}}/3$	3/4	$1/2$	3/4
ZZHH	1/2	1/2	1/2	1/2	1/2	1/2
ZHHH	1/3	1/3	1/3	1/3	$3 R_{(4),3}^{\text{HEFT}}$	1/3
HHHH	25/12	49/12	$25 R_{(4),2}^{\text{SMEFT}}/12$	1225/48	$12 R_{(4),4}^{\text{HEFT}}$	0



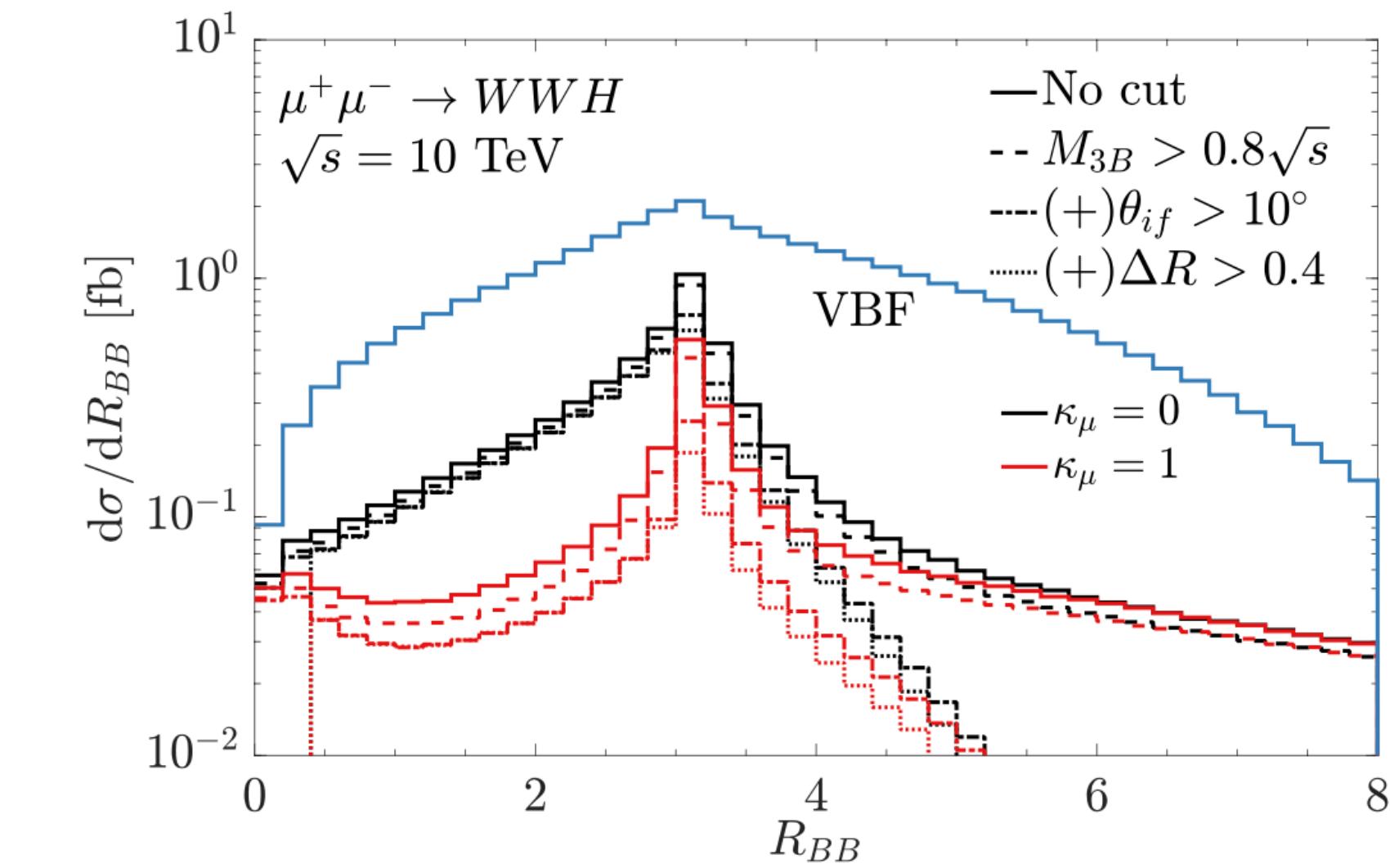
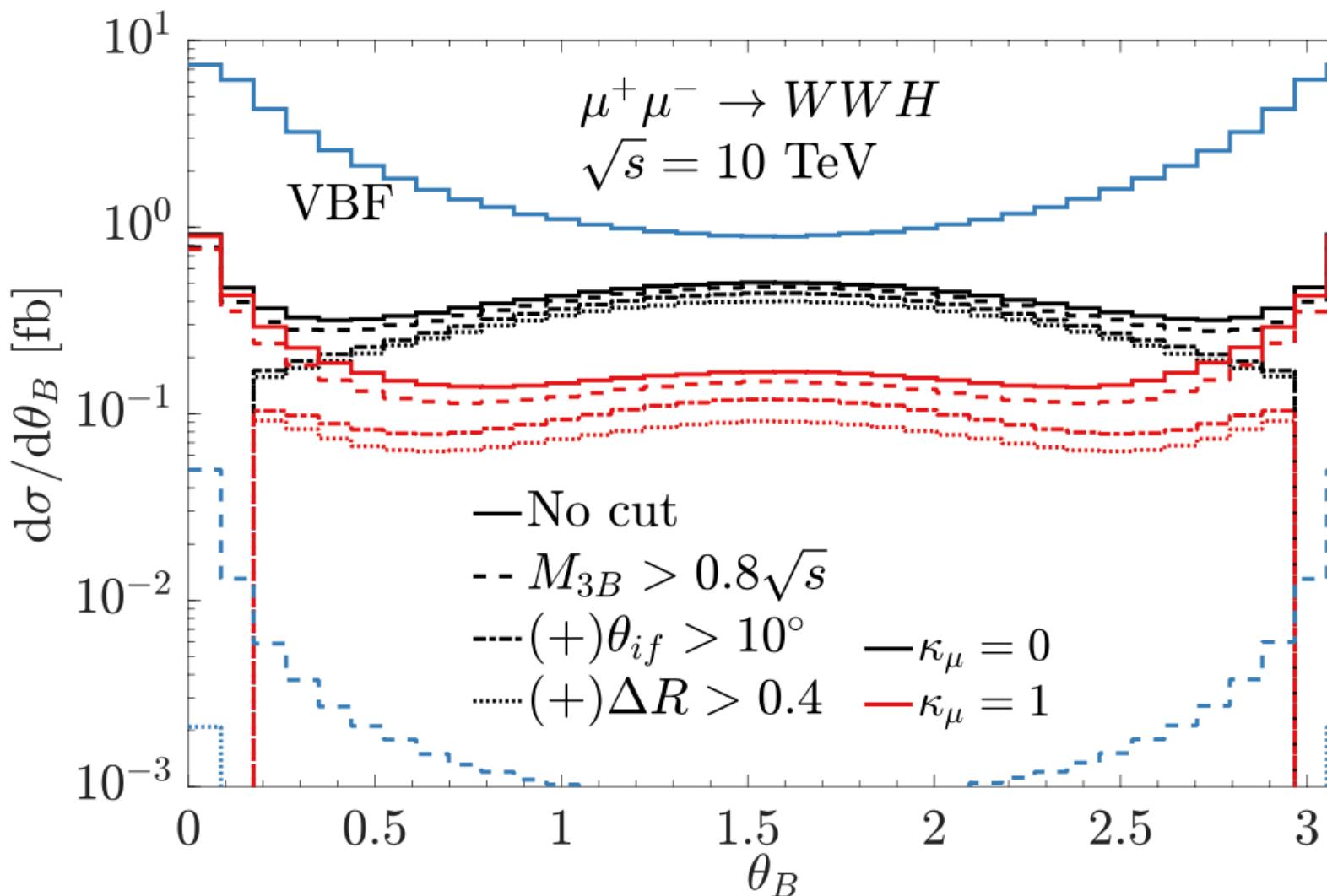
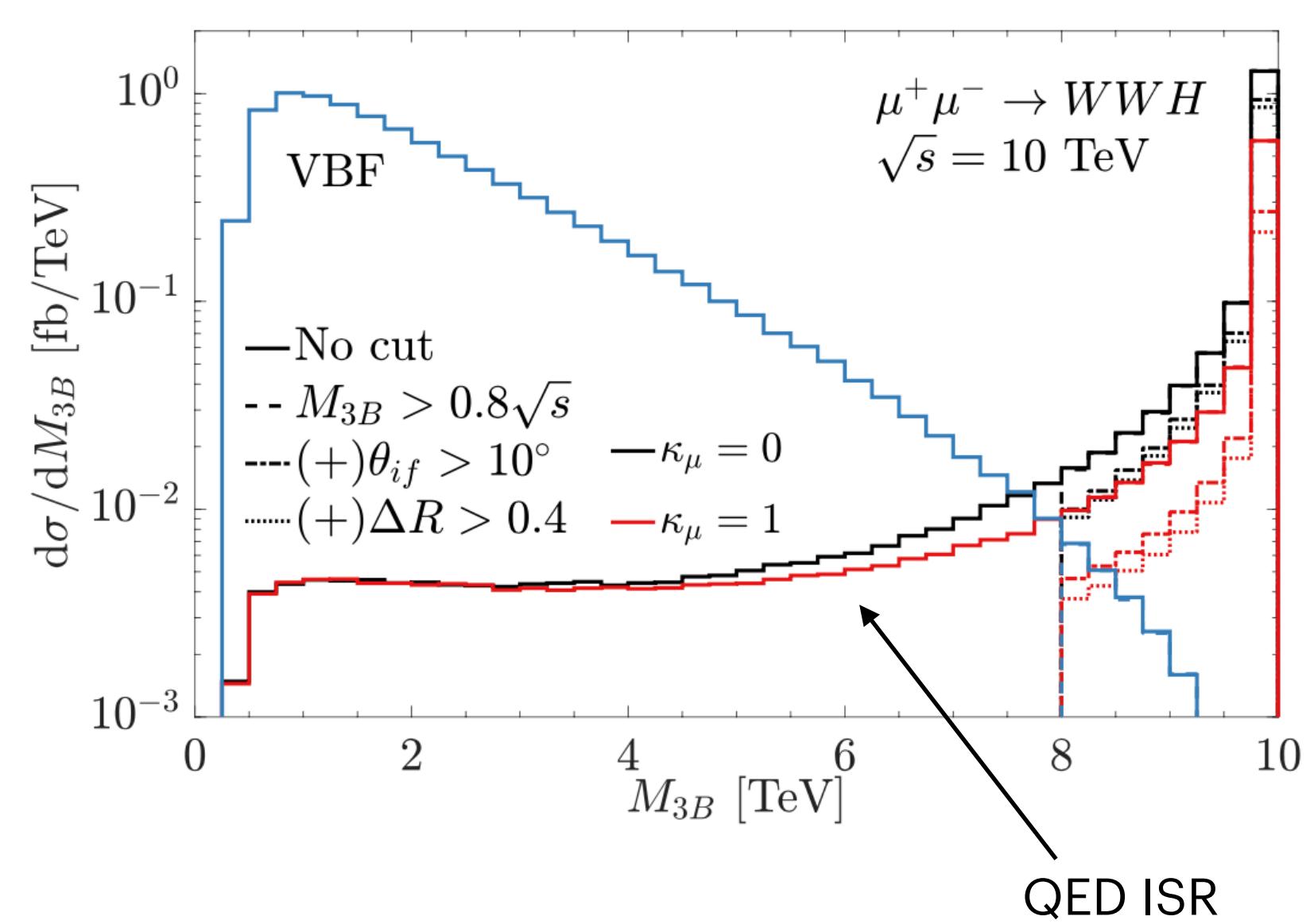
$$\sum_X \sigma_{\mu^+ \mu^- \rightarrow X}(s) \leq \frac{4\pi}{s}$$

