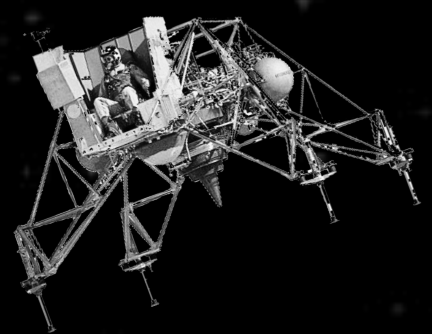
UPPSALA UNIVERSITET

Imperial College London

Phase transitions: A source of cosmic-frontier phenomenology

Eliel Camargo-Molina



THE STANDARD MODEL
Of Particle Physics

No Dark Matter

No Inflation

Predicted vacuum energy is huge!

No reason for so much more matter than anti-matter

Origin of its very special parameter values

No gravity

Why is the Higgs so light?

Vacuum Stability

The Scalar Potential and Phase Transitions

THE STANDARD MODEL
Of Cosmology

What is Dark Matter?

Flatness problem: Initial conditions for a present-day flat universe are < $O(10^{-60})$

Inflation?

Vacuum energy is tiny!

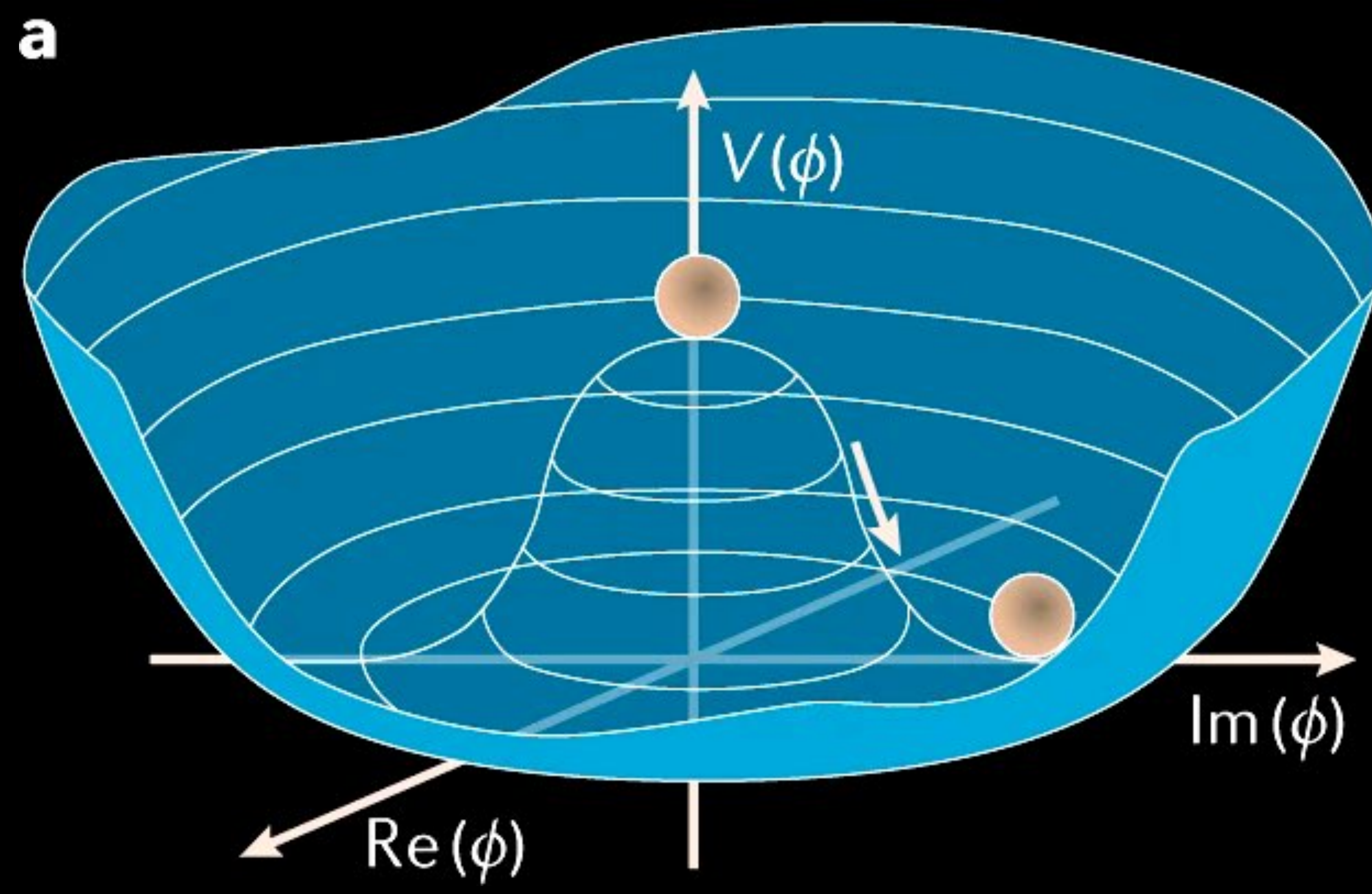
CC, is it constant?

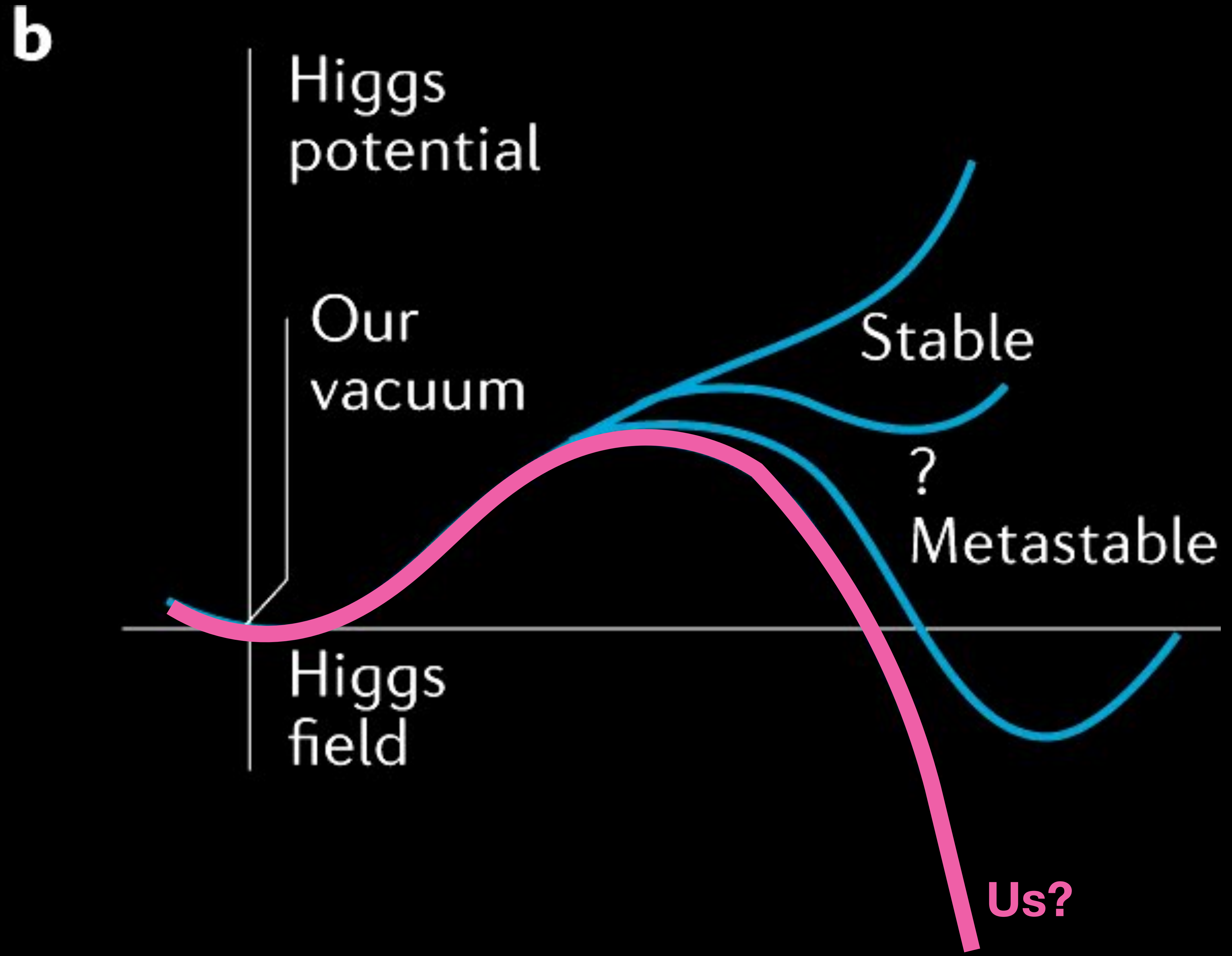
Hubble constant disagreement

6 free parameters, too much?

