Origin of Neutrino Mass on Convex Cone of Positivity

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Based on 2202.12907 with Shun Zhou

Motivation

- Non-zero mass of neutrinos → $O^{(5)} = \overline{L}\tilde{H}\tilde{H}^T L^c$

  Oscillations of Atmospheric neutrinos, Solar neutrinos, Reactor neutrinos ...

  Effective operator

- SM is not complete theory but an effective theory

  Tree-level completions for dim-5: type-I seesaw, type-II seesaw, type-III seesaw,...

  [P. Minkowski,PLB(1977)] [W. Konetschny and W. Kummer, PLB(1977)] [R. Foot, et al, ZPC 44(1989)]

- New physics (UV) is very likely to couple to L and H doublets

- The seesaws are not distinguishable up to dim-5

  Model-depended predictions for low-energy observables appear at higher dim.

Target: A model-independent way to distinguish seesaws at higher dim. space

- **Recently developed geometry perspective of positivity bounds will help in the SMEFT framework**

SMEFT, positivity and cone

- **SMEFT framework**: describe the IR behavior of some "UV completion", by integrating out its heavy dofs, with high-dimensional operators

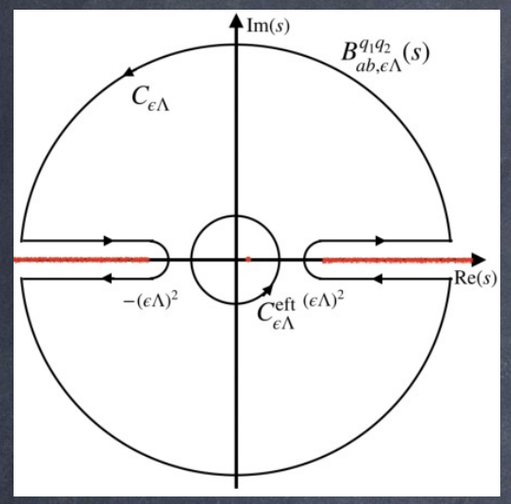
$$\mathcal{L}_{\rm EFT} = \mathcal{L}_{\rm SM} + \frac{C_i^{(5)} O_i^{(5)}}{\Lambda} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(7)} O_i^{(7)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \cdots$$

- **Positive structure**: axiomatic principles of QFT, including causality, unitarity, Lorentz symmetry... -> bounds for Wilson coefficients
  - 2-to-2 amplitude: $\mathcal{M}_{2\to 2}(s,t) = c_0 + c_2 s^2 + c_{2,1} s^2 t + \cdots c_{n,m} s^n t^m$
  - $c_2 > 0$; or in SMEFT: $C^{(8)} > 0$ [A. Adams et al., JHEP 06] [see also talk by S. D. Bakshi]
  - More bounds on higher-s dependence.

    Recent development: [S. Ghosh et al., 2204.07617] [L.-Y. Chiang et al., 2204.07140] [S. D. Bakshi et al., 2205.03301] [D. Chowdhury et al., 2205.13762] [G. N. Remmen, N. L. Rodd, 2206.13524]

- **Geometry perspective:** UVs as tree-completions for high-dim. operators, are distinguishable in the convex cone. [C.Zhang and S.-Y. Zhou, PRL(2020)]

Master formula



2-to-2 forward scattering($t=0$) for $ij \to kl$: [C.Zhang and S.-Y. Zhou, PRL(2020)]

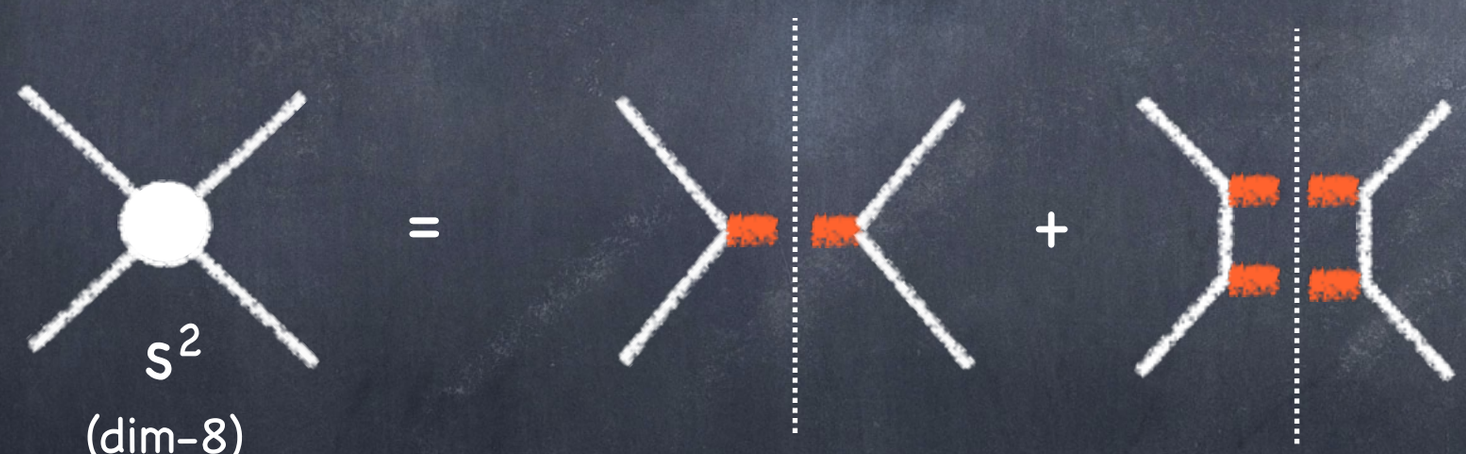
$$M^{ijkl} \equiv \frac{d^2}{ds^2}\mathcal{M}_{ij\to kl}(s,0)$$

$$= \frac{1}{2\pi i}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\,\mathrm{Disc}\mathcal{M}_{ij\to kl}(\mu) + (j \leftrightarrow l)$$

$$+ \text{ residues at poles}$$

Causality -> Cauchy's integral formula

Unitarity -> Optical theorem: $\mathrm{Disc}\mathcal{M}^{ijkl}(s) = \sum_X \mathcal{M}^{ij\to X}(s)\left(\mathcal{M}^{kl\to X}(s)\right)^*$



$$M^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\sum_X[\mathcal{M}_{ij\to X}\,\mathcal{M}^*_{kl\to X} + (j\leftrightarrow l)]$$