hep-ph/0106281

# Kinematic separation of signal

$\mu^+ \mu^- \rightarrow W^+ W^- H$

Kinematic separation between multi-boson direct production and VBF, e.g. 10 TeV:



- WWZ largest cross section, but small deviation
- WWH large cross section and considerable deviation
- ZZH smaller/-ish cross section, but largest (relative) deviation
- Direct production has almost full energy (except for ISR)  $\Rightarrow M_{3B}$
- VBF generates mostly forward bosons  $\Rightarrow \theta_B$
- Separation criterion for final state bosons  $\Rightarrow \Delta R_{BB}$

Cut flow	$\kappa_\mu = 1$	w/o ISR	$\kappa_\mu = 0$ (2)	CVBF	NVBF
$\sigma$ [fb]	<b>WWH</b>				
No cut	0.24	0.21	0.47	2.3	7.2
$M_{3B} > 0.8\sqrt{s}$	0.20	0.21	0.42	$5.5 \cdot 10^{-3}$	$3.7 \cdot 10^{-2}$
$10^\circ < \theta_B < 170^\circ$	0.092	0.096	0.30	$2.5 \cdot 10^{-4}$	$2.7 \cdot 10^{-4}$
$\Delta R_{BB} > 0.4$	0.074	0.077	0.28	$2.1 \cdot 10^{-4}$	$2.4 \cdot 10^{-4}$
# of events	740	770	2800	2.1	2.4
$S/B$	2.8				

# Results and final projections

Muon collider with energy range  $1 < \sqrt{s} < 30$  TeV and luminosity  $\mathcal{L} = \left(\frac{\sqrt{s}}{10 \text{ TeV}}\right)^2 10 \text{ ab}^{-1}$

