Higgs Alignment and the Top Quark

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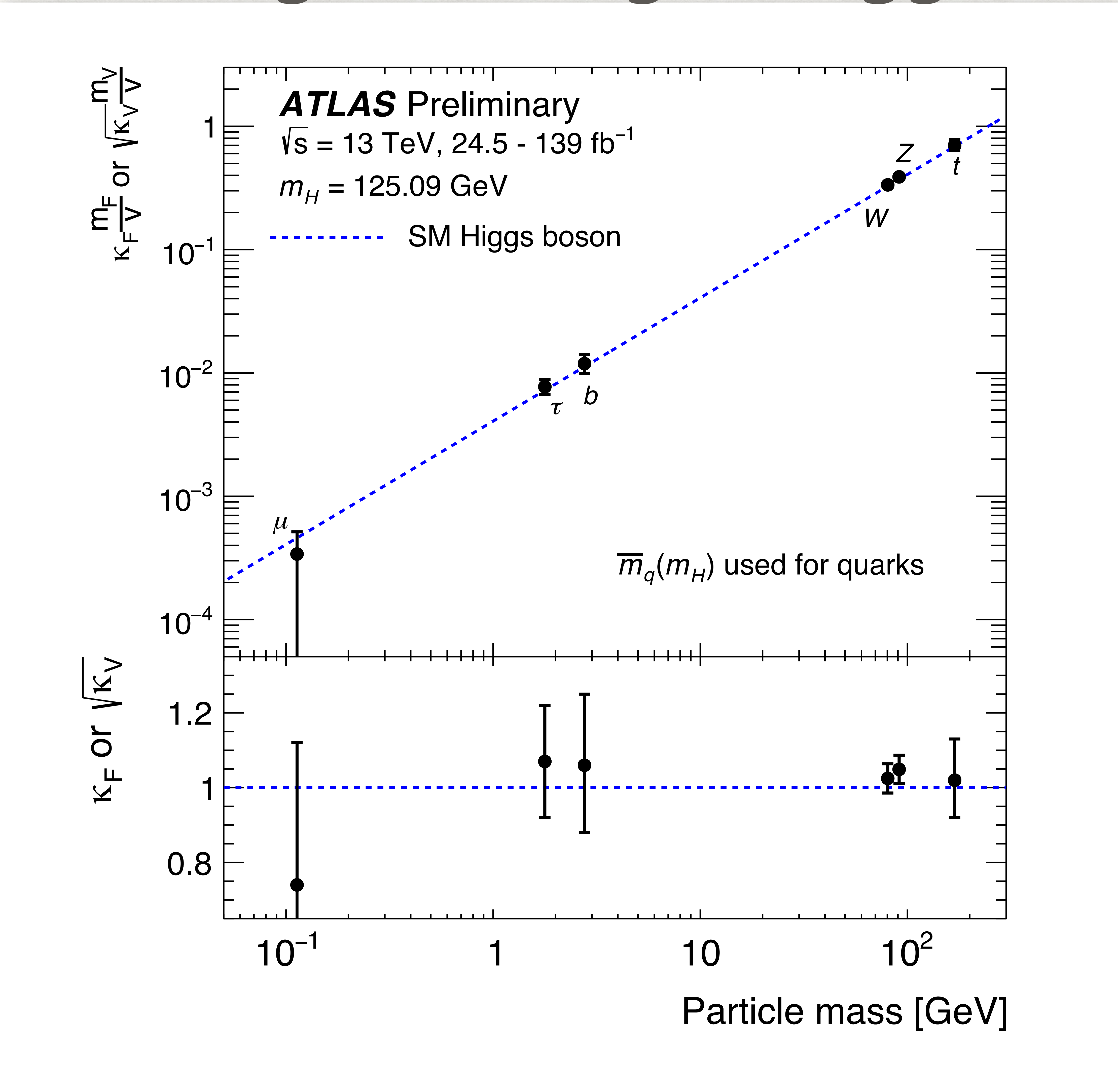
Outline (as time permits):

