Electroweak Penguin Decays at LHCb

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## Electroweak penguin decays

- Processes including $b \rightarrow s \ell \ell$ transition are sensitive to New Physics (NP) contribution
- Suppressed in the SM (they can happen only via loop or boxes): small BR ~10-7 - 10-6
- New physics mediators can enter in the loops and modify the amplitudes
- SM gauge interactions have the same amplitude for all the families: Lepton Flavour Universality (LFU)

Neutral Current


New Physics


## Electroweak penguin decays

- Rare $b$ decays can be described by an effective theory:

$$
\mathrm{H}_{\mathrm{eff}} \propto \frac{4 G_{F}}{\sqrt{2}} V_{\mathrm{tb}} V_{\mathrm{ts}}^{*} \sum C_{i} \mathrm{C}_{i} \longrightarrow \begin{aligned}
& \text { Effective coupling } \\
& \text { Wilson Coefficients (WC) } \\
& \text { Local Operators }
\end{aligned}
$$



- NP can introduce new operators or modify the WCs depending on its structure: $C_{i}=C_{i}^{S M}+C_{i}^{N P}$


## Electroweak penguin decays

- Large variety of observables available:
- Relative rates of $b \rightarrow s \mu^{+} \mu^{-}$and $b \rightarrow s e^{+} e^{-}$, of the form

$$
R_{K^{(*)}}=\frac{\mathcal{B}\left(B \rightarrow K^{(*)} \mu^{+} \mu^{-}\right)}{\mathcal{B}\left(B \rightarrow K^{(*)} e^{+} e^{-}\right)} \stackrel{\text { SM }}{=} 1 \pm \mathcal{O}\left(10^{-2}\right)
$$

* are clean: QCD uncertainties cancels out in the ratio
* are predicted by the SM with very high precision


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* Reduced form factor uncertainties
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- Branching fractions:
* $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$very clean SM predictions $\sim \mathcal{O}(4 \%) \quad$ Marten's talk
* $B \rightarrow K^{*} \mu^{+} \mu^{-}, B_{s} \rightarrow \phi \mu^{+} \mu^{-}$. Suffer the most from theory uncertainties


## $b$ decays @LHCb

LHCb detector in Runs 1-2 (2010-2018)


- LHCb forward detector: $27 \%$ of $b$ hadrons produced from $p p$ collision inside acceptance ( $B^{+}, B^{0}, B_{s}, B_{c}, \Lambda_{b} \ldots$ )
- Good trigger on displaced tracks especially for di-muons channel ( $\sim 90 \%$ efficiency)
- Good PID performances from RICH 1,2, CALO and Muon Stations
- Electron ID $\sim 90 \%$ for $\sim 5 \% h \rightarrow e$
- Kaon ID $\sim 95 \%$ for $\sim 5 \% \pi \rightarrow K$
- Muon ID $\sim 97 \%$ for $\sim 1-3 \% \pi \rightarrow \mu$

Excellent tracking performances
( $\sim 96 \%$ efficiency)

- $\Delta \mathrm{p} / \mathrm{p}=0.5(1.0) \%$ at low(high) momentum
- Impact parameter resolution: $(15+29$ / pT[GeV]) $\mu \mathrm{m}$


## LFU ratio status

- Relative rates are measured as double ratios: $\quad R_{X}=\frac{B F\left(B \rightarrow X \mu^{+} \mu^{-}\right)}{B F\left(B \rightarrow X J / \psi\left(\rightarrow \mu^{+} \mu^{-}\right)\right)} \cdot \frac{B F\left(B \rightarrow X J / \psi\left(\rightarrow e^{+} e^{-}\right)\right)}{B F\left(B \rightarrow X e^{+} e^{-}\right)}$ Run1 3fb-1) JHEP 08 (2017) $055 \quad X=K, K^{*}, \Lambda_{b}$..

$$
R_{K^{* 0}}= \begin{cases}0.66_{-0.07}^{+0.11} \pm 0.03 & \text { for } 0.045<q^{2}<1.1 \mathrm{GeV}^{2} \\ 0.69_{-0.07}^{+0.11} \pm 0.05 & \text { for } 1.1 \quad<q^{2}<6.0 \mathrm{GeV}^{2}\end{cases}
$$

$$
\begin{aligned}
& \text { Run } 1+20164.7 \text { fb-D JHEP } 05 \text { (2020) } 040 \\
& R_{p K}=0.86_{-0.11}^{+0.14} \pm 0.05(1 \sigma)
\end{aligned}
$$



- Near future:


$m\left(K_{S}^{0} e^{+} e^{-}\right)\left[\mathrm{MeV} / c^{2}\right]$
- Update of $R_{p K}$ and combined $R_{K}-R_{K^{*}}$ analysis with the full dataset
- Ratio measurements with many more decay channels: $R_{\phi}, R_{K \pi \pi} \ldots$


## Angular distributions: $K^{*} \mu^{+} \mu^{-}$

- The angular distributions of the $B^{0} \rightarrow K^{*} \mu^{+} \mu^{-}$decay is described by $\Omega=\left(\theta_{\ell}, \theta_{K}, \phi\right)$
- The coefficients $F_{L}, A_{F B}, S_{i}$ are related to WCs

- New basis of $P_{i}^{\prime}$ operator to reduce form factors uncertainties:
e.g. $P_{5}^{\prime}=S_{5} / \sqrt{F_{L}\left(1-F_{L}\right)}$
- Observables are extracted from a multidimensional fits in the angles, $m(K \pi), m(K \pi \mu \mu)$



## Angular distributions: $K^{*+} \mu^{+} \mu^{-}, \phi \mu^{+} \mu^{-}$

- Recent angular analysis of $B^{+} \rightarrow K^{*+} \mu^{+} \mu^{-}$showed tension in the SM consistent with that found in $B^{0} \rightarrow K^{*} \mu^{+} \mu^{-}$
- Angular observables are also studied for $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$
- Not all observable accessible (flavour symmetric final state)
- Results found to be compatible with SM predictions
- Near future:
- Update of $B^{0} \rightarrow K^{*} \mu^{+} \mu^{-}$with the full $9 \mathrm{fb}^{-1}$ dataset
- Angular analysis with electrons: $B^{0} \rightarrow K^{*} e^{+} e^{-}$, $B^{+} \rightarrow K^{+} e^{+} e^{-}$
- Direct fits to WCs via amplitude analysis



## Differential branching fractions

- $b \rightarrow s \mu \mu \mathrm{BF}$ are measured to be consistently lower than the SM prediction
- Large hadronic form factors uncertainties (20-30\%)




 JHEP 04 (2017) 142


## Differential branching fractions: $B_{s}^{0} \rightarrow \phi \mu^{+} \mu^{-}$

- Recent update of differential $B F\left(B_{s}^{0} \rightarrow \phi(\rightarrow K K) \mu^{+} \mu^{-}\right)$
- Relative to $B_{s}^{0} \rightarrow J / \psi \phi$
- Main systematics
* Model of the simulation sample (depending on $\Delta \Gamma_{s}$ and form factors)
* Normalisation BF
- In $1.1<q^{2}<6$. GeV, $3.6 \sigma$ below the SM



- First observation of

$$
B_{s}^{0} \rightarrow f_{2}^{\prime}(1525) \mu^{+} \mu^{-}(9 \sigma)
$$

- Consistent with SM

$$
B F\left(B_{s}^{0} \rightarrow f_{2}^{\prime}(1525) \mu^{+} \mu^{-}\right)=(1.57 \pm 0.19 \pm 0.06 \pm 0.06 \pm 0.08) \times 10^{-7}
$$

$$
B_{s, d}^{0} \rightarrow \mu^{+} \mu^{-}(\gamma)
$$

PRL 128 (2021) 041801

- Helicity suppressed, very rare decays
- Precise SM predictions

$$
\begin{aligned}
& B F\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right) \stackrel{\mathrm{SM}}{=}(3.66 \pm 0.14) \times 10^{-9} \\
& B F\left(B^{0} \rightarrow \mu^{+} \mu^{-}\right) \stackrel{\mathrm{SM}}{=}(1.03 \pm 0.05) \times 10^{-10}
\end{aligned}
$$

- Sensitive to axial-vector coupling $C_{10}$

$$
\left.\begin{array}{c}
\mathcal{B}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)=\left(3.09_{-0.43-0.11}^{+0.46+0.15}\right) \times 10^{-9}, \\
\mathcal{B}\left(B^{0} \rightarrow \mu^{+} \mu^{-}\right)<2.3(2.6) \times 10^{-10}, \\
\mathcal{B}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-} \gamma\right)<1.5(2.0) \times 10^{-9} \longleftarrow \\
\text { with } m_{\mu \mu}>4.9 \mathrm{GeV} / c^{2} .
\end{array}\right] \text { First limit }
$$

- $B_{s}^{0}$ lifetime is sensitive to NP

$$
\tau_{\mu^{+} \mu^{-}}=2.07 \pm 0.29 \pm 0.03 \mathrm{ps}
$$

Consistent with the SM at $1.5 \sigma$

## Searches: $B_{(s)}^{0} \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-}$

- Helicity suppressed