[1901.06150](#); [2001.04431](#);  
PoS(ICHEP2020)703; Nat.Phys.17, 289-292

- Sensitivity to (deviations of) the muon Yukawa coupling
- Definition of # signal events:  $S = N_{\kappa_\mu} - N_{\kappa_\mu=1}$
- Definition of # background events:  $B = N_{\kappa_\mu=1} + N_{\text{VBF}}$
- Statistical significance of anom. muon Yukawa couplings:

$$\mathcal{S} = \frac{S}{\sqrt{B}}$$

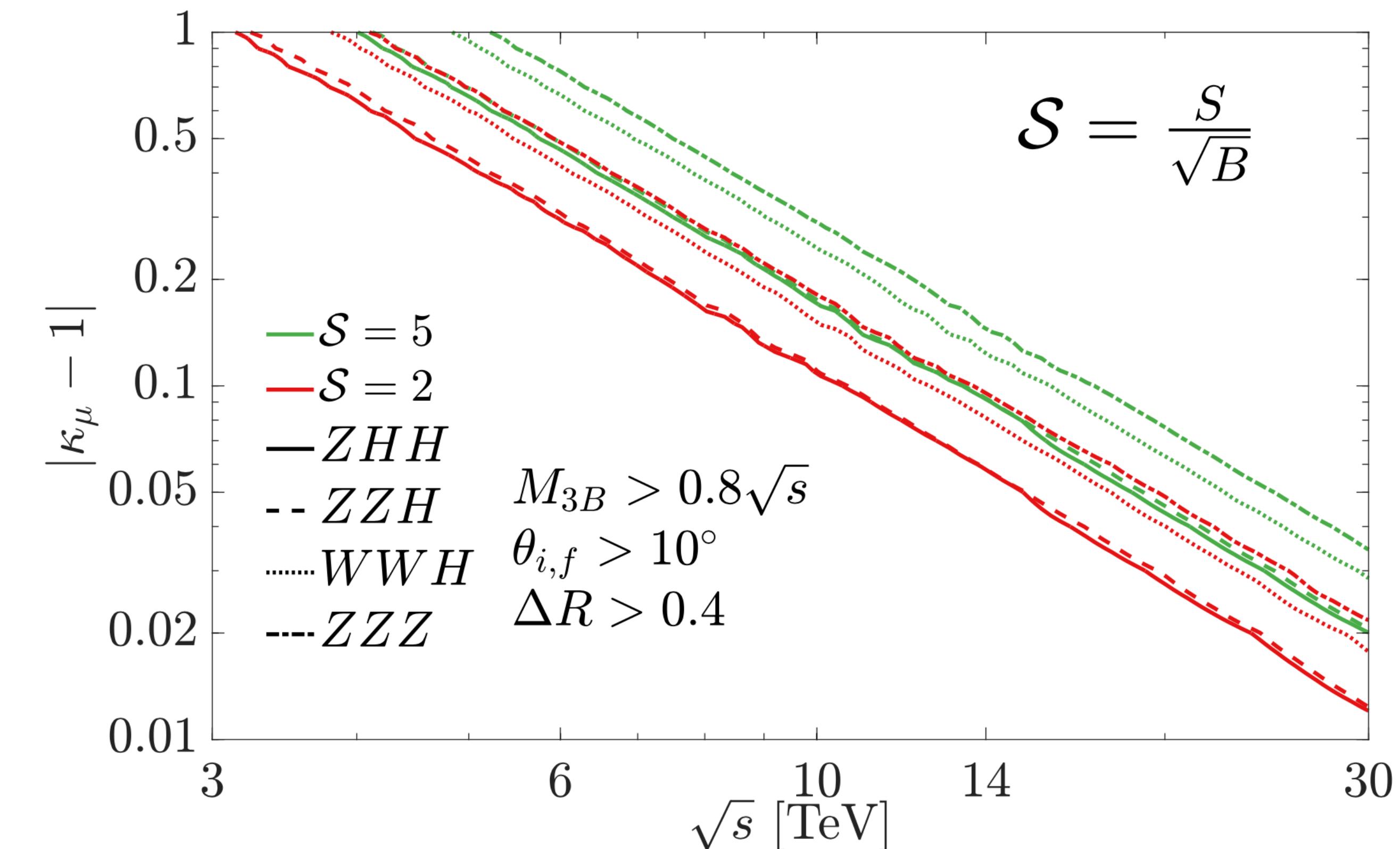
(note that always:  $N_{\kappa_\mu} \geq N_{\kappa_\mu=1}$ )

$$\sigma|_{\kappa_\mu=1+\delta} = \sigma|_{\kappa_\mu=1-\delta}; \quad \Rightarrow \quad \mathcal{S}|_{\kappa_\mu=1+\delta} = \mathcal{S}|_{\kappa_\mu=1-\delta}$$

⌚ 5 $\sigma$  sensitivity to 20% @ 10 TeV ... 2% @ 30 TeV

⌚ Sensitivity to  $\kappa$  translates to new physics scale  $\Lambda$

$$\Lambda > 10 \text{ TeV} \sqrt{\frac{g}{\Delta \kappa_\mu}}$$



# Conclusions & Outlook

- ⦿ Muon collider highly interesting Energy Frontier option
- ⦿ Recent technological progress: muon cooling, beam dump etc.
- ⦿ Huge potential for Higgs and electroweak physics as well as BSM sensitivity (multi bosons)
- ⦿ Example: sensitivity to anomalous muon Yukawa couplings
- ⦿ Deviations grow with number of final state (EW/Higgs) bosons
- ⦿ Optimal: tri-boson processes (diboson less sensitivity, quartic bosons smaller xsec.)
- ⦿ Separation direct production from VBF:  $BBB$  invariant mass and  $B$  angular cuts
- ⦿ **Muon Yukawa coupling testable with sensitivity 20% @ 10 TeV .... 2% @ 30 TeV**
- ⦿ Translates to  $5\sigma$  sensitivities to new physics of  $\Lambda \sim 20 - 70$  TeV
- ⦿ Work in progress: multi-Higgs final states & trilinear/quartic Higgs coupling

# Multi-Boson Interactions (MBI) 2022

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Hybrid format: T.D. Lee Institute Shanghai, 22.-25.8.2022

<https://indico.cern.ch/event/932480/>

The screenshot shows the homepage of the conference website. It features a large image of the Shanghai skyline at sunset. To the right of the image, the text "Multi-Boson Interactions 2022" is displayed. Below the image, the dates "Aug 22 – 25, 2022" and the location "Tsung-Dao Lee Institute" are listed. A search bar with the placeholder "Enter your search term" and a magnifying glass icon is located at the bottom right. On the left side, there is a sidebar with links for "Overview", "Timetable", and "Contribution List".