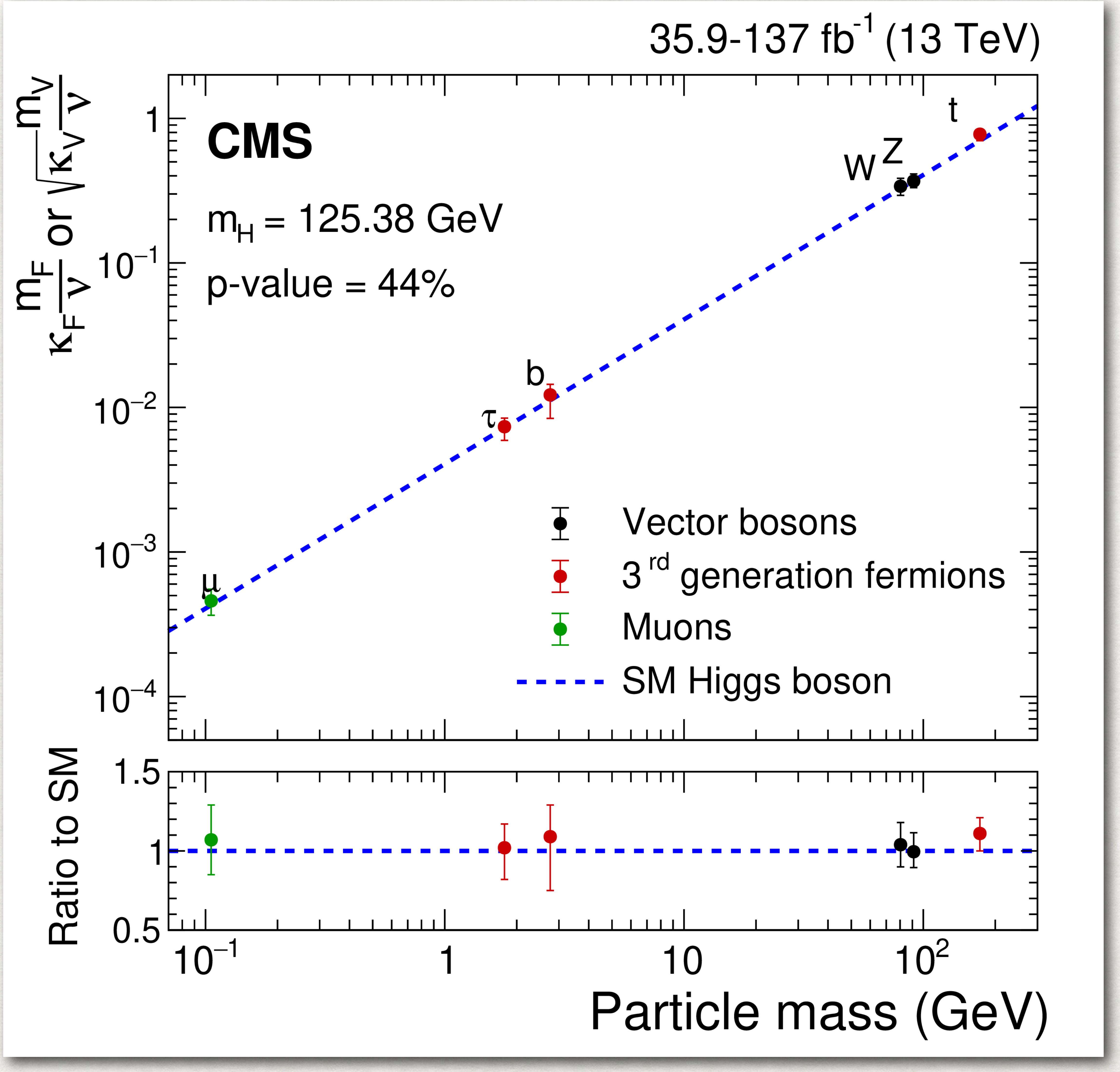
1. Introduction.
2. Alignment and the top quark.
3. The Higgs mass and sum rule for BSM Higgs masses
4. Experimental consequences.
5. What happens in two loops?