Convex cone nature

Two observations: $M^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\sum_X \boxed{[\mathcal{M}_{ij\to X}\,\mathcal{M}^*_{kl\to X} + (j\leftrightarrow l)]}$

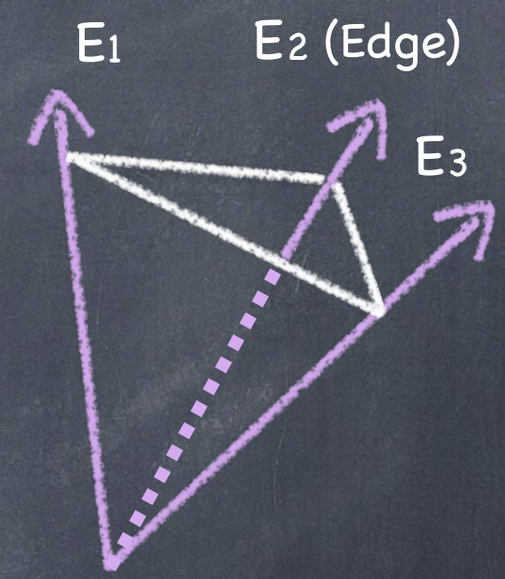
[C.Zhang and S.-Y. Zhou, PRL(2020)]

M^{ijkl} is the positive linear combination of $\mathcal{M}_{ij\to X}\,\mathcal{M}^*_{kl\to X} + (j\leftrightarrow l)$

1. M^{ijkl} is inside the convex cone

Any vector inside cone can always be written as positive linear combination of E_i

$$\vec{x}: \quad \vec{x} = \sum_i w_i \vec{E}_i \ (w_i \geq 0)$$



$\{\vec{x}\} = \mathrm{cone}\left(\left\{\vec{E}_i\right\}\right)$

2. X belongs to the irreps of symmetry, M^{ijkl} can be the edge

$$\mathbf{r}_i \otimes \mathbf{r}_j = \sum_\alpha C^{i,j}_{\mathbf{r},\alpha}\mathbf{r}$$

Clebsch-Gordan (CG) coeffs.

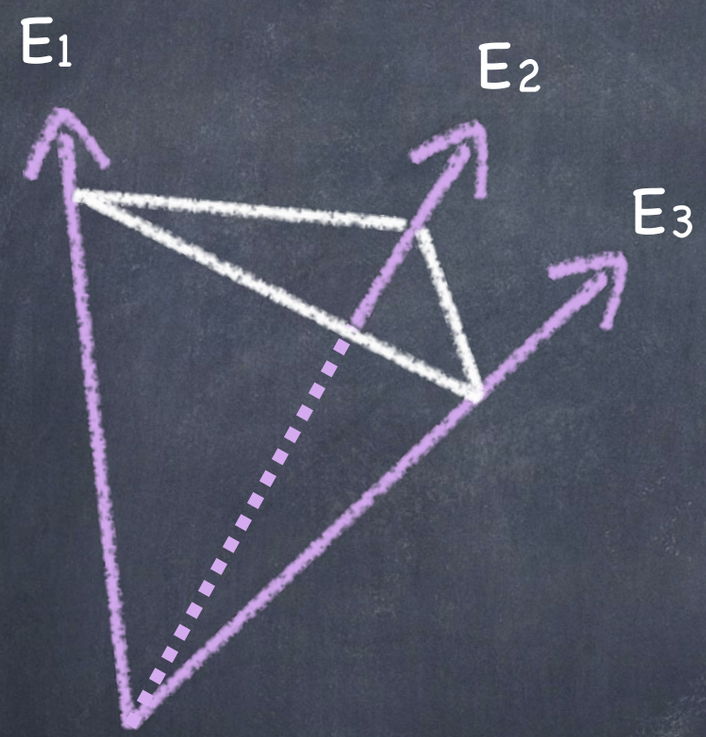
$$\boxed{M^{ijkl} = \frac{1}{2\pi}\int_{(\epsilon\Lambda)^2}^{\infty}\frac{d\mu}{\mu^3}\sum_X |\langle X|\mathcal{M}|\mathbf{r}\rangle|^2 \mathcal{G}^{ijkl}_{\mathbf{r}}}$$

Generator :

$$\mathcal{G}^{ijkl}_{\mathbf{r}} \equiv \Sigma_\alpha C^{i,j}_{\mathbf{r},\alpha}(C^{k,l}_{\mathbf{r},\alpha})^* + (j\leftrightarrow l)$$