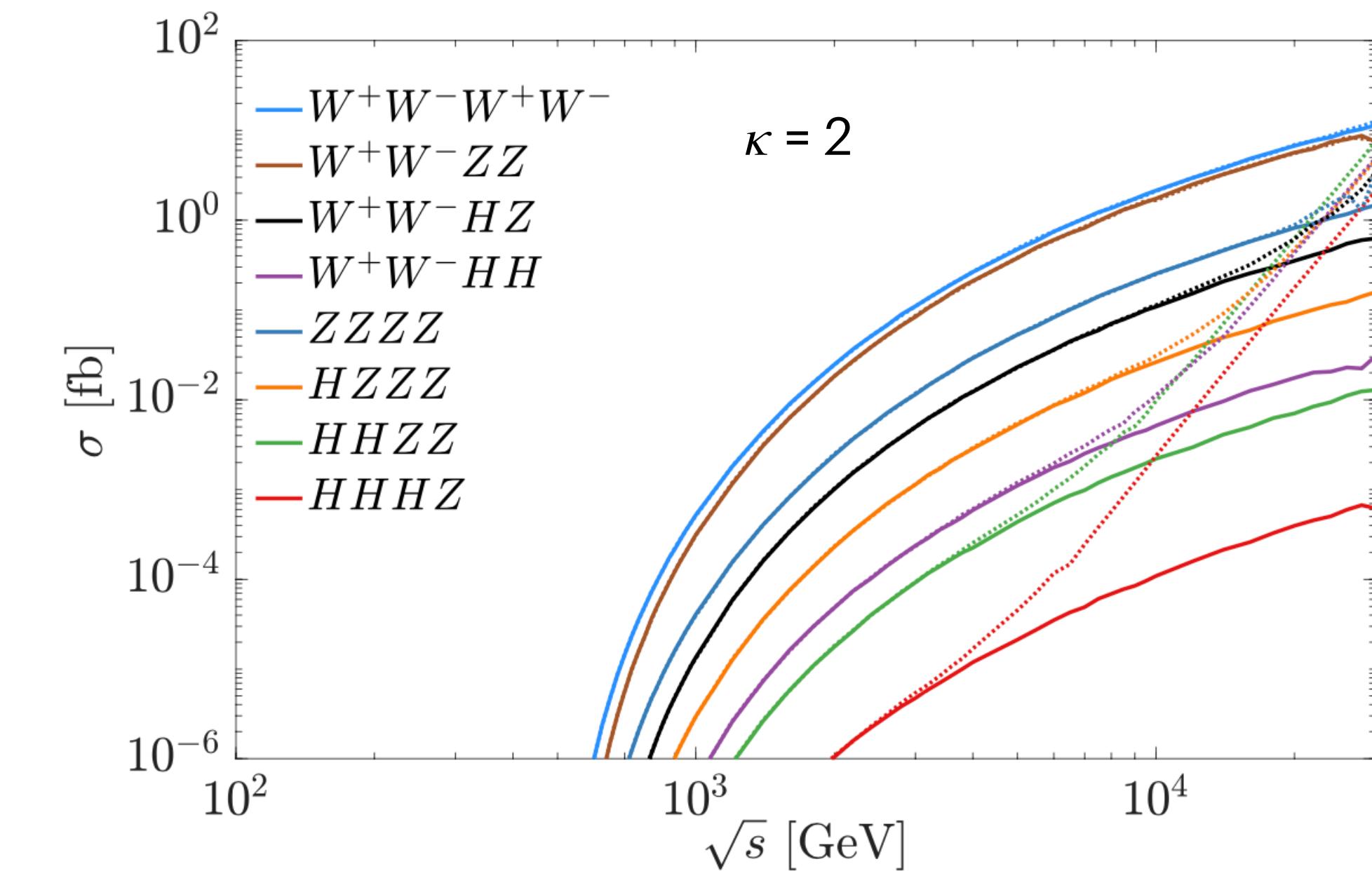
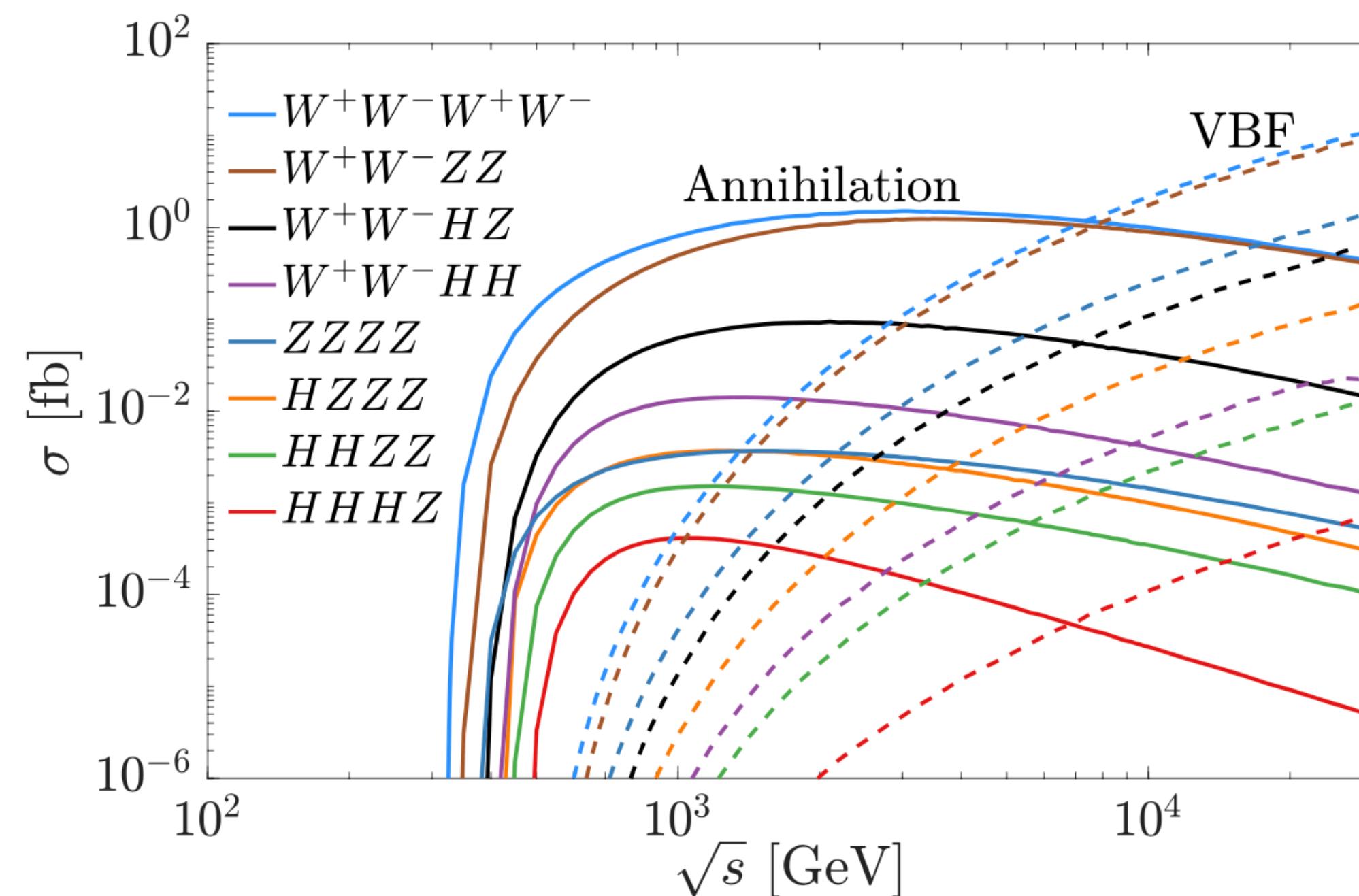
**Theory speakers include:** [I. Brivio](#), [A. Carmona](#), [R. Gomez](#), [J. Gu](#), [M. Mangano](#), [M. McCullough](#), [B. Mistlberger](#), [D. Pagani](#), [M. Pellen](#), [R. Poncelet](#), [P. Skands](#), [A. Wulzer](#), [K. Xie](#), [T. Yanagida](#)