The Higgs Potential

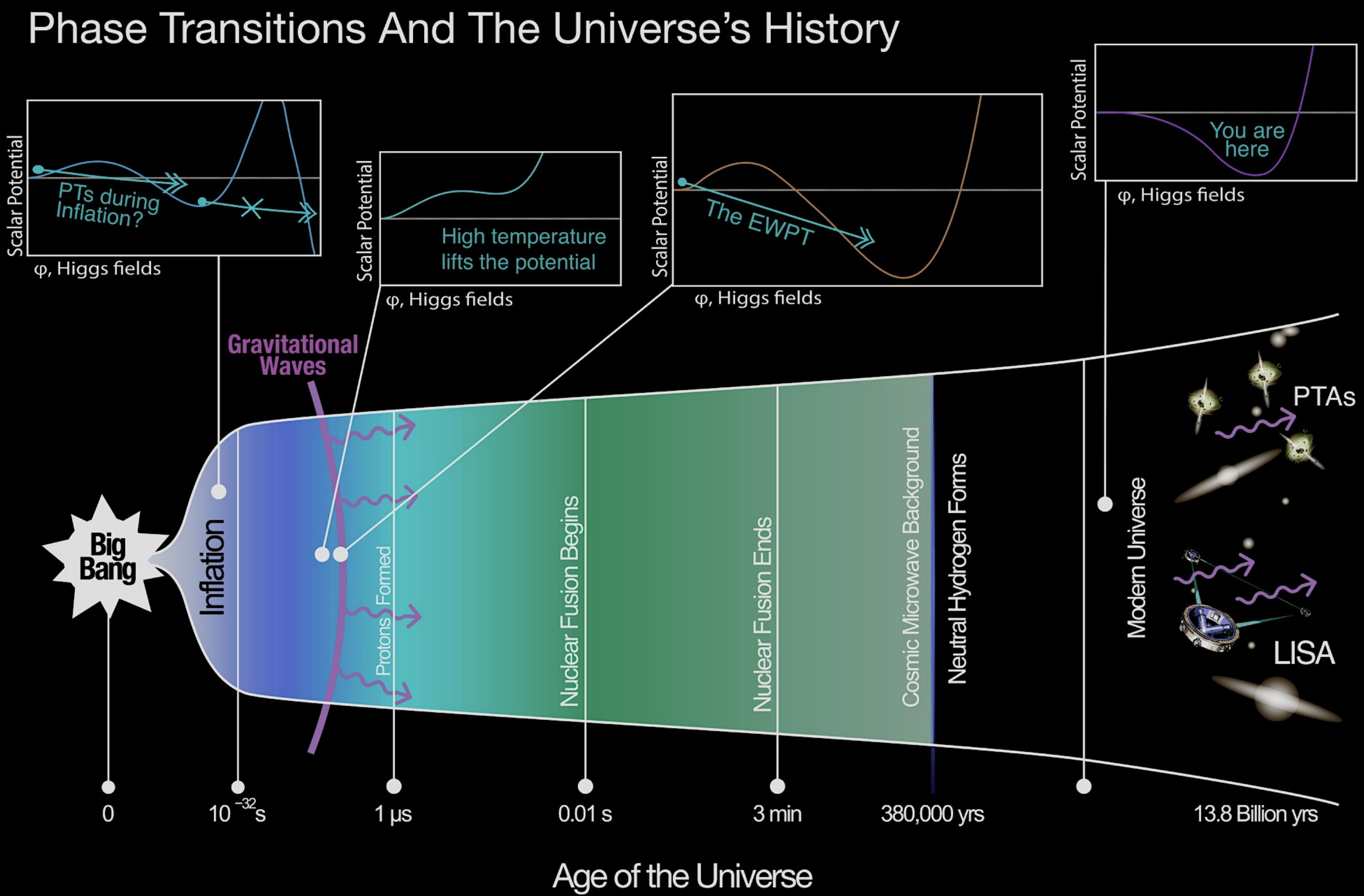
From Bass, De Roeck and Kado. Nature rev.phys.





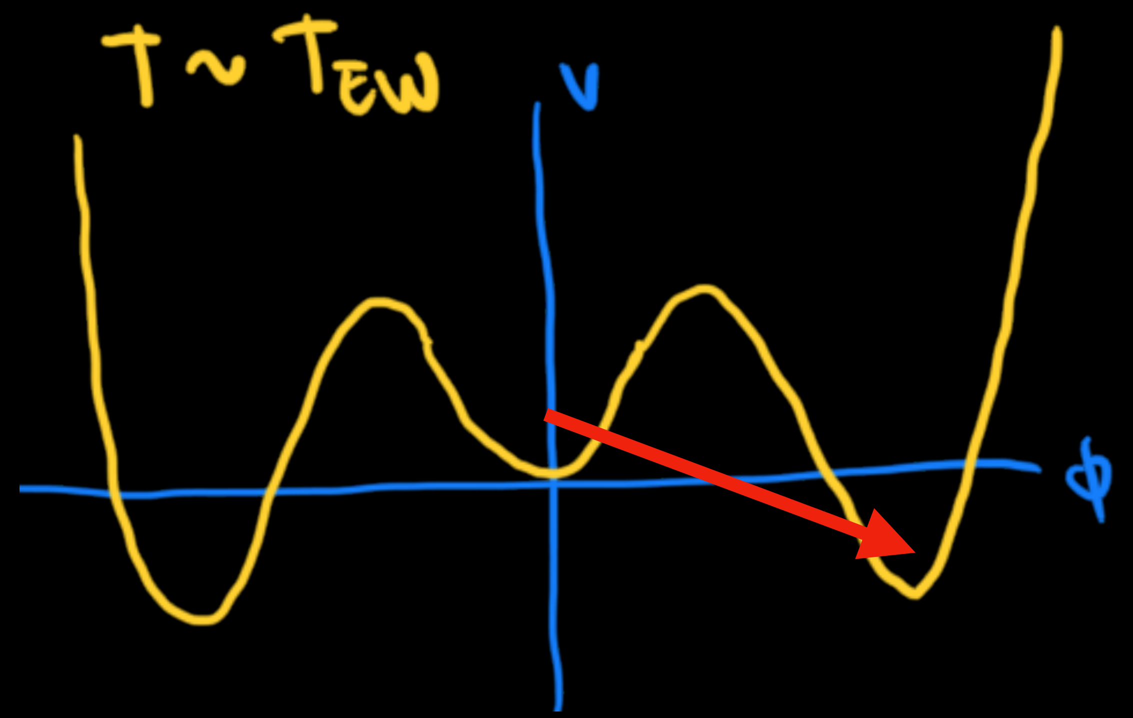
- How did we get here?
- The masses of fermions
- Massive gauge bosons
- …