Higgs Alignment and the Top Quark: The Intro

In Gildener-Weinberg (GW) models of the electroweak interactions, the BSM Higgs bosons are surprisingly light. Yet, in these models, the 125 GeV Higgs boson **H** has **Standard Model** couplings to W, Z, quarks and leptons; that is, **H is aligned!**. Were it not for the top quark — its large mass and the Glashow-Weinberg constraint on quark-Higgs couplings — these couplings would be **experimentally the same** as the SM Higgs. The top quark's coupling to a single Higgs doublet disrupts this near-perfect "alignment", but the effect is very small, $< O(0.1\%)$, and inaccessible — now, anyway. A consequence of this is that many popular searches for BSM Higgs bosons will **remain fruitless and in vain!** The only hope to test these models is to discover their **light** BSM Higgses — in LHC Run 3 or the HL-LHC.

So far, all LHC data is consistent with H(125) being the single Higgs boson of the Standard Model.





This is puzzling: practically all attempts to cure the ills of the Standard Model — most famously, naturalness — require two or more Higgs multiplets, usually doublets. But, why should one CP-even mass eigenstate have SM couplings?

The usual answer is Higgs alignment: Something — originally decoupling of heavier Higgses (Boudjema & Semenov, PRD **66**, 095007); Gunion & Haber, PRD **67**, 075019) — causes the lightest CP-even H to be the linear combination

$$H \cong \sum_i v_i \rho_i / v \quad \text{where} \quad v = \sqrt{\sum_i v_i^2}$$

(Note: I'm assuming N Higgs doublets b/c of the rho parameter.)

But, is decoupling natural?

BUT — is decoupling natural? Is there a global symmetry to prevent large radiative corrections to alignment? Yes: there have been a few proposals, but the symmetries are rather elaborate and artificial, or related to supersymmetry.

There is one very simple and attractive exception:
The Higgs is a (pseudo-) Goldstone boson of spontaneously broken scale invariance — a **dilaton**. Then, Higgs alignment is automatic! (Exact in tree approximation.) But what is the dilaton scale (VEV)? The answer has been in front of us since 1976!

E. Gildener and S. Weinberg, PRD **13**, 3333 (1976).

The GW-2HDM

The key assumption of GW models is that the classical Lagrangian is <u>**scale-invariant**</u>: the Higgs potential V_0 is <u>purely quartic</u> **and** all fermion hard masses arise from their <u>dim'n-4 Yukawa</u> couplings to Higgs bosons. (Hence the need for <u>complex</u> Higgs doublets (SW, PRL **19**, 1264, 1967) — and **no** vectorlike quarks or leptons with electroweak interactions!)

Use a simple 2HDM (Lee & Pilaftsis, PRD **86**, 035004; also W. Shepherd & K.L., PRD **99**, 055015):

$$\begin{aligned} V_0 = {} & \lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) \\ & + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \tfrac{1}{2}\lambda_5\left[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2\right] \end{aligned}$$

$$\text{where } \underline{\lambda_i = \lambda_i^*;\ \lambda_1 > 0, \lambda_2 > 0.}$$

A nontrivial **flat minimum** of V_0 can occur on the ray

$$\Phi_{1\beta} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi\cos\beta \end{pmatrix}, \quad \Phi_{2\beta} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi\sin\beta \end{pmatrix}$$

where $0 < \phi < \infty$ and $0 < \beta < \pi/2$.

The extremal conditions are

$$\lambda_1 + \tfrac{1}{2}\lambda_{345}\tan^2\beta = \lambda_2 + \tfrac{1}{2}\lambda_{345}\cot^2\beta = 0;$$

$$\lambda_1, \lambda_2 > 0 \Longrightarrow \lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5 < 0.$$

Use the "aligned basis":

$$\Phi = \Phi_1 c_\beta + \Phi_2 s_\beta \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}w^+ \\ H + iz \end{pmatrix} \Longrightarrow \langle\Phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \phi \end{pmatrix}$$

$$\Phi' = -\Phi_1 s_\beta + \Phi_2 c_\beta \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H^+ \\ H' + iA \end{pmatrix} \Longrightarrow \langle\Phi'\rangle = 0$$

Tree-level Higgs mass matrices are <u>diagonal</u> in this basis —

$$\mathcal{M}^2_{0^-} = \begin{pmatrix} M_z^2 & 0 \\ 0 & M_A^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_5\phi^2 \end{pmatrix}$$

$$\mathcal{M}^2_{\pm} = \begin{pmatrix} M_{w^\pm}^2 & 0 \\ 0 & M_{H^\pm}^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\frac{1}{2}\lambda_{45}\phi^2 \end{pmatrix}$$

$$\mathcal{M}^2_{0^+} = \begin{pmatrix} M_H^2 & 0 \\ 0 & M_{H'}^2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -\lambda_{345}\phi^2 \end{pmatrix}$$

— and they'll remain nearly diagonal <u>thru 2nd order</u> in the loop expansion of the effective potential of S. Coleman and E. Weinberg, PRD 7, 1888.