# BACKUP

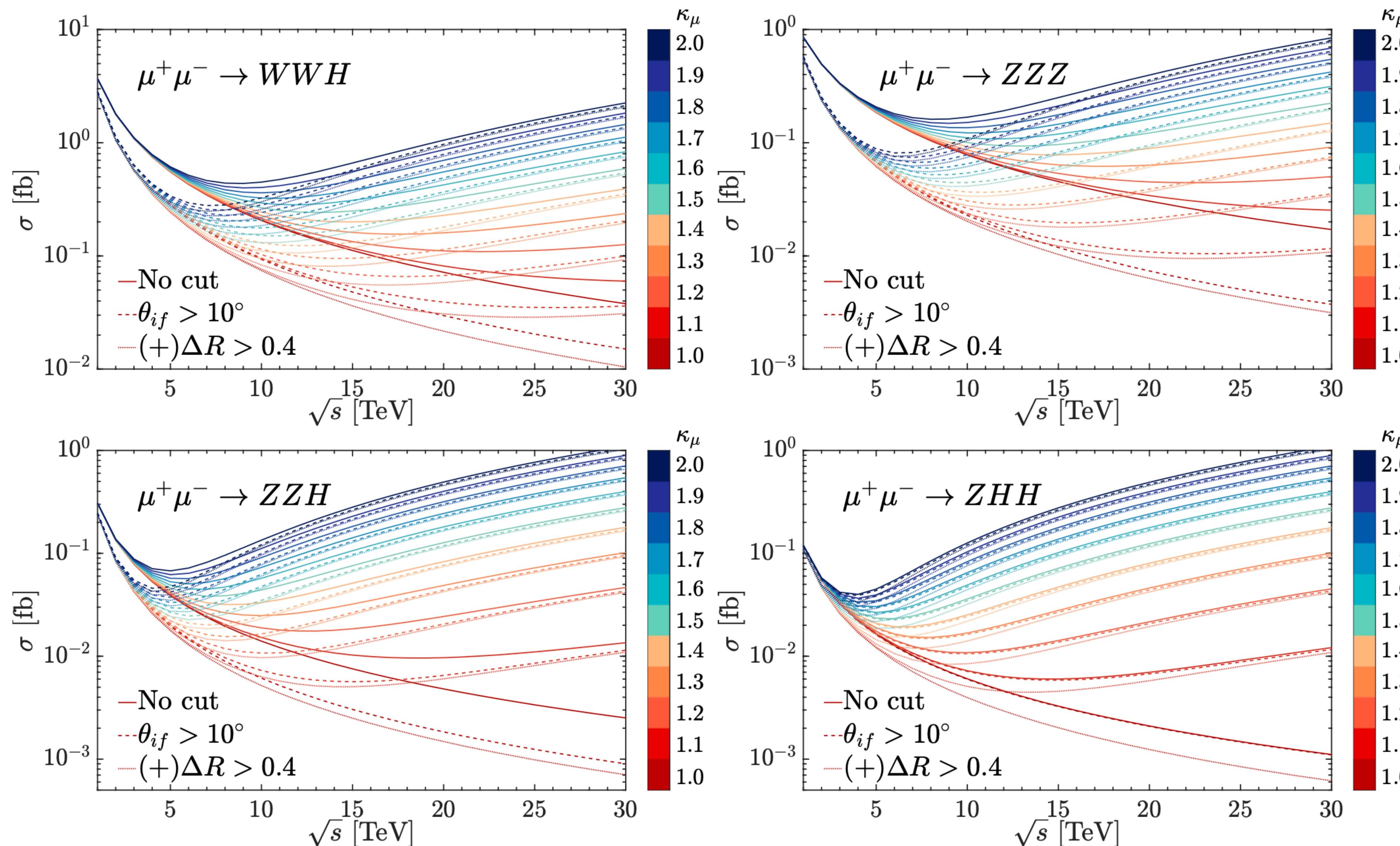
# Additional cross sections

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