UV completions and Cone

Edge of cone



May correspond to

UV states in irreps

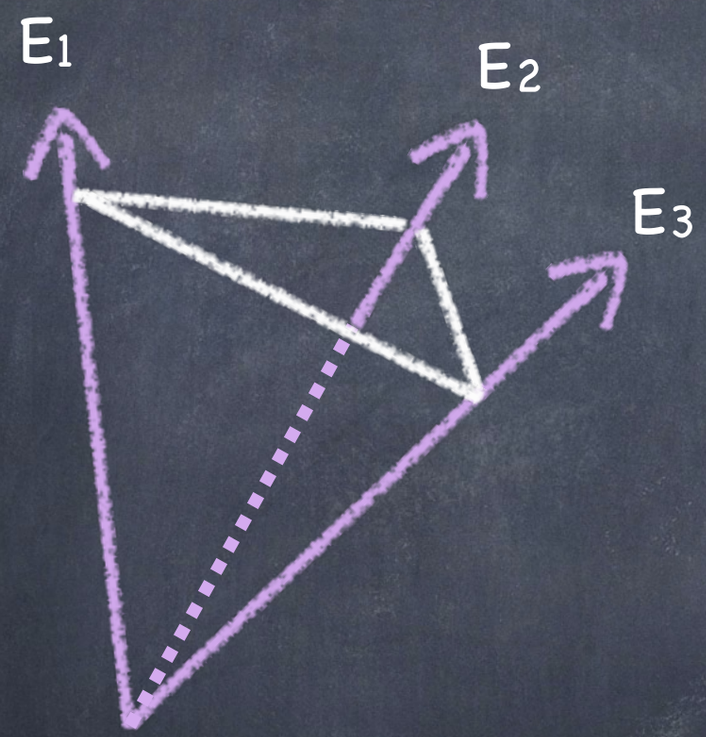
Cone constructed by Generators

$$\mathcal{G}^{ijkl}_{\mathbf{r}} \equiv \Sigma_\alpha C^{i,j}_{\mathbf{r},\alpha}(C^{k,l}_{\mathbf{r},\alpha})^* + (j \leftrightarrow l)$$

Tree-level completions for Higher dim. operators

UV completions and Cone

Edge of cone



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May correspond to

May seesaws appear at dim-8?

UV states in irreps

$$O^{(5)} = \overline{L}\tilde{H}\tilde{H}^T L^c$$

$\overline{L^c}\otimes H$	$\overline{L^c}\otimes L$	$\overline{L^c}\otimes H$	
1	**3**	**3**	(irreps)
$\overline{L^c}\otimes H$	$H\otimes H$	$\overline{L^c}\otimes H$	
Type-I	**Type-II**	**Type-III**	

Seesaw models

[L.-F. Li, PRD(1980)] [E. Ma, PRL(1998)]

[H.-L. Li, et al. PRD(2021)],
[C. W. Murphy, JHEP(2020)]

- $LLHH$:

$$\mathcal{O}_1 = (\bar{L}\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu L)\left(D^\mu H^\dagger D^\nu H\right),$$
$$\mathcal{O}_2 = (\bar{L}\gamma_\mu \sigma^I \mathrm{i}\overleftrightarrow{D}_\nu L)\left(D^\mu H^\dagger \sigma^I D^\nu H\right) \;;$$

- LLLL:

$$\mathcal{O}_3 = \partial_\nu\left(\bar{L}\gamma^\mu L\right)\partial^\nu\left(\bar{L}\gamma_\mu L\right),$$
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L,H are assigned as the irrep **2** :
thus **X** may be assigned as **1** or **3**

$$\mathbf{2}\times\mathbf{2} = \mathbf{1}+\mathbf{3}$$
$$\mathbf{2}\times\bar{\mathbf{2}} = \mathbf{1}+\mathbf{3}$$

CG coeffs.:

$$C^{ab}_{\mathbf{1},c} = \epsilon^{ab}\;, \quad C^{ab}_{\mathbf{3},c} = \left(\epsilon\sigma^I\right)^{ab}\;;$$
$$\bar{C}^{ab}_{\mathbf{1},c} = \delta^a_b\;, \quad \bar{C}^{ab}_{\mathbf{3},c} = \left(\sigma^I\right)^a_b\;,$$

X is Boson, $i,j = \mathbf{2}, \mathbf{r} = \mathbf{1}$ or $\mathbf{3}$

$\mathcal{M}_{ij\to\mathbf{r}} =$

	H_b	L_b	$H^\dagger_b$	$\bar{L}_b$
H_a	$C^{ab}_{\mathbf{1/3},c}$	$C^{ab}_{\mathbf{1/3},c}$	$\bar{C}^{ab}_{\mathbf{1/3},c}$	$\bar{C}^{ab}_{\mathbf{1/3},c}$
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$H^\dagger_a$	$\pm\bar{C}^{ab}_{\mathbf{1/3},c}$	$C^{ab}_{\mathbf{1/3},c}$	$C^{ab}_{\mathbf{1/3},c}$	$C^{ab}_{\mathbf{1/3},c}$
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x being the relative size of coupling between HH and LL

Get Generators!

$$\mathcal{G}^{ijkl}_{\mathbf{r}} \equiv \Sigma_\alpha C^{i,j}_{\mathbf{r},\alpha}\left(C^{k,l}_{\mathbf{r},\alpha}\right)^* + (j\leftrightarrow l)$$
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[C. Zhang, 2112.11665]

[H.-L. Li, et al. PRD(2021)],
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[C. Zhang, 2112.11665]

Tree Completions for dim-8

Characteristic vectors

UV State	Spin	$SU(2)_L \otimes U(1)_Y$	Interaction	Seesaw	Extremal Ray	$\vec{c}$
E	1/2	$\mathbf{1}_{-1}$	$g\bar{E}\left(H^\dagger L\right)$		✓	$\frac{1}{2}(-1,-1,0,0,0,0,0)$
Σ_1	1/2	$\mathbf{3}_{-1}$	$g\bar{\Sigma}_1^I\left(H^\dagger\sigma^I L\right)$		✗	$\frac{1}{2}(-3,1,0,0,0,0,0)$
N	1/2	$\mathbf{1}_0$	$g\bar{N}\left(H^{\mathrm{T}}\epsilon L\right)$	Type-I	✓	$\frac{1}{2}(-1,1,0,0,0,0,0)$
Σ	1/2	$\mathbf{3}_0$	$g\bar{\Sigma}^I\left(H^{\mathrm{T}}\epsilon\sigma^I L\right)$	Type-III	✗	$\frac{1}{2}(-3,-1,0,0,0,0,0)$
$\mathcal{B}_1$	1	$\mathbf{1}_1$	$g\mathcal{B}_1^\mu\left[(H^\dagger\epsilon\mathrm{i}\overleftrightarrow{D}_\mu H^*)+x(\bar{L}^c\epsilon\mathrm{i}\overleftrightarrow{D}_\mu L)\right]$		✗	$\frac{1}{2}(0,0,x^2,-x^2,16,0,-16)$
Ξ_1	0	$\mathbf{3}_1$	$g\Xi_1^I\left[M(H^\dagger\epsilon\sigma^I H^*)+x(\bar{L}^c\epsilon\sigma^I L)\right]$	Type-II	✗	$\frac{1}{2}(0,0,-3x^2,-x^2,0,16,0)$
$\mathcal{S}$	0	$\mathbf{1}_{0S}$	$gM\mathcal{S}\left(H^\dagger H\right)$		✓	$2(0,0,0,0,0,0,1)$
$\mathcal{B}$	1	$\mathbf{1}_{0A}$	$g\mathcal{B}^\mu\left[H^\dagger\mathrm{i}\overleftrightarrow{D}_\nu H+x(\bar{L}\gamma_\mu L)\right]$		✗	$\frac{1}{2}(0,0,-x^2,0,-4,4,0)$
Ξ	0	$\mathbf{3}_{0S}$	$gM\Xi^I(H^\dagger\sigma^I H)$		✗	$2(0,0,0,0,2,0,-1)$
$\mathcal{W}$	1	$\mathbf{3}_{0A}$	$g\mathcal{W}^{I\mu}\left[(H^\dagger\sigma^I\mathrm{i}\overleftrightarrow{D}_\mu H)+x(\bar{L}\gamma_\mu\sigma^I L)\right]$		✗	$\frac{1}{2}(0,0,0,-x^2,4,4,-8)$