<u>**This means that H keeps its very nearly SM couplings thru O(V_2)**</u> **!**

(E. Eichten and K.L. to appear <u>soon</u>.)

Higgs Alignment and the Top Quark: The Result

To establish the top quark's role in Higgs alignment,
extremize the one-loop effective potential of the GW-2HDM:

$$V_1 = \frac{1}{64\pi^2}\sum_n \alpha_n \overline{M}_n^4 \left(\ln \frac{\overline{M}_n^2}{\Lambda_{GW}^2} - k_n \right)$$ (S. Martin, PRD **65**, 116003)

$$(\alpha_n, k_n) = (6, \tfrac{5}{6}),\ (3, \tfrac{5}{6}),\ (-12, \tfrac{3}{2}),\ (1, \tfrac{3}{2}),\ (1, \tfrac{3}{2}),\ (2, \tfrac{3}{2})$$

for $n = W^\pm,\ Z,\ t,\ H',\ A,\ H^\pm.$ (all heavy particles)

N.B.: The renormalization scale Λ_{GW} explicitly breaks scale invariance.

R. Jackiw, PRD **9**, 1686 (1974)

$$\overline{M}_n^2 = \begin{cases} M_n^2\left(2\left(\Phi^\dagger\Phi + \Phi'^\dagger\Phi'\right)/\phi^2\right) = M_n^2\left((H^2 + H'^2 + \cdots)/\phi^2\right), & n \neq t \\ M_t^2\left(2\Phi_1^\dagger\Phi_1/(\phi^2 c_\beta^2)\right) = M_t^2\left((H - H'\tan\beta)^2 + \cdots)/\phi^2\right), \end{cases}$$

<— Note bene ! Glashow-Weinberg constraint.

where $M_W^2 = \frac{1}{4}g^2\phi^2,\ M_{H'}^2 = -\lambda_{345}\phi^2,\ M_t^2 = \frac{1}{2}\Gamma_t^2\phi^2 c_\beta^2,$ etc.

$$\mathbf{First, H:}$$

$$0 = \frac{\partial V_1}{\partial H}\bigg|_{\langle\rangle} \propto \sum_n \alpha_n M_n^4 \left(\ln\frac{M_n^2}{\Lambda_{\rm GW}^2} + \tfrac{1}{2} - k_n\right) = A + B\left(\ln\frac{v^2}{\Lambda_{\rm GW}^2} + \tfrac{1}{2}\right)$$

where $A = \sum_n \alpha_n M_n^4 \left(\ln\frac{M_n^2}{v^2} - k_n\right)$ and $B = \sum_n \alpha_n M_n^4$.

This fixes the R-scale $\boldsymbol{\Lambda_{\rm GW} = v\exp\left[(A + \tfrac{1}{2}B)/2B\right]}$

in terms of $\boldsymbol{\phi = v}$ at which $\boldsymbol{V_1 = (V_1)_{\rm min} = -B/128\pi^2}$.

Identify $\langle H\rangle = v = 2^{-1/4}G_F^{-1/2} = 246\,\mathrm{GeV}$, and $M_W^2 = \frac{1}{4}g^2v^2$, etc.

This is a **deeper** minimum (< 0) than $V_0 = 0$ **iff**

$$\boldsymbol{B = \sum_n \alpha_n M_n^4 = 6M_W^4 + 3M_Z^4 + 2M_{H^\pm}^4 + M_A^4 + M_{H'}^4 - 12M_t^4 > 0.}$$

Then, $\mathbf{H'}$:

$$0 = \frac{\partial(V_0 + V_1)}{\partial H'}\bigg|_{\langle\,\rangle + \delta_1 H + \delta_1 H'} = M_{H'}^2 \delta_1 H' + \frac{\partial V_1}{\partial H'}\bigg|_{\langle\,\rangle}$$