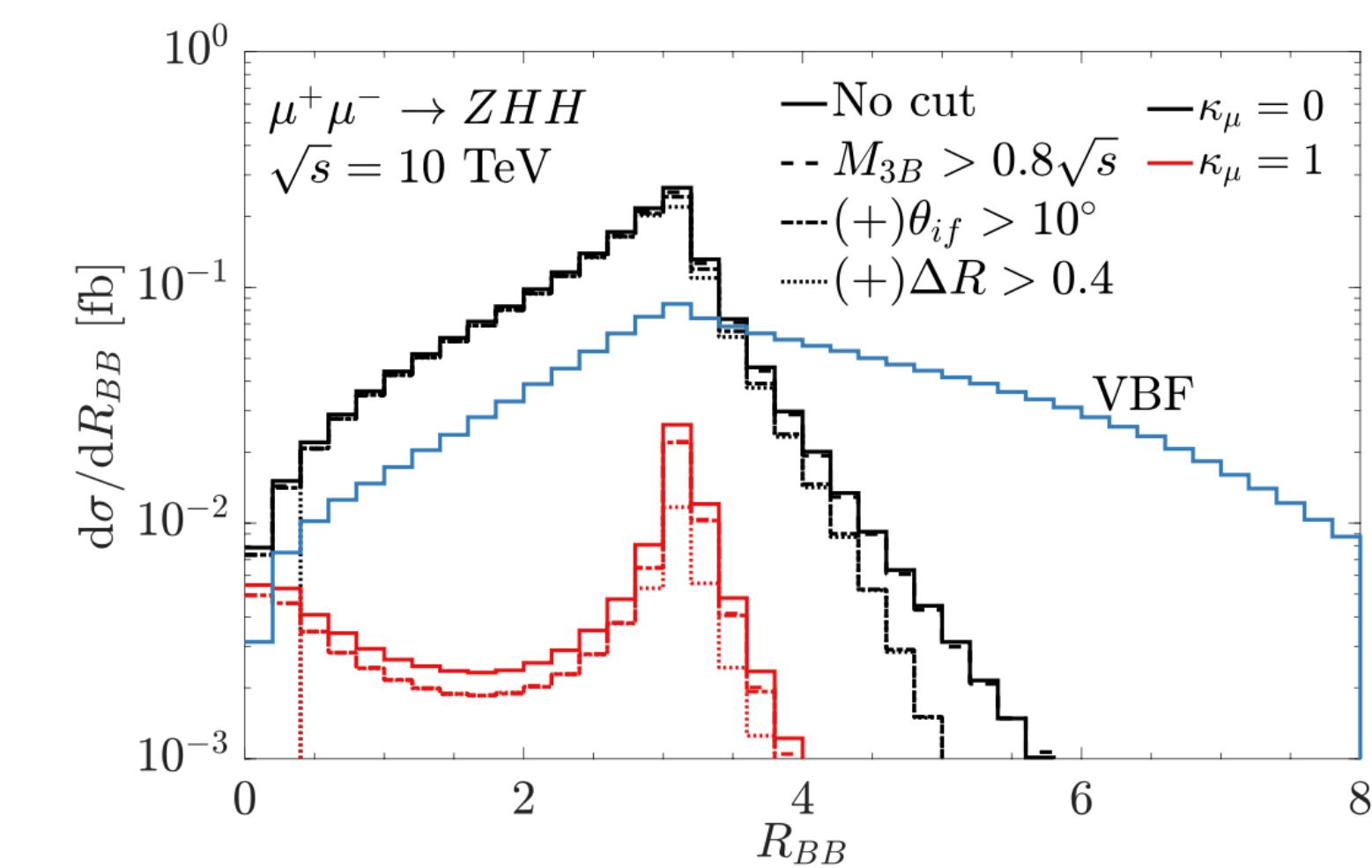
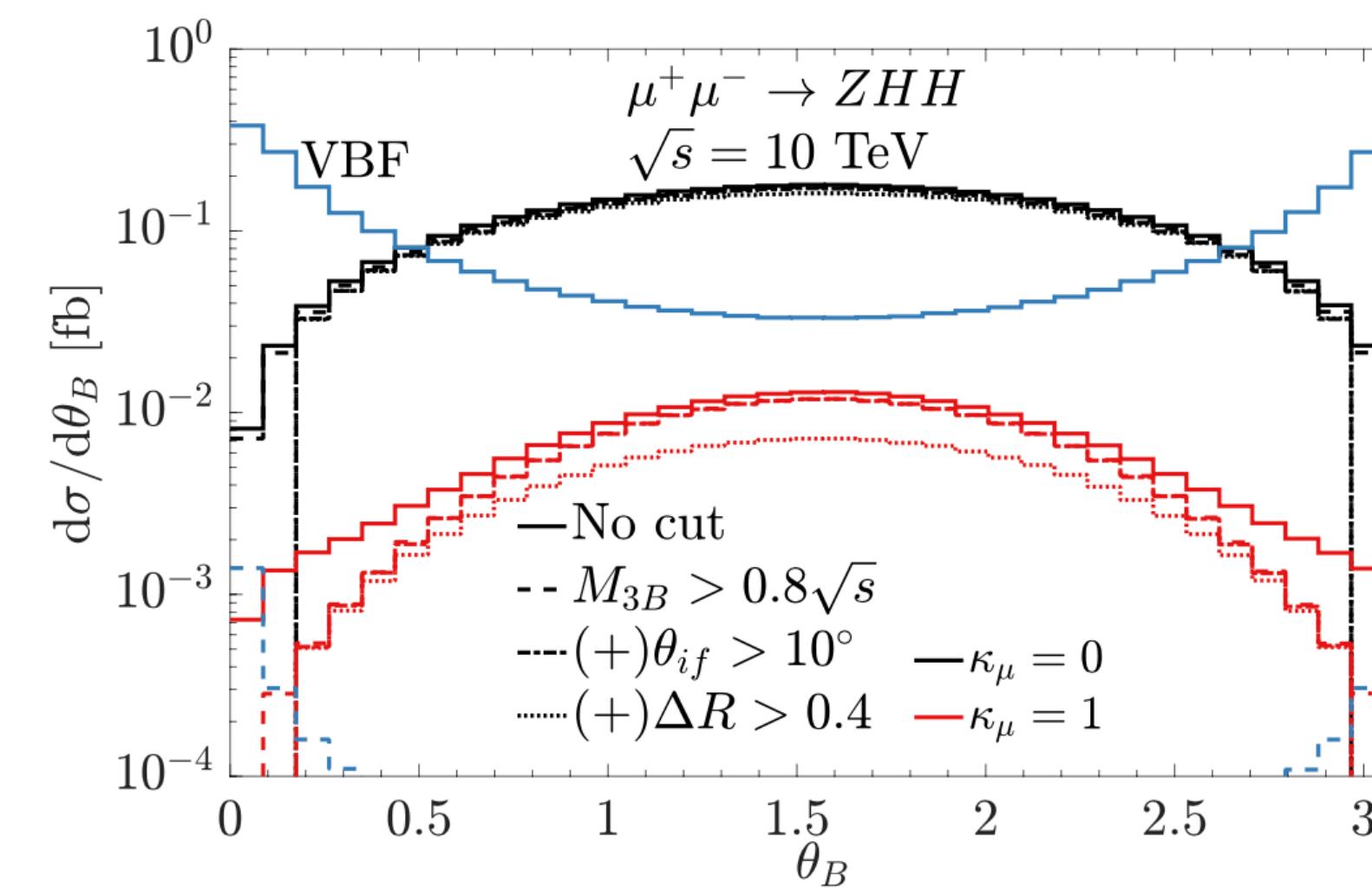
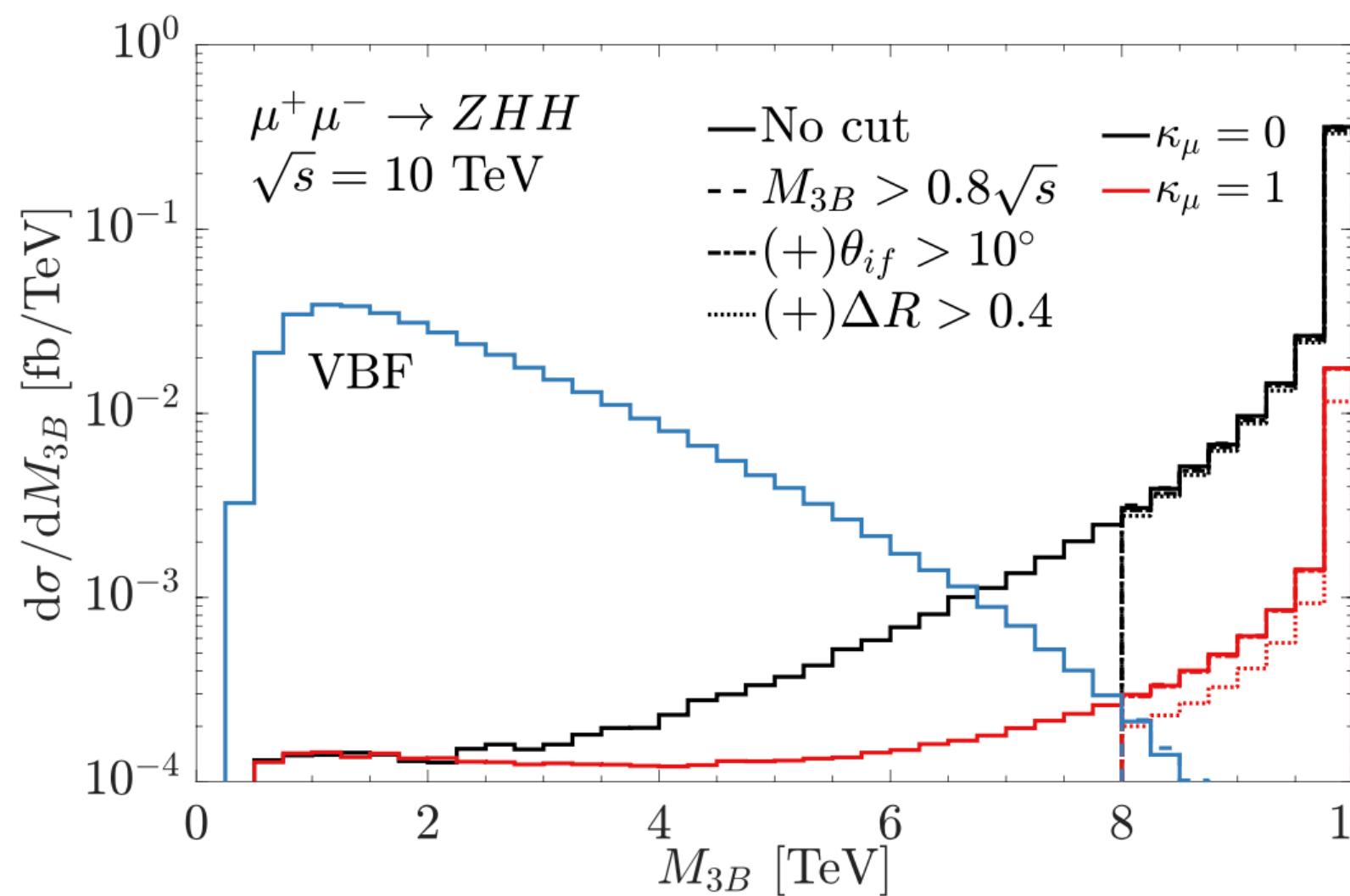