The three seesaws show up as a subset of completions

Tree Completions for dim-8

Characteristic vectors

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N	1/2	$\mathbf{1}_0$	$g\bar{N}\left(H^{\mathrm{T}}\epsilon L\right)$	Type-I	✓	$\frac{1}{2}(-1,1,0,0,0,0,0)$
Σ	1/2	$\mathbf{3}_0$	$g\bar{\Sigma}^I\left(H^{\mathrm{T}}\epsilon\sigma^I L\right)$	Type-III	✗	$\frac{1}{2}(-3,-1,0,0,0,0,0)$
$\mathcal{B}_1$	1	$\mathbf{1}_1$	$g\mathcal{B}_1^\mu\left[(H^\dagger\epsilon\mathrm{i}\overleftrightarrow{D}_\mu H^*)+x(\bar{L}^c\epsilon\mathrm{i}\overleftrightarrow{D}_\mu L)\right]$		✗	$\frac{1}{2}(0,0,x^2,-x^2,16,0,-16)$
Ξ_1	0	$\mathbf{3}_1$	$g\Xi_1^I\left[M(H^\dagger\epsilon\sigma^I H^*)+x(\bar{L}^c\epsilon\sigma^I L)\right]$	Type-II	✗	$\frac{1}{2}(0,0,-3x^2,-x^2,0,16,0)$
$\mathcal{S}$	0	$\mathbf{1}_{0S}$	$gM\mathcal{S}\left(H^\dagger H\right)$		✓	$2(0,0,0,0,0,0,1)$
$\mathcal{B}$	1	$\mathbf{1}_{0A}$	$g\mathcal{B}^\mu\left[H^\dagger\mathrm{i}\overleftrightarrow{D}_\nu H+x(\bar{L}\gamma_\mu L)\right]$		✗	$\frac{1}{2}(0,0,-x^2,0,-4,4,0)$
Ξ	0	$\mathbf{3}_{0S}$	$gM\Xi^I(H^\dagger\sigma^I H)$		✗	$2(0,0,0,0,2,0,-1)$
$\mathcal{W}$	1	$\mathbf{3}_{0A}$	$g\mathcal{W}^{I\mu}\left[(H^\dagger\sigma^I\mathrm{i}\overleftrightarrow{D}_\mu H)+x(\bar{L}\gamma_\mu\sigma^I L)\right]$		✗	$\frac{1}{2}(0,0,0,-x^2,4,4,-8)$