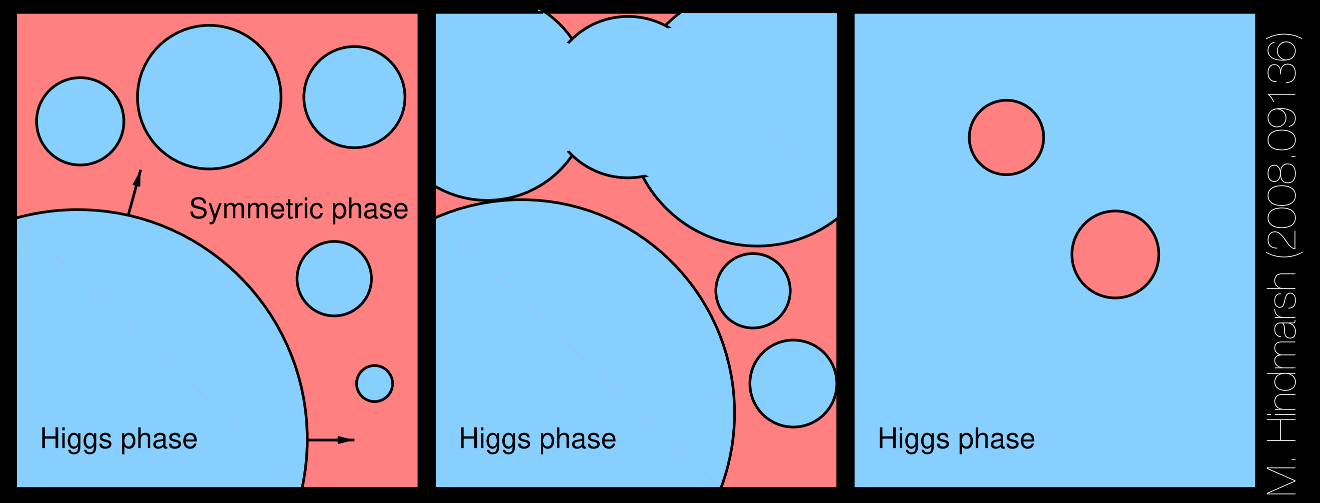
- Are we staying here?
- A case for BSM
- Early Universe insight
- …


Phase Transitions And The Universe's History
Scalar Potential
PTs during Inflation?
φ, Higgs fields
Scalar Potential
High temperature lifts the potential
φ, Higgs fields
Scalar Potential
The EWPT
φ, Higgs fields
Scalar Potential
You are here
φ, Higgs fields
Gravitational Waves
Big Bang
Inflation
Protons Formed
Nuclear Fusion Begins
Nuclear Fusion Ends
Cosmic Microwave Background
Neutral Hydrogen Forms
Modern Universe
PTAs
LISA
0
10^{-32} s
1 μs
0.01 s
3 min
380,000 yrs
13.8 Billion yrs
Age of the Universe


The EPWT → BSM?

The Electroweak Phase transition (schematically)





- The SM predicts a second order transition!
- Bubbles of the new phase collide and turbulence in the plasma + sounds waves + … leads to GW spectrum
- With a bit of luck, the peak might be at LISA's reach
- Early days in precision calculations …

Sakharov conditions

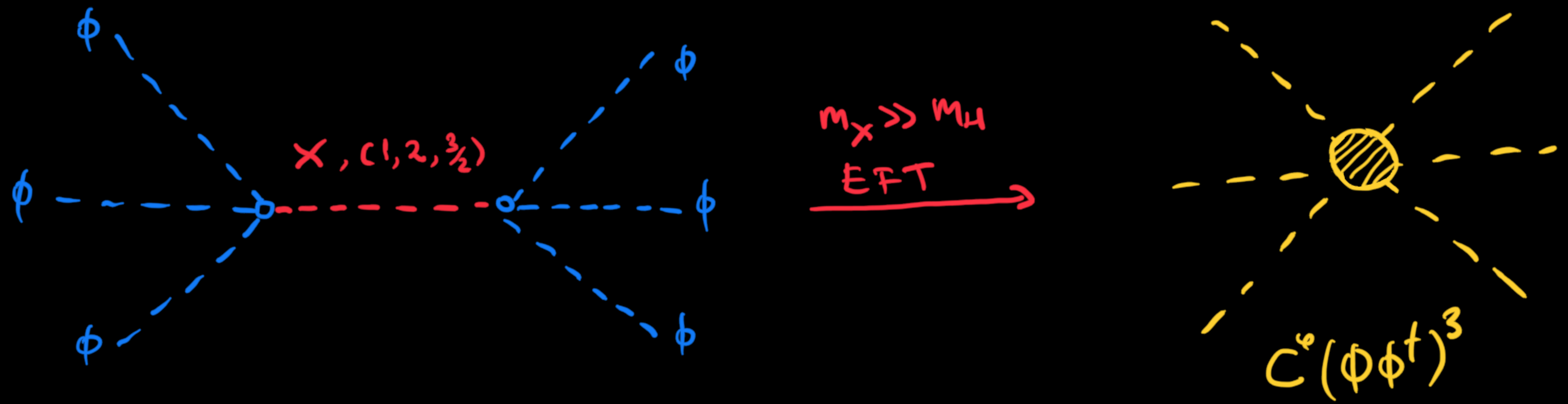
First-order EWPT +

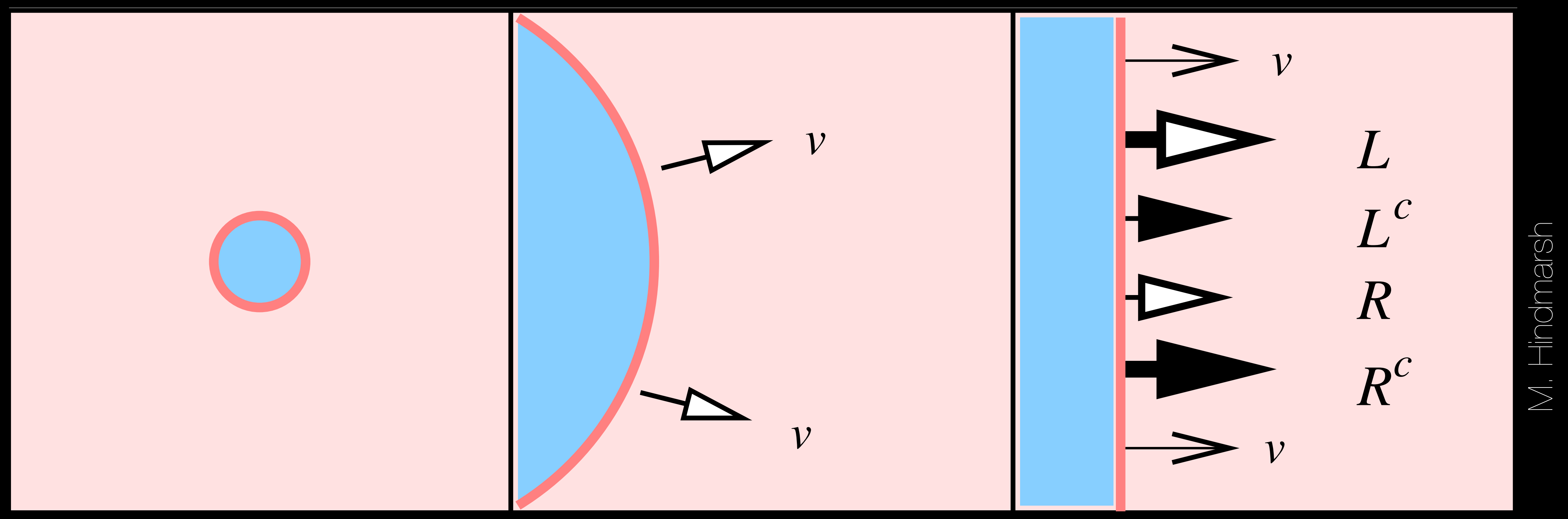
- B violation
- C and CP violation
- Non-equilibrium

- CP violation in bubble wall field profile
- CP-asymmetry in reflection of fermions
- Sphaleron processes that lead to baryogenesis