# Variations of cross sections with $\kappa$

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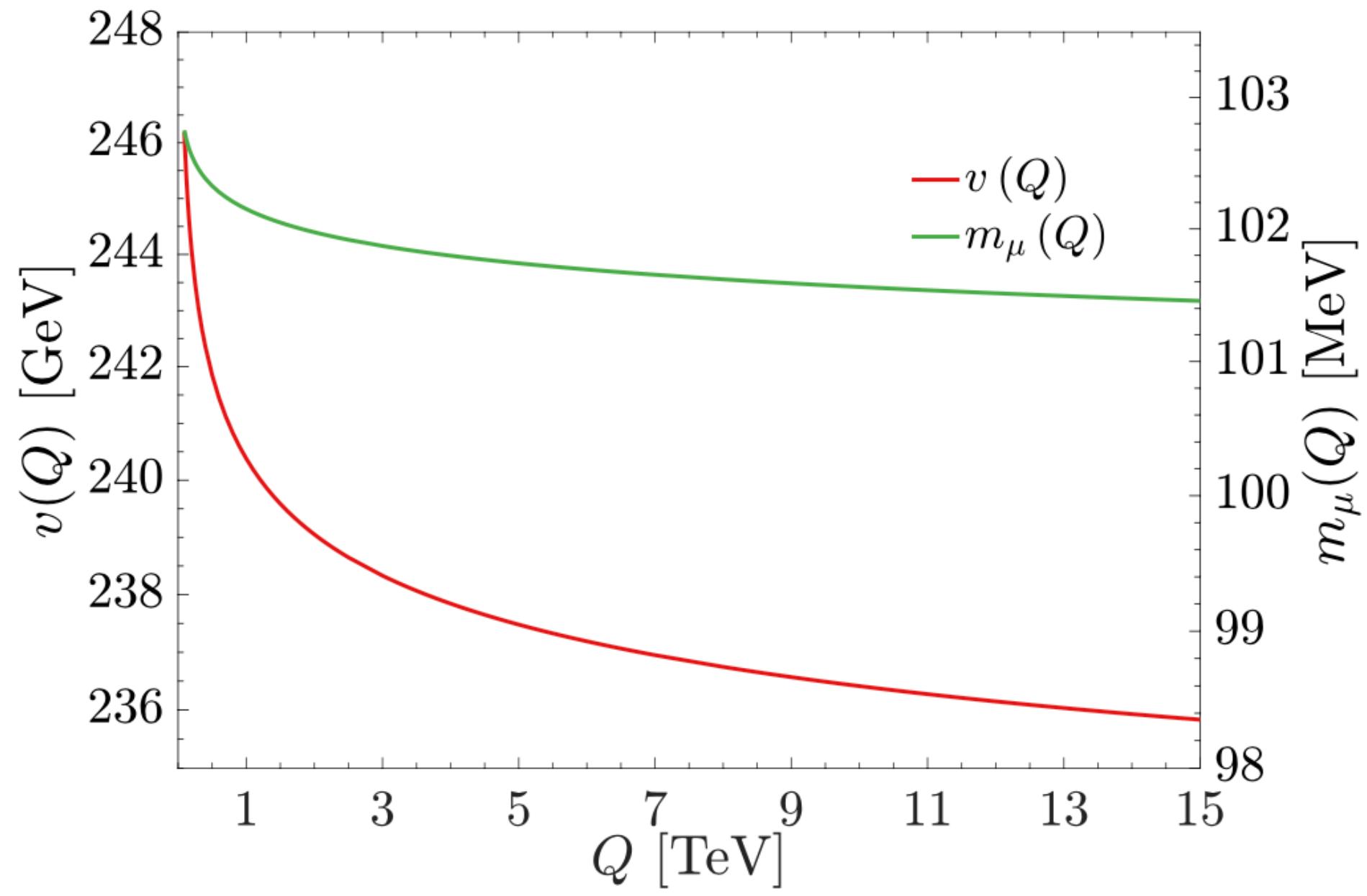
# Kinematic separation of signal

 $\mu^+ \mu^- \rightarrow ZZH$ 


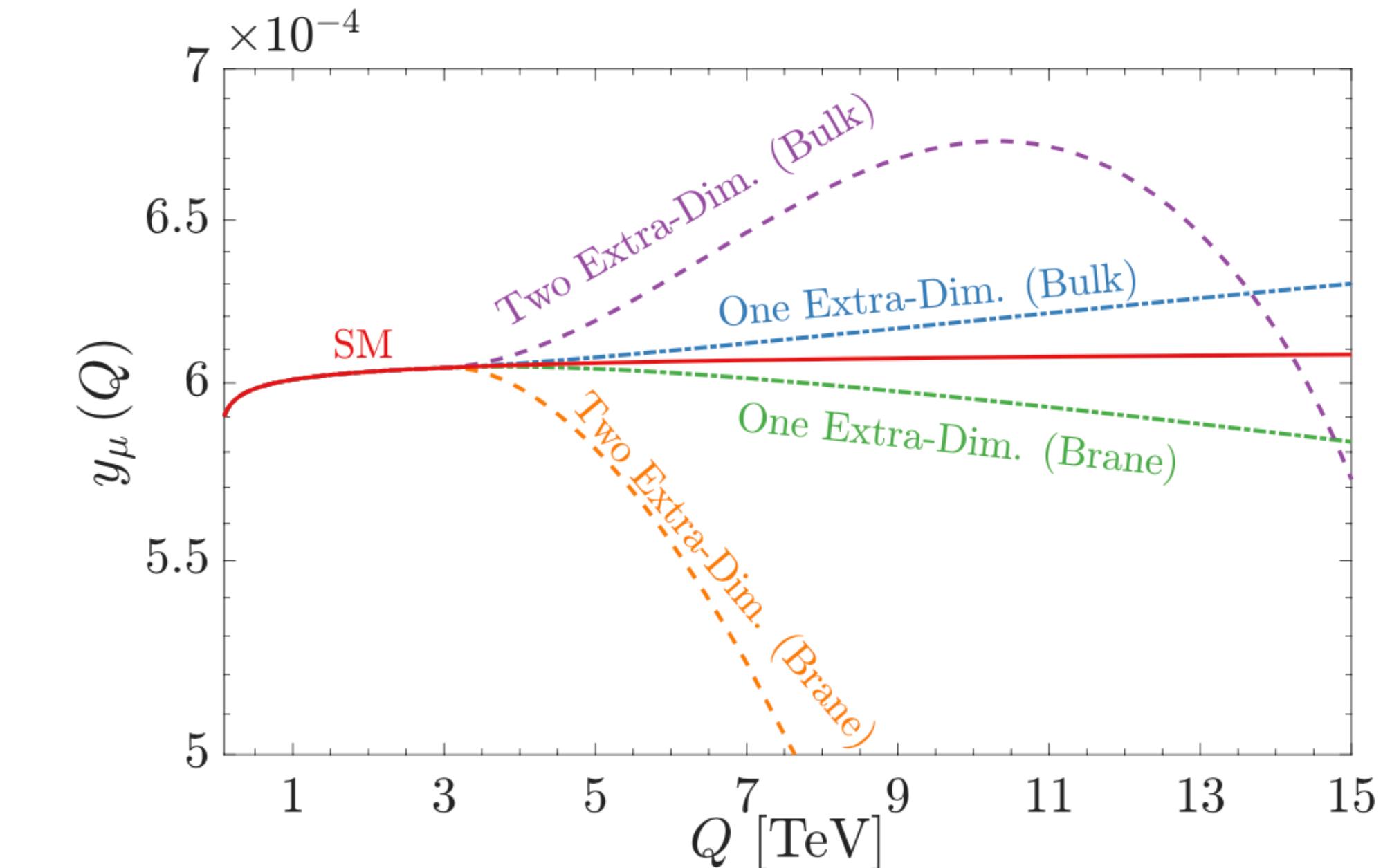
$\sigma$ [fb]	ZHH				
	6.9 · 10 <sup>-3</sup>	6.1 · 10 <sup>-3</sup>	0.119	9.6 · 10 <sup>-2</sup>	6.7 · 10 <sup>-4</sup>
No cut	6.9 · 10 <sup>-3</sup>	6.1 · 10 <sup>-3</sup>	0.119	9.6 · 10 <sup>-2</sup>	6.7 · 10 <sup>-4</sup>
$M_{3B} > 0.8\sqrt{s}$	5.9 · 10 <sup>-3</sup>	6.1 · 10 <sup>-3</sup>	0.115	1.5 · 10 <sup>-4</sup>	7.4 · 10 <sup>-6</sup>
$10^\circ < \theta_B < 170^\circ$	5.7 · 10 <sup>-3</sup>	6.0 · 10 <sup>-3</sup>	0.110	8.8 · 10 <sup>-6</sup>	7.5 · 10 <sup>-7</sup>
$\Delta R_{BB} > 0.4$	3.8 · 10 <sup>-3</sup>	4.0 · 10 <sup>-3</sup>	0.106	8.0 · 10 <sup>-6</sup>	5.6 · 10 <sup>-7</sup>
# of events	38	40	1060	—	—
$S/B$	27				