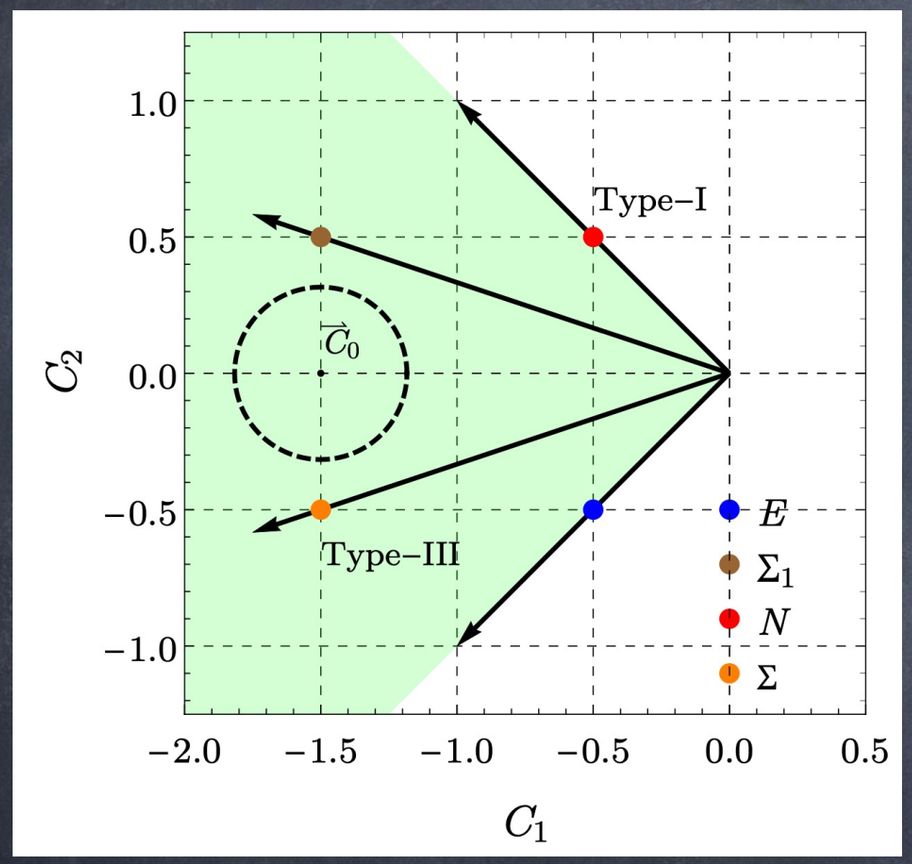
Fermion

Boson

The Boson and Fermion UVs live in different subspaces of the dim-7 Coeff. space

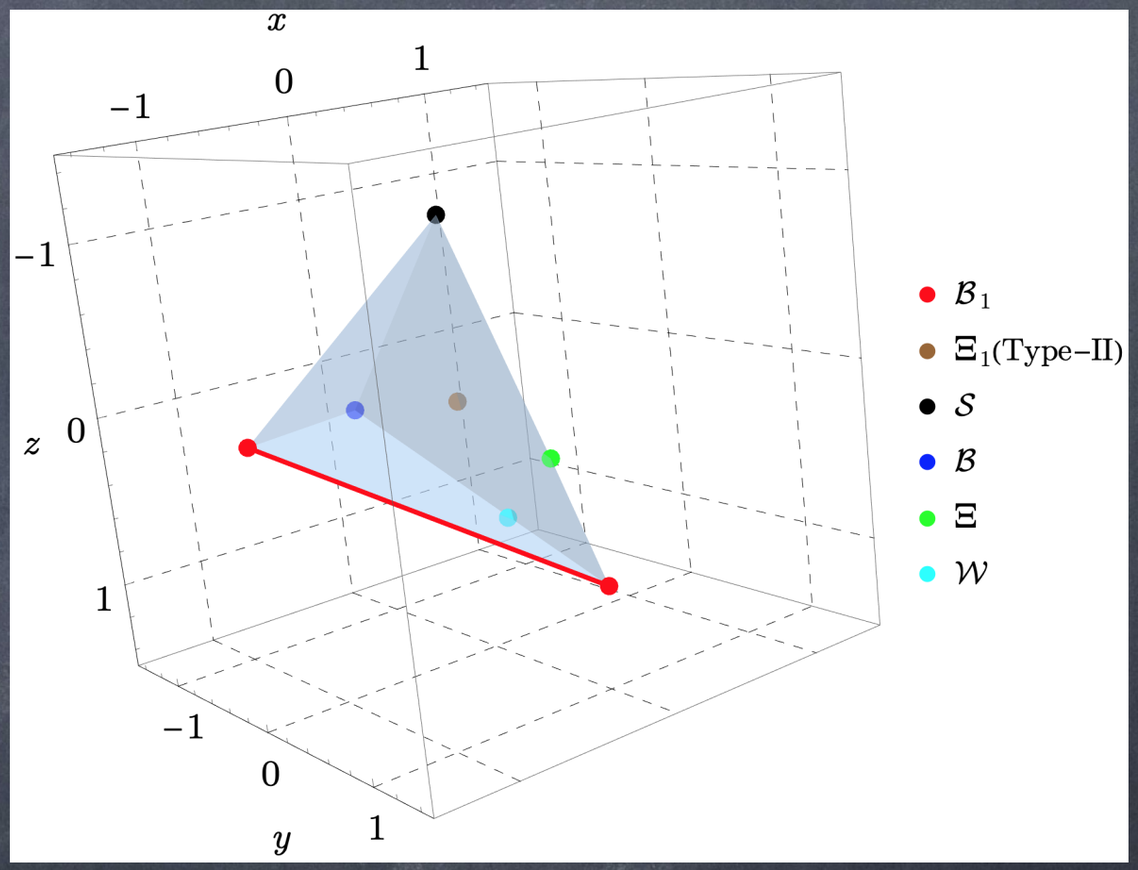
Cross Section of the Cone

UV states are Fermions



$$\mathcal{O}_1 = (\bar{L}\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu L)\,(D^\mu H^\dagger D^\nu H)\,,$$
$$\mathcal{O}_2 = (\bar{L}\gamma_\mu \sigma^I \mathrm{i}\overleftrightarrow{D}_\nu L)\,(D^\mu H^\dagger \sigma^I D^\nu H)$$