If the EWPT was first order, it could be due to

- Light new physics
- Heavy new physics -> SMEFT





BSM effects

SM particles

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + C_5 O_5 + \sum_i C_i O_i + O\left(\frac{v^3}{\Lambda^3}\right)$$

$[\mathrm{GeV}]^{-2}$

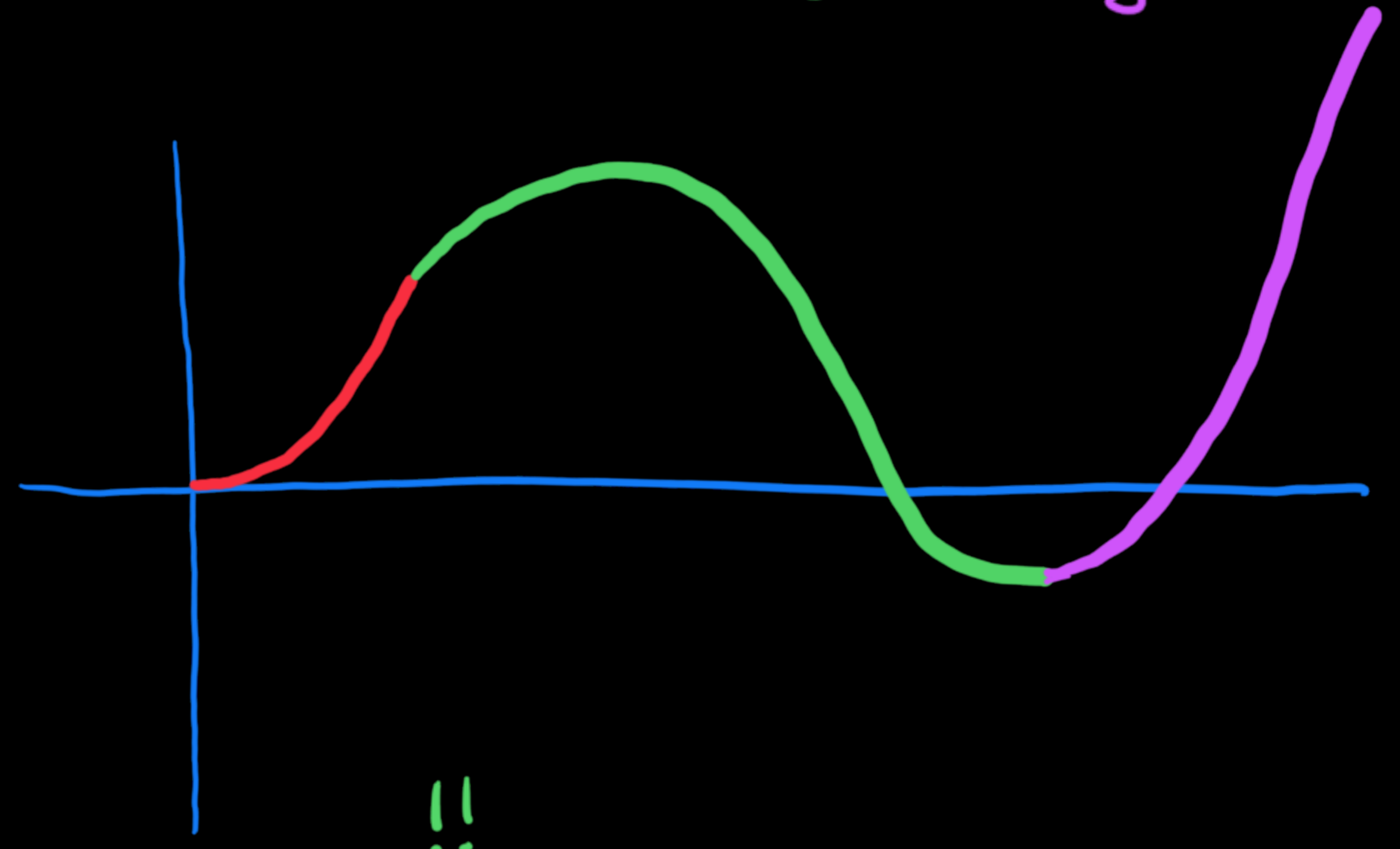
The EWPT in (heavy) BSM

- The new operator allows us to have a barrier already at tree level if

  $\lambda < 0$
- A little strange, but nothing in principle against it
- The size of Wilson Coefficients dictates the size of the barrier and the order of the EWPT

$\lambda < 0$ $\quad$ $c^{\varphi} < 0$

$$V_{tree} = -m_{eff}(t)\,\varphi^2 + \frac{\lambda}{8}\varphi^4 - \frac{c^{\varphi}}{8}\varphi^6$$



- Large coefficients needed for FOPT
- They suggest a sub-TeV scale of NP
- Or the need to go beyond dim 6
- … unlikely

- $\lambda > 0$ and tiny also works! Just as the SM’s case.
- The barrier is generated radiatively
- The WC plays a more indirect role

  (at least at LO)
- Potentially the NP scale can be higher

!!

$$V^{LO} = -m_{eff}\frac{\varphi^2}{2} - \frac{Te^3}{12\pi}\varphi^3 + \frac{\lambda}{8}\varphi^4$$

- Smaller coefficients do the trick
- TeV-scale NP
- Testable at future colliders

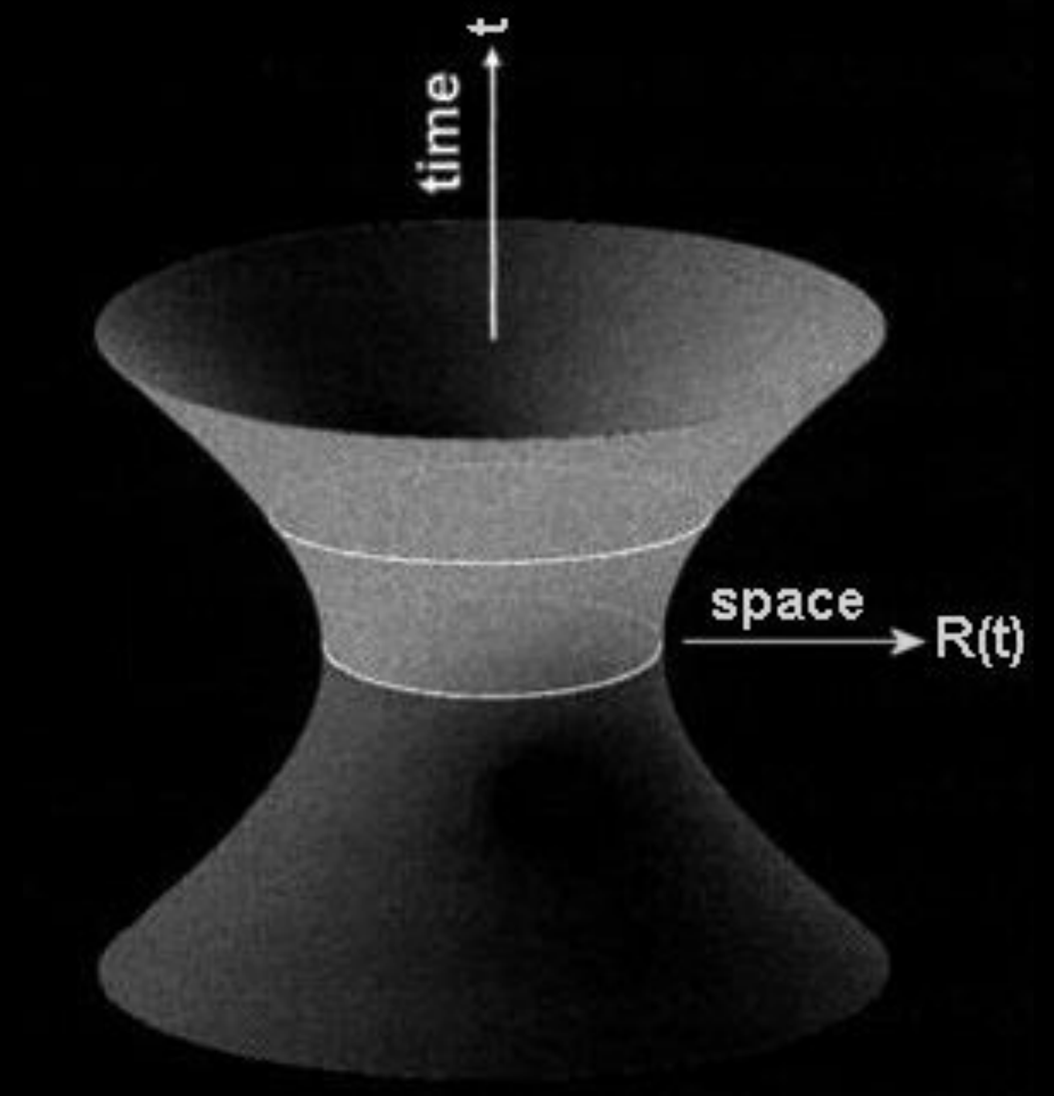
This + global fit : [arXiv:2103.14022 ECM, Enberg, Löfgren]

The SM instability

BSM?