$\Rightarrow$ the shift of $\langle H' \rangle$ from zero in $\mathcal{O}(V_1)$ is: $\quad \delta_1 H' = -\dfrac{1}{M_{H'}^2}\dfrac{\partial V_1}{\partial H'}\bigg|_{\langle\,\rangle}$

(the familiar tadpole formula)

$$\Rightarrow \delta_1 H' = \frac{\alpha_t M_t^4 \tan\beta}{16\pi^2 M_{H'}^2 v}\left(\ln\frac{M_t^2}{\Lambda_{\rm GW}^2} + \frac{1}{2} - k_t\right) = \delta_1 H' \cong 1\text{-}3\,\text{GeV} \ll \text{v}$$

$\delta_1 H'$ establishes the connection of the top quark to Higgs alignment:
If $\delta_1 H' = 0$, $H = \rho_1 c_\beta + \rho_2 s_\beta$ is still a mass eigenstate.
$\rightarrow$ Large $M_t \Rightarrow$ its appearance in V_1.
$\rightarrow$ The Glashow-Weinberg no-FCNC $\Rightarrow \delta_1 H' \neq 0$ in $\mathcal{O}(V_1)$.

The $\mathcal{O}(V_1)$ H and BSM Higgs masses:

$$\mathcal{M}^2_{HH} = \frac{B}{8\pi^2 v^2} \cong M_H^2 = (125\,\mathrm{GeV})^2,$$ (> 0, as promised)

$$\mathcal{M}^2_{HH'} = -\frac{\alpha_t M_t^4 \tan\beta}{16\pi^2 v^2}\left(\ln\frac{M_t^2}{\Lambda^2_{\mathrm{GW}}} + \frac{5}{2} - k_t\right) \cong \text{few GeV},$$

$$\mathcal{M}^2_{H'H'} = M^2_{H'} + \frac{\alpha_t M_t^4 \tan\beta}{8\pi^2 v^2}\left(\ln\frac{M_t^2}{\Lambda^2_{\mathrm{GW}}} + \frac{1}{2} - k_t + \tan^2\beta\right) \cong M^2_{H'}.$$

The one-loop sum rule for the BSM Higgs boson masses

$$M_H^2 = \frac{B}{8\pi^2 v^2} = \frac{1}{8\pi^2 v^2}\left(6M_W^4 + 3M_Z^4 + 2M_{H^\pm}^4 + M_A^4 + M_{H'}^4 - 12M_t^4\right)$$

$$\Longrightarrow \left(M_{H'}^4 + M_A^4 + 2M_{H^\pm}^4\right)^{1/4} = 540\,\mathrm{GeV}$$

(W. Shepherd & K.L., ibid;
E. Pilon & K.L., ibid)

The significance of this sum rule is obvious: All the BSM Higgs bosons in the GW-2HDM must lie below about 500 GeV! (**N.B.:** $M_A = M_{H^\pm}$ is used to eliminate the BSM Higgs contribution to the T-parameter.)

Two-loop preview:
(but no sum rule)

$$180\,\mathrm{GeV} \lesssim \mathrm{M_A} = \mathrm{M_{H^\pm}} \lesssim 360\text{–}420\,\mathrm{GeV}$$

with

$$550\text{–}700\,\mathrm{GeV} \gtrsim \mathrm{M_{H'}} \gtrsim \mathrm{M_H} = 125\,\mathrm{GeV}$$

<u>Higgs Alignment: Experimental consequences</u>

ATLAS & CMS discovered H(125) relatively easily because of its rather strong coupling to WW and ZZ: <u>production</u> via WW and ZZ fusion and <u>decay</u> to WW* and ZZ*, (ZZ* —> 4 leptons was much more convincing than $\gamma\gamma$). gg fusion via the top-quark loop is important because the Htt coupling is "full strength" M_t /v.

Many ATLAS and CMS searches for BSM Higgs bosons — (generically H') — rely on the **<u>lamp-post strategy</u>**:

They assume that VV -> H' and H' -> VV are important modes of the H'; also H' -> HZ, H+ -> HW+; and the H'tt coupling ~M_t /v.

<u>The rates for these H'VV and H'HV couplings are suppressed by ~1/1000 !!</u>