UV states are Bosons

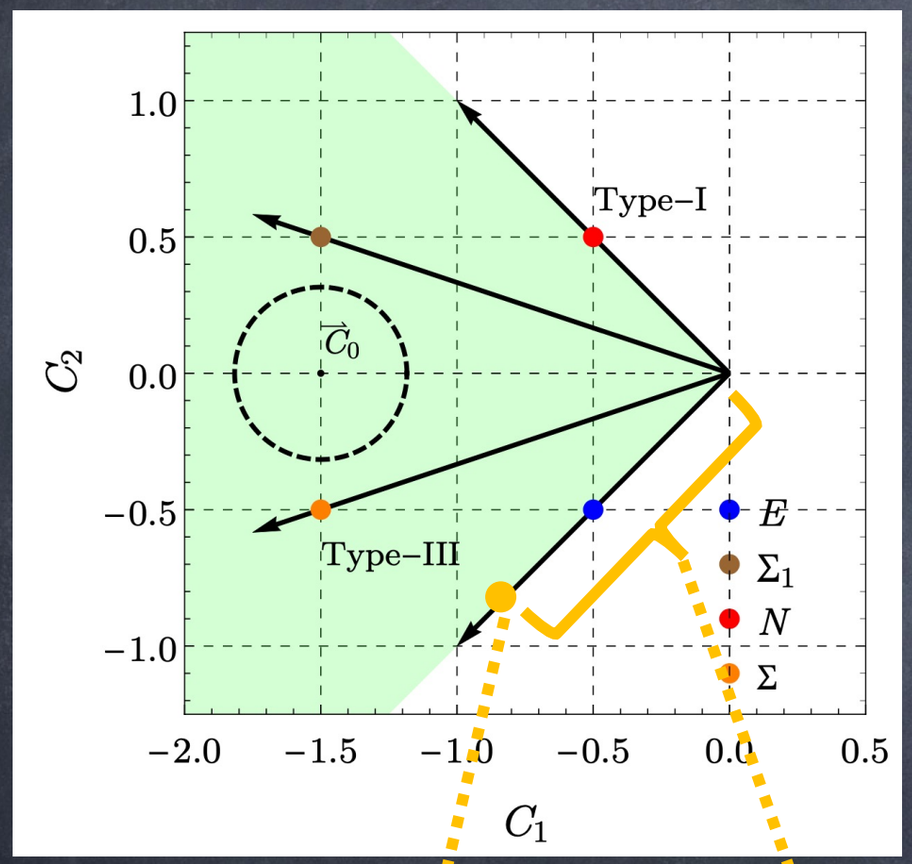


$$(x, y, z) = \frac{A(C_i)}{C_3 + 2C_4 - 2C_5 - 3C_6 - C_7}$$

$$A(C_i) = (-0.7C_3 + 0.69C_4 + 0.11C_5 + 0.13C_6 + 0.057C_7\,, 0.031C_3 + 0.85C_5 - 0.5C_6 - 0.16C_7, -0.019C_3 + 0.31C_6 - 0.95C_7)$$

Convex Optimization

UV states are Fermions



measured point

Length corresponds to the $M/\sqrt{g}$

Conversely extract the lower bounds on the scale of UV [C. Zhang, 2112.11665]

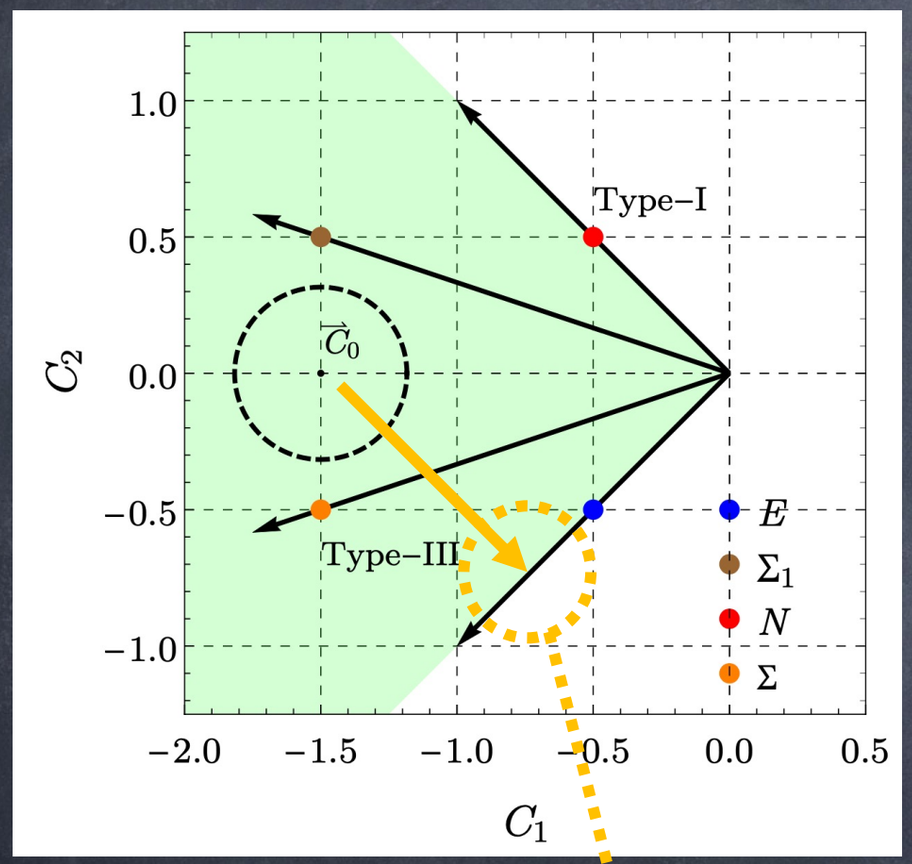
$$\vec{C}(\lambda) \equiv \vec{C}_0 - \lambda \vec{c}_i = \sum_{j \neq i} \omega_j \vec{c}_j + (\omega_i - \lambda)\vec{c}_i \ .$$

Conic optimization:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & \vec{C} - \lambda \vec{c}_i \subset \mathcal{C} \\ & (\vec{C} - \vec{C}_0) \cdot A \cdot (\vec{C} - \vec{C}_0) \leq \Delta \end{aligned}$$

Convex Optimization

UV states are Fermions



Exceed the bound

Conversely extract the lower bounds on the scale of UV [C. Zhang, 2112.11665]

$$\vec{C}(\lambda) \equiv \vec{C}_0 - \lambda \vec{c}_i = \sum_{j \neq i} \omega_j \vec{c}_j + (\omega_i - \lambda)\vec{c}_i \ .$$

Conic optimization:

$$\begin{aligned} \text{maximize} \quad & \lambda \\ \text{subject to} \quad & \vec{C} - \lambda \vec{c}_i \subset \mathcal{C} \\ & (\vec{C} - \vec{C}_0) \cdot A \cdot (\vec{C} - \vec{C}_0) \leq \Delta \end{aligned}$$

$\vec{C}_0$	E	Σ_1	N	Σ
$(-1/2, 1/2)$	∞	∞	≥ 1.0	∞
$(-3/2, 0)$	≥ 0.9	≥ 1.07	≥ 0.9	≥ 1.07
$(-3/2, 0)$ with $\Delta = 0.1$	≥ 0.85	≥ 1.0	≥ 0.85	≥ 1.0
$(0, 0)$ with $\Delta = 0.1$	≥ 1.22	≥ 1.5	≥ 1.22	≥ 1.5

Summary

- Positive structures arise at the dim-8 level in EFT coefficient space, as a consequence of axiomatic QFT principles.
- To give Weinberg operator, the new physics is very likely to couple to L and H, thus building the cone structure of LLHH dim-8 space is helpful to probe new physics.
  - the irreps of symmetry form the cone, seesaw models can naturally appear.
- Conversely probing the scale of UV states is realized by conic optimization.