Inflation

The earliest stage in the evolution of the Universe that we have some evidence for is inflation, a period of accelerating expansion, which made the Universe spatially flat, homogeneous and isotropic and also generated the initial seeds for structure formation.



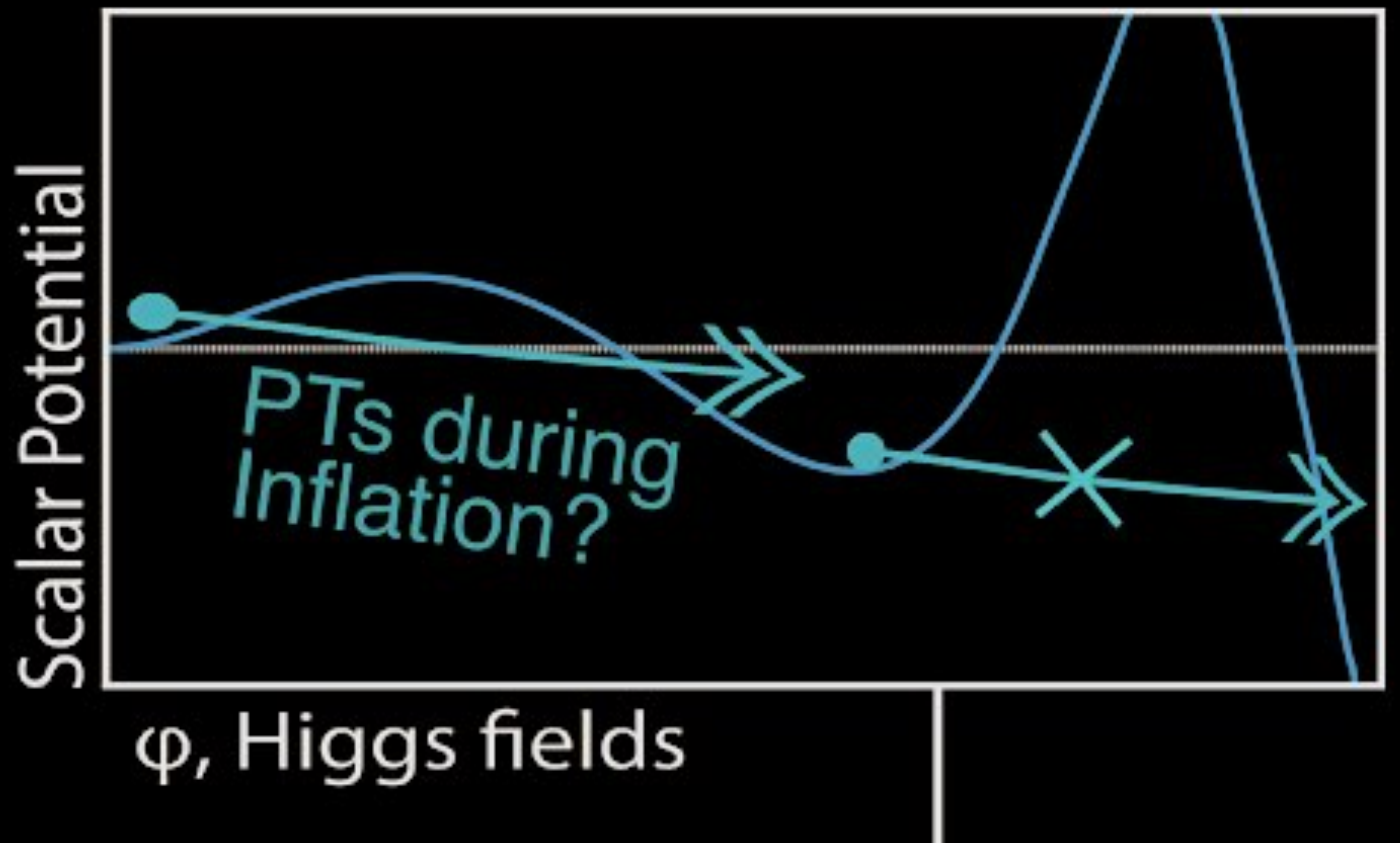
The decay rate of the SM's vacuum is enhanced during this period. At the same time, we are here, so there is a limit on how large that decay rate can be for whatever the theory of nature is.

$$\Gamma_{\rm inf} \lesssim 0.02\, e^{-3N} H_{\rm inf}^4 \sim 10^{-80} \left(\frac{V_{\rm inf}^{1/4}}{10^{16}\ {\rm GeV}} \right)^{-3} H_{\rm inf}^4 .$$

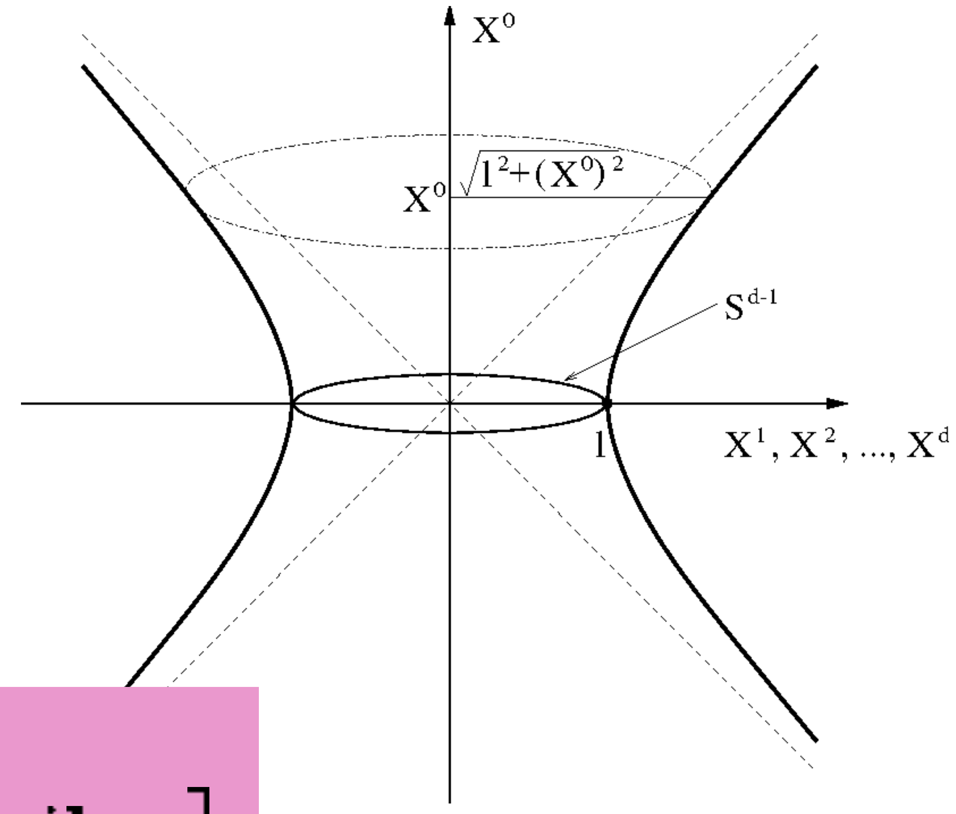
Cosmological Aspects of Higgs Vacuum Metastability