# Running of muon Yukawa

VeV and muon mass in the SM



Muon Yukawa in different BSM models



$$\beta_{y_t} = \frac{dy_t}{dt} = \frac{y_t}{16\pi^2} \left( \frac{9}{2}y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2 \right),$$

$$\beta_{y_\mu} = \frac{dy_\mu}{dt} = \frac{y_\mu}{16\pi^2} \left( 3y_t^2 - \frac{9}{4}(g_2^2 + g_1^2) \right),$$

$$\beta_v = \frac{dv}{dt} = \frac{v}{16\pi^2} \left( \frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2 \right),$$

$$\beta_{g_i} = \frac{dg_i}{dt} = \frac{b_i g_i^3}{16\pi^2},$$

$$b_i^{\text{SM}} = (41/10, -19/6, -7)$$

$$\frac{dy_t}{dt} = \beta_{y_t}^{\text{SM}} + \frac{y_t}{16\pi^2} 2(S(t) - 1) \left( \frac{3}{2}y_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2 \right),$$

$$\frac{dy_\mu}{dt} = \beta_{y_\mu}^{\text{SM}} - \frac{y_\mu}{16\pi^2} 2(S(t) - 1) \left( \frac{9}{4}g_2^2 + \frac{9}{4}g_1^2 \right),$$

$$\frac{dy_t}{dt} = \beta_{y_t}^{\text{SM}} + \frac{y_t}{16\pi^2} (S(t) - 1) \left( \frac{15}{2}y_t^2 - \frac{28}{3}g_3^2 - \frac{15}{8}g_2^2 - \frac{101}{120}g_1^2 \right),$$

$$\frac{dy_\mu}{dt} = \beta_{y_\mu}^{\text{SM}} + \frac{y_\mu}{16\pi^2} (S(t) - 1) \left( 6y_t^2 - \frac{15}{8}g_2^2 - \frac{99}{40}g_1^2 \right),$$

5D Brane,

5D Brane,

5D Bulk,

5D Bulk.

# Collection of useful formulæ

Unitarity violation for operator insertions at  $d = 6, 8, 10$ :

corresponds to 95 TeV, 17 TeV, 11 TeV, respectively

$$\Lambda_d = 4\pi \kappa_d \left( \frac{v^{d-3}}{m_\mu} \right)^{1/(d-4)}, \quad \text{where} \quad \kappa_d = \left( \frac{(d-5)!}{2^{d-5}(d-3)} \right)^{1/(2(d-4))}$$


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$$R_{(3),1}^{\text{SMEFT}} = \left( \frac{v^2 c_{\ell\varphi}^{(2)} + c_{\ell\varphi}^{(1)}}{3v^2 c_{\ell\varphi}^{(2)} + c_{\ell\varphi}^{(1)}} \right)^2, \quad R_{(3),2}^{\text{SMEFT}} = \left( \frac{5v^2 c_{\ell\varphi}^{(2)} + c_{\ell\varphi}^{(1)}}{3v^2 c_{\ell\varphi}^{(2)} + c_{\ell\varphi}^{(1)}} \right)^2$$

$$m_\mu^{(8)} = \frac{v}{\sqrt{2}} \left( y_\mu - \frac{v^2}{2} c_{\ell\varphi}^{(1)} - \frac{v^4}{4} c_{\ell\varphi}^{(2)} \right),$$

$$\lambda_\mu^{(8)} = \left( y_\mu - \frac{3v^2}{2} c_{\ell\varphi}^{(1)} - \frac{5v^4}{4} c_{\ell\varphi}^{(2)} \right),$$

$$R_{(3),1}^{\text{HEFT}} = \left( \frac{y_\mu}{y_1} \right)^2, \quad R_{(3),2}^{\text{HEFT}} = \left( \frac{y_2}{y_1} \right)^2, \quad R_{(3),3}^{\text{HEFT}} = \left( \frac{y_3}{y_1} \right)^2$$

$$R_{(4),1}^{\text{SMEFT}} = \left( \frac{3v^2 c_{\ell\varphi}^{(3)} + 2c_{\ell\varphi}^{(2)}}{5v^2 c_{\ell\varphi}^{(3)} + 2c_{\ell\varphi}^{(2)}} \right)^2, \quad R_{(4),2}^{\text{SMEFT}} = \left( \frac{7v^2 c_{\ell\varphi}^{(3)} + 2c_{\ell\varphi}^{(2)}}{5v^2 c_{\ell\varphi}^{(3)} + 2c_{\ell\varphi}^{(2)}} \right)^2$$

$$R_{(4),1}^{\text{HEFT}} = \left( \frac{y_\mu}{y_2} \right)^2, \quad R_{(4),2}^{\text{HEFT}} = \left( \frac{y_1}{y_2} \right)^2, \quad R_{(4),3}^{\text{HEFT}} = \left( \frac{y_3}{y_2} \right)^2, \quad R_{(4),4}^{\text{HEFT}} = \left( \frac{y_4}{y_2} \right)^2$$