Backups

To formulate this approach, **symmetries of the system help**
(will also discuss cases without symmetries)

- Make use of symmetries of the problem (SM symmetries, helicities)
- Dispersion relation: $$M^{ijkl} = \sum_X{}' \int_{(\epsilon\Lambda)^2}^{\infty} \frac{d\mu\, m_X{}^{ij} m_X{}^{kl}}{\pi(\mu - \frac{1}{2}M^2)^3} + (j \leftrightarrow l)$$
- Becomes: $$M^{ijkl} = \sum_{X\in r}{}' \int_{(\epsilon\Lambda)^2}^{\infty} d\mu \frac{|< X|M|r >|^2}{\pi\left(\mu - \frac{1}{2}M^2\right)^3} P_r^{i(j|k|l)}$$

Dynamics

Symmetry

i(j|k|l): j,l symmetrized

- P_r^{ijkl} is the projective operator of an irrep r, obtained by CG coefficients.

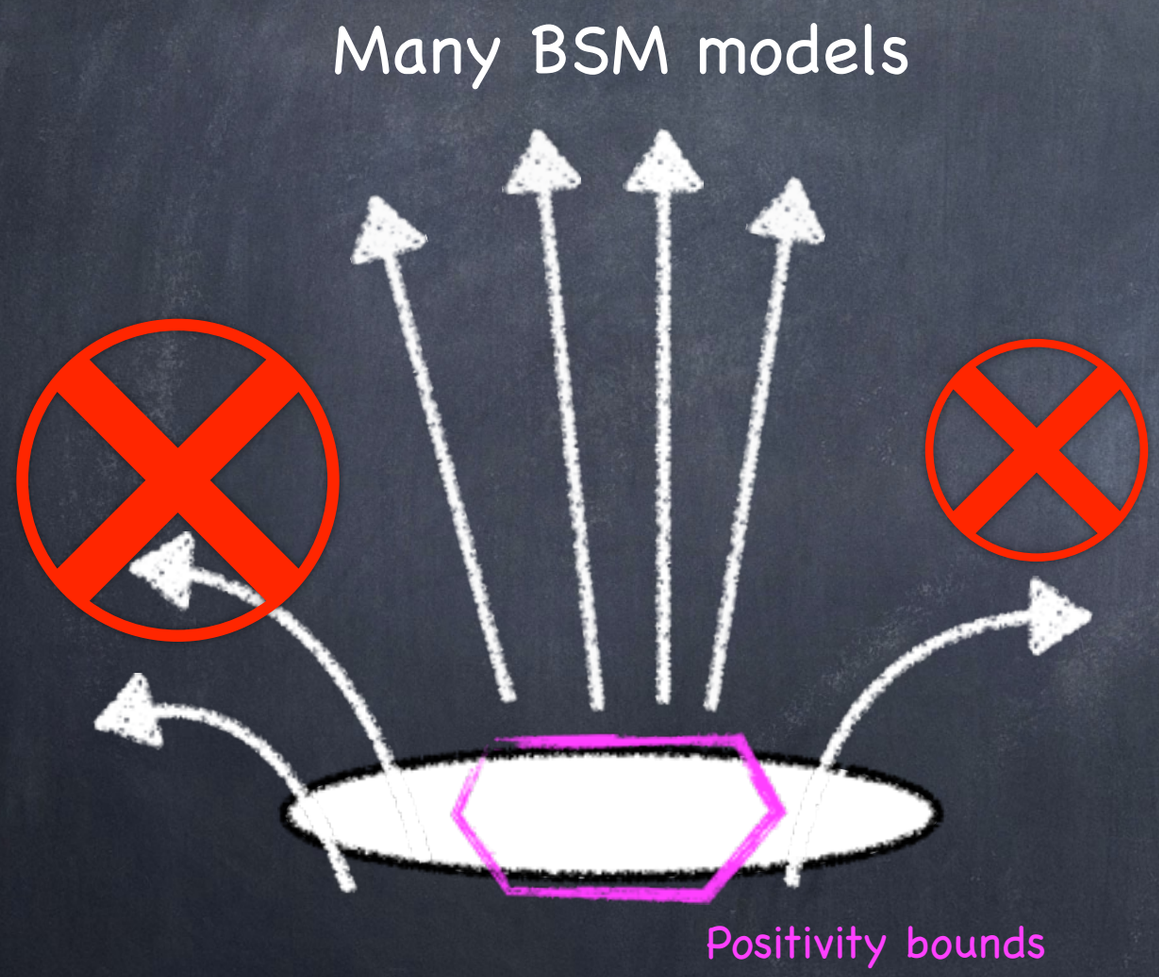
$$P_r^{ijkl} = \sum_\alpha C_{i,j}^{r,\alpha} \left(C_{k,l}^{r,\alpha}\right)^*$$