Tommi Markkanen[a] **Arttu Rajantie**[b] **Stephen Stopyra**[c]

One can use the bounds (5.22) or (5.24) to constrain the Hubble rate during inflation $H_{\rm inf}$ and other parameters of the theory. This computation can be done in two ways, either using the instanton calculation of the tunneling rate discussed in Section 4, or using the stochastic Starobinsky-Yokoyama approach discussed in Section 3.4. The instanton calculation includes both quantum tunneling and classical excitation, and it can incorporate interactions and gravitational backreaction at short distances. Because it requires analytic continuation, it only works with constant Hubble rate $H_{\rm inf}$, but it can still be expected to be a good approximation when the Hubble rate is slowly varying. In contrast, the stochastic approach can describe a time-dependent Hubble rate and gives a more detailed picture of the time evolution, but it includes only the classical excitation process and does not include interactions on sub-Hubble scales.



The Stochastic formalism for Inflation



QFT calculations in curved spacetimes are hard!
Calculations can be done by separating scalar fields in long- and short-wave (quantum) Modes,

$$\bar{\phi}(\mathbf{x},t) = \underbrace{\phi(\mathbf{x},t)}_{\text{LONG-WAVE}} + \underbrace{\int \frac{d^3k}{(2\pi)^{\frac{3}{2}}}\theta(k-\epsilon a(t)H)\left[a_{\mathbf{k}}\bar{\phi}_{\mathbf{k}}(t)e^{-i\mathbf{k}\mathbf{x}} + a^\dagger_{\mathbf{k}}\bar{\phi}^*_{\mathbf{k}}(t)e^{i\mathbf{k}\mathbf{x}}\right]}_{\text{QUANTUM}}.$$

Which leads to a jump from quantum to classical by studying field averages over Hubble volumes

$$\frac{d}{dt}\phi = -\frac{1}{3H}V'(\phi) + \xi(t) \quad \text{with } \langle \xi(t_1)\xi(t_2)\rangle = \frac{H^3}{4\pi^2}\delta(t_1-t_2).$$

White noise!

And writing everything in terms of probability distributions for the scalar fields (used to calculate exp. values)

$$P_n(t,\phi) = e^{-v(\phi)}e^{-\Lambda_n t}\psi_n(\phi),$$

where ψ_n and Λ_n are eigenfunctions and eigenvalues, respectively, of the differential equation

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial\phi^2} + W(\phi)\right]\psi_n = \frac{4\pi^2\Lambda_n}{H^3}\psi_n,$$

$$P(t;\phi) = P_{\rm eq}(\phi) + c_0\psi_0(\phi)\psi_1(\phi)e^{-\Lambda_1 t} + O\left(e^{-\Lambda_2 t}\right)$$

As long as the potential is bounded from below!

The Stochastic formalism

$$\frac{3\Omega}{\pi^2}v^+ = 4\beta\hat{\phi} + 2\bar{\alpha}\hat{\phi}^2 + \hat{\phi}^4.$$

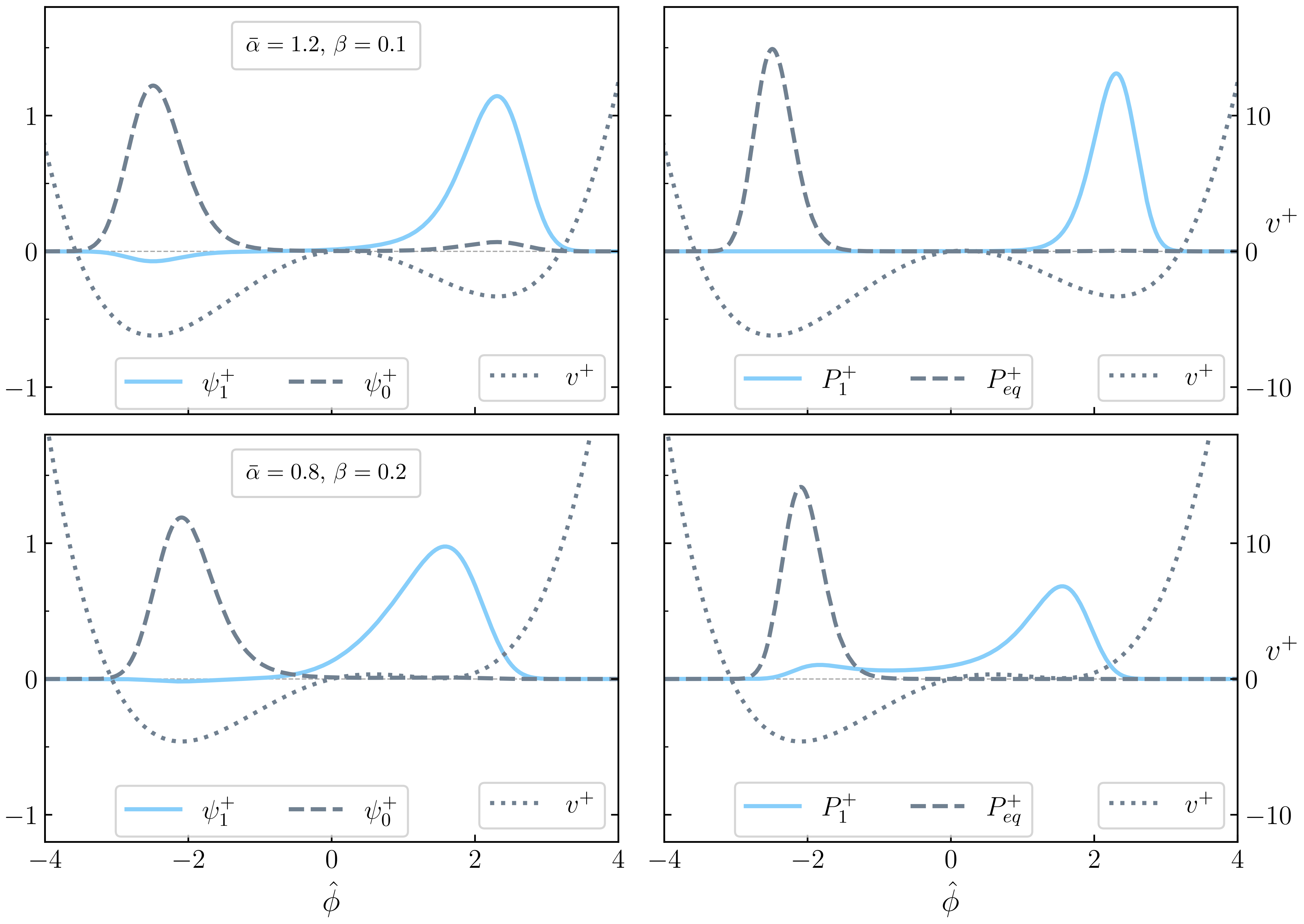


FIG. 4. Comparison between ψ_0^+, ψ_1^+ (left) and $P_{\rm eq}$, P_1^+ (right) for two sets of parameter values. The corresponding potentials are shown for illustration.

If the potential is bounded from below, and there is an excited (unstable) state with PDF P_1 then

$$\begin{aligned} P(t;\phi) &= (1-p_1(t))P_{\rm eq}(\phi) + p_1(t)P_1(\phi) \\ &= P_{\rm eq}(\phi) + p_1(t)(P_1(\phi) - P_{\rm eq}(\phi)). \end{aligned}$$

$$p_1(t) = \exp(-\Gamma(t-t_0))$$

Which compared to

$$P(t;\phi) = P_{\rm eq}(\phi) + c_0\psi_0(\phi)\psi_1(\phi)e^{-\Lambda_1 t} + O\left(e^{-\Lambda_2 t}\right)$$

Makes it easier to see that

$$\Gamma = \Lambda_1$$

[arXiv:2204.02875 ECM, Rajantie]

The Stochastic formalism

$$\frac{3\Omega}{\pi^2}v^+ = 4\beta\hat{\phi} + 2\bar{\alpha}\hat{\phi}^2 + \hat{\phi}^4.$$

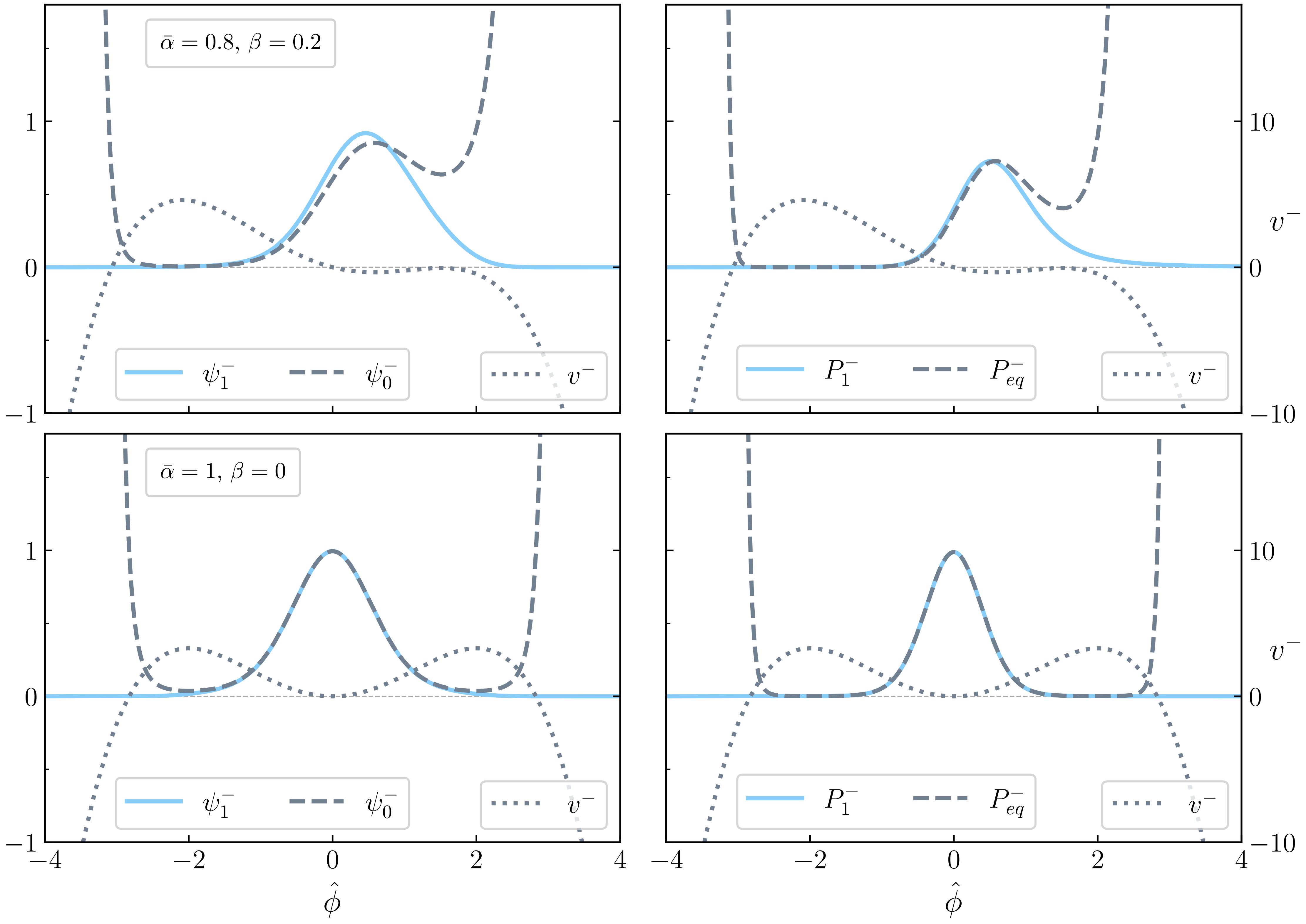


FIG. 5. Comparison between ψ_0^-, ψ_1^- (left) and $P_{\rm eq}^-$, P_1^- (right) for a non-symmetric (top) and symmetric (bottom) potentials. The corresponding potentials are shown for illustration.

Not so easy with unbounded potentials! There is no equilibrium probability. However

$$P_1(\phi) = \frac{1}{N}\psi_0(\phi)\psi_1(\phi),$$

Which we found has the nice property

$$P(t;\phi) = e^{-\Lambda_1(t-t_0)}P_1(\phi).$$

And is normalizable and looks like the "would-be" $P_{\rm eq}$ For small field values!

If we consider an observer whose existence requires the field ϕ to have a finite value, and which gets destroyed if the field ever reaches infinity, then the observer will only ever observe the conditional probability distribution which assumes that the field is finite. At any time t, this is given by

$$P\left(t;\phi \middle| |\phi| < \infty\right) = \frac{P(t;\phi)}{\int_{-\infty}^{\infty} d\phi' P(t;\phi')} = P_1(\phi). \qquad (28)$$

Therefore, the observer actually observes the field in an time-independent probability distribution $P_1(\phi)$.

$$\Gamma = \Lambda_1$$

[arXiv:2204.02875 ECM, Rajantie]

The Stochastic formalism, quantum corrections

PERTURBATION THEORY

The one-loop decay rate is

$$\frac{\Gamma}{\mathcal{V}} = \left(\frac{B}{2\pi}\right)^2 \left|\frac{\det' S''\left(\phi_b\right)}{\det S''\left(\phi_{\mathrm{fv}}\right)}\right|^{-1/2} e^{-B_R}$$

And the thermal interpretation of de Sitter spacetime: $T = H/(2\pi)$

Which is very hard to calculate even in simple theories. Using among other approximations, the saddle point approximation and treatment of different scales with statistical/field theory approaches one gets:

$$\boxed{\Gamma = \frac{\kappa}{2\pi}\sqrt{\frac{V''_{\mathrm{fv}}}{|V''_{\mathrm{top}}|}}\, e^{-\frac{8\pi^2 \Delta U^{1\ \mathrm{loop}}}{3H^4}} ,}$$

With $U^{1\mathrm{loop}}$ the constraint effective potential

$$e^{-\int \mathrm{d}^4x U^{1\ \mathrm{loop}}} = \int \mathrm{d}\phi\, e^{-S[\phi]}\, \delta\left(\frac{1}{\mathcal{V}}\int \phi\, \mathrm{d}^4x - \phi_b\right)$$

Which looks a lot like the escape rate of a particle with the potential $U^{1\mathrm{loop}}$

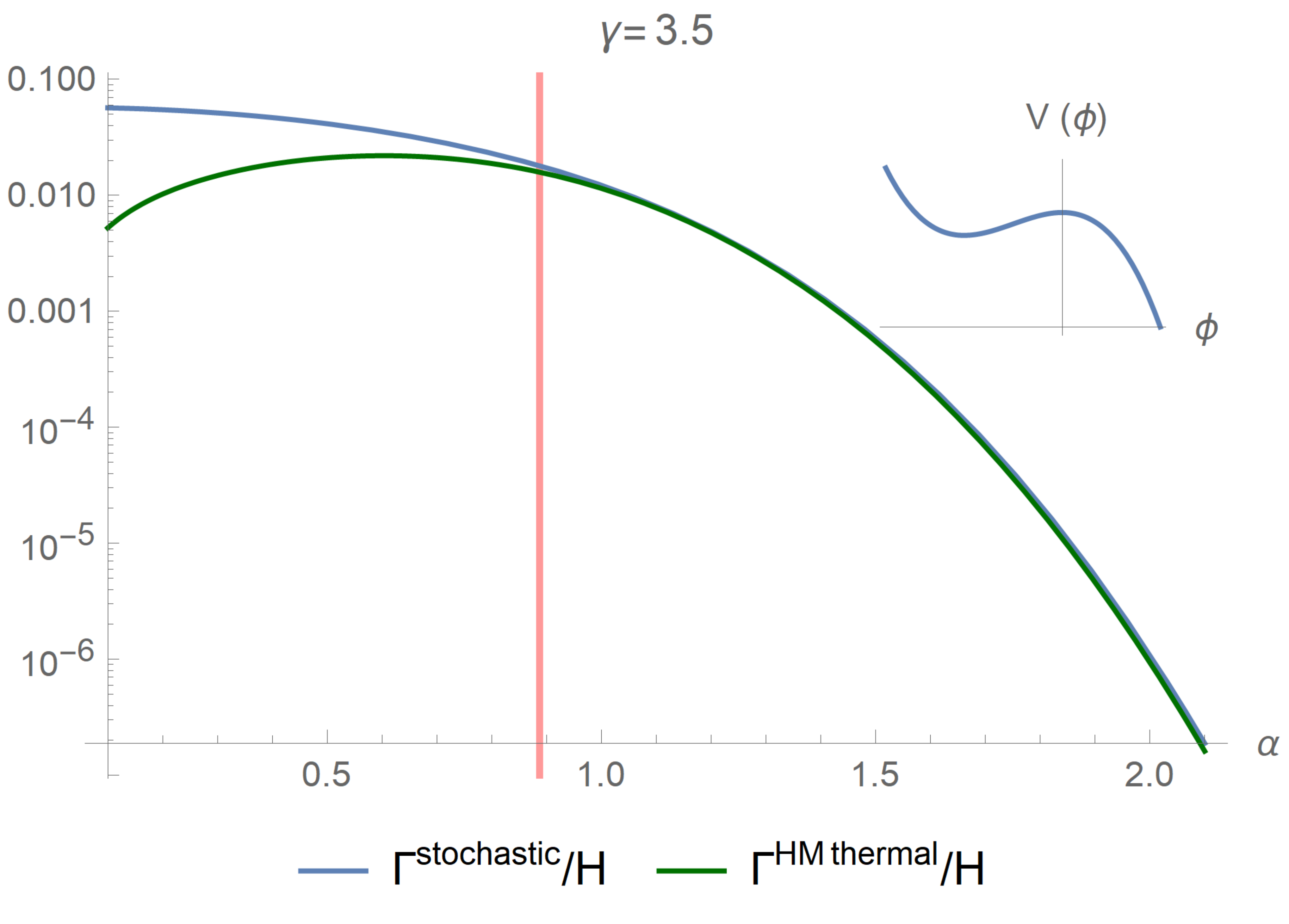
STOCHASTIC APPROACH

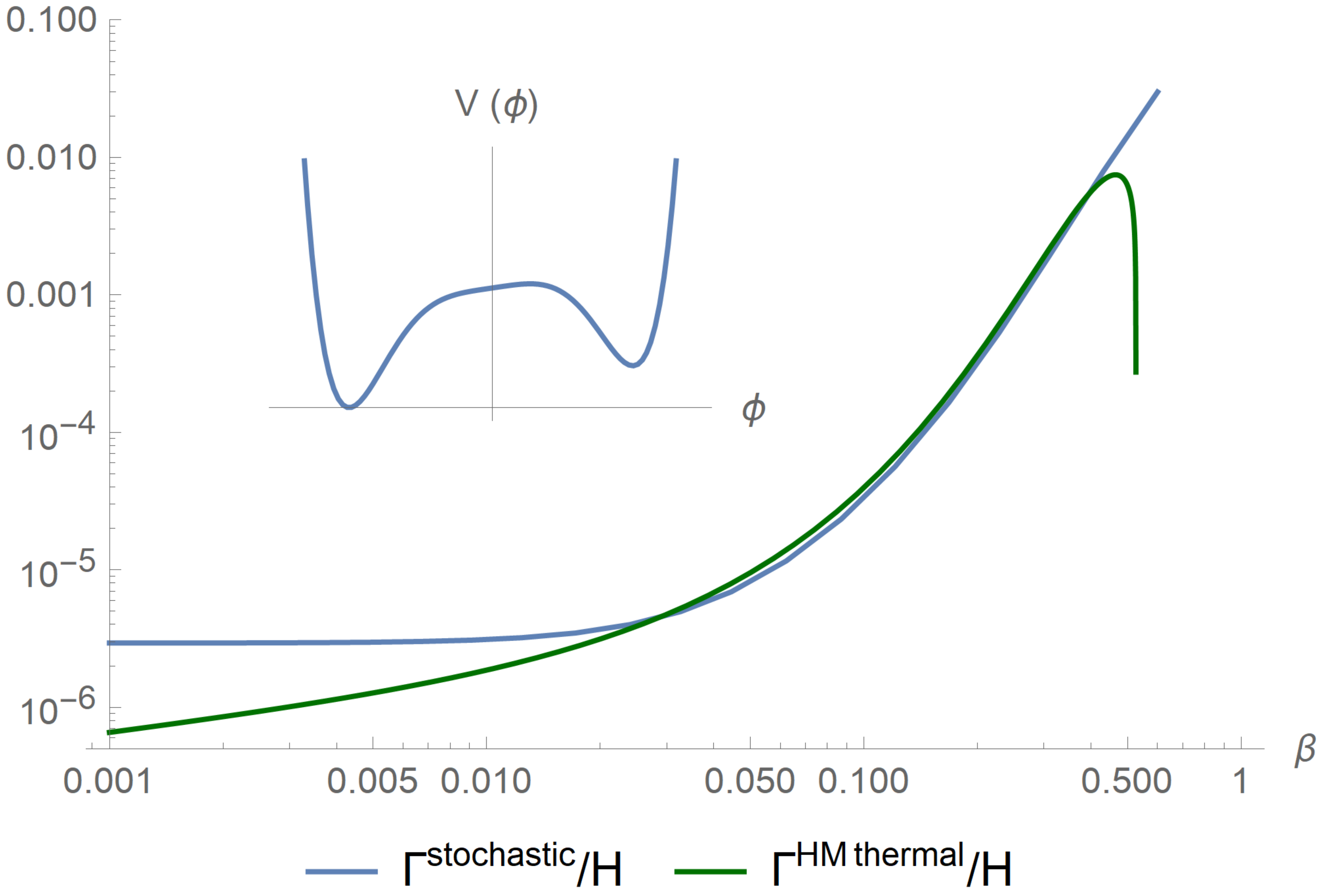
The Stochastic approach is classical but non-perturbative. It captures vacuum decay using the LO potential but no quantum corrections.

What if we use the constraint effective potential inside the Stochastic formalism to capture them?

Can we then get a formalism capturing both non perturbative and quantum corrections for decay rates in de Sitter?

The Stochastic formalism, quantum corrections





Our results suggest that the stochastic approach with the constraint effective potential can give a non-perturbative way of computing vacuum decay rates when the saddle-point approximation, which the Hawking-Moss calculation relies on, is not valid.

$$\alpha = \frac{m^2}{H^2\lambda^{1/2}}\ , \quad \beta = \frac{\mu^3}{H^3\lambda^{1/4}}$$

[arXiv:2204.03480 ECM, Carrillo González, Rajantie]

Conclusions

- Phase transitions are a powerful bridge between the early Universe and particle physics
- Early days, many interesting questions to address
- The order of the EWPT will be probed experimentally and a GW signal could be the first ”proof” for the need of physics BSM. Light NP? Heavy TeV scale NP? Both will be tackled at colliders
- Another motivation for BSM can come from inflation. Is the SM stable?
- New methods to perform calculations being developed. Combining stochastic and peturbative approaches seems to be one way forward!
- Stay tuned! Many interesting themes in combining early universe + scalars + PTs!