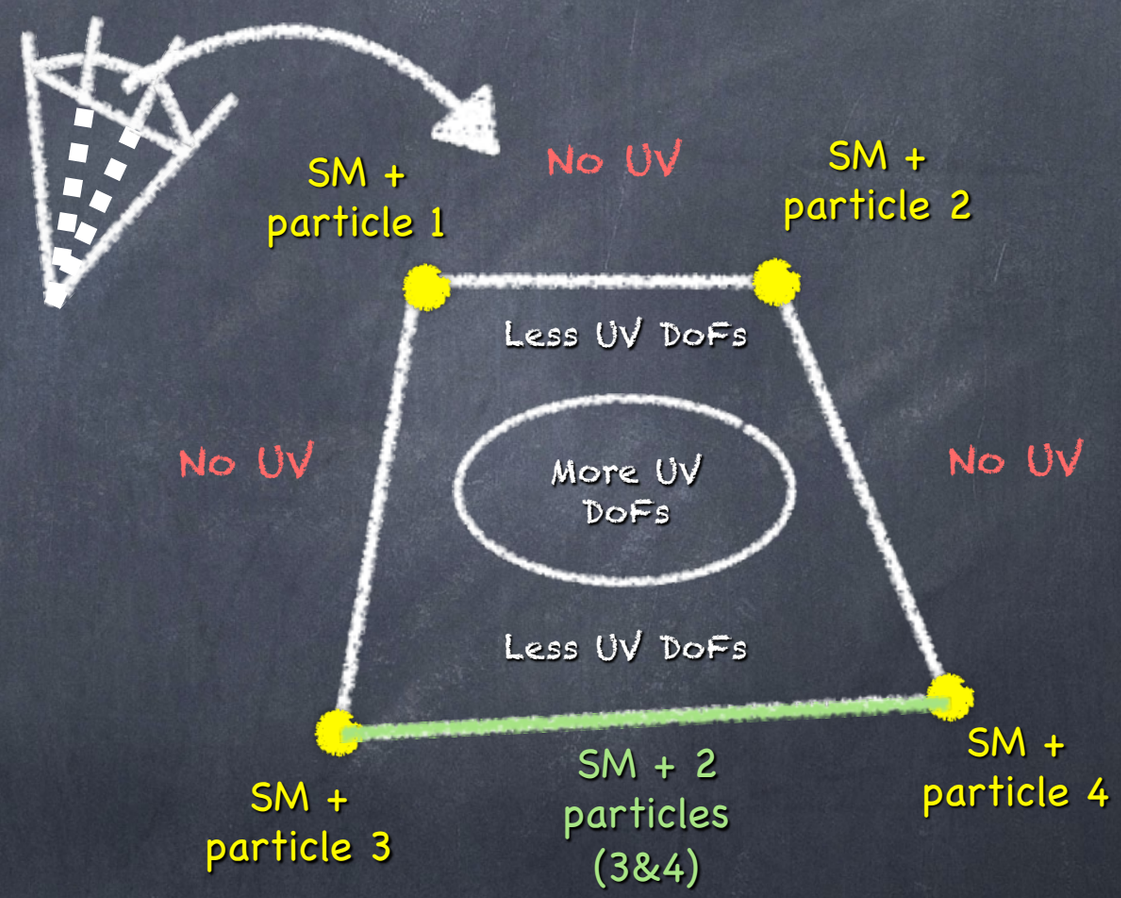
- The generators are simply (subset of) $P_r^{i(j|k|l)}$

Talk by Cen Zhang (HEP 2021)

- Infer UV model from EFT measurements

Inverse problem: Given the measured values of the operator coefficients around the electroweak scale, to what extend can we possibly determine the nature of the new physics beyond the SM? [Gu, Wang, 2008.07551]
see also [S. Dawson et al. 2007.01296] [N. Arkani-Hamed et al. hep-ph/0512190]





[CZ and S.-Y. Zhou 2005.03047]
[2009.02212 B. Fuks, Y. Liu, CZ, S.-Y. Zhou]

Talk by Cen Zhang (HEP 2021)

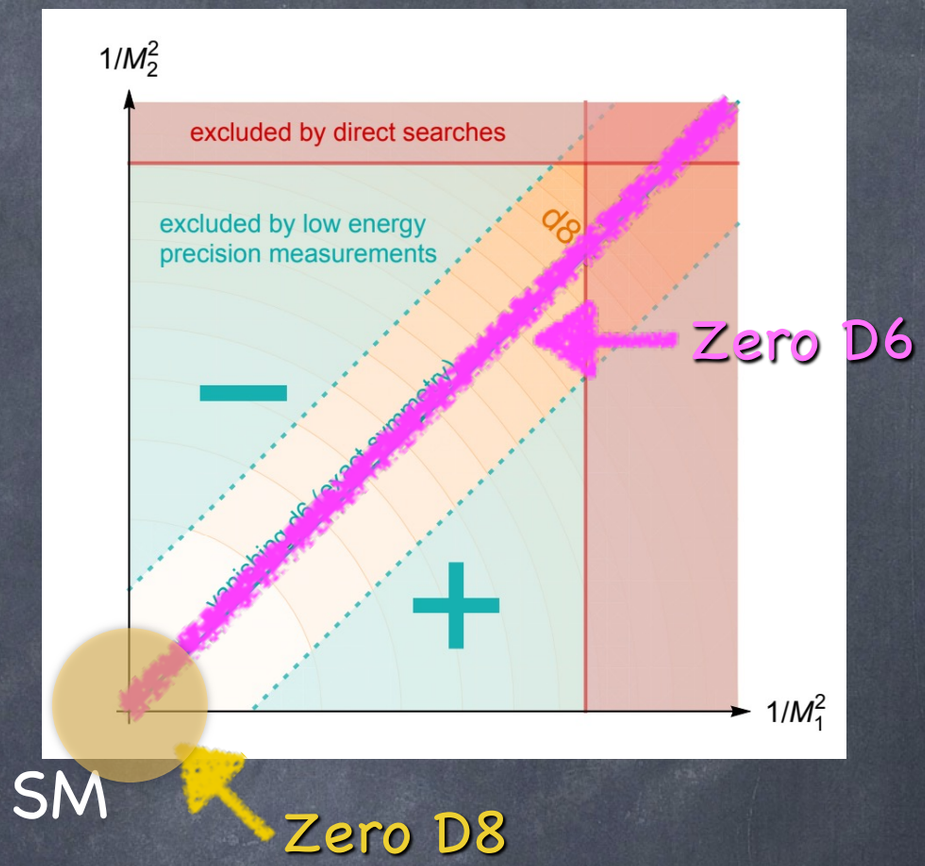
- Testing and confirming the SM: Null result of measurements at dim-6 does not exclude all BSM, but does at dim-8 by using positivity bounds

- Dim-6: no positivity, different states may cancel each other's effects.
  - E.g., scalar and vector generate 4-fermion operators with opposite signs.
  - No UV particle can be absolutely excluded.

- Dim-8: with positivity, different states are not allowed to cancel.
  - All states can be exclude to some absolute scale. (by using posi. bound)
  - Unlike dim-6 cannot lift this limit by adding more and more BSM states.
  - A robust confirmation of the SM.

[2009.02212 B. Fuks, Y. Liu, CZ, S.-Y. Zhou]

[Gu, Wang, 2008.07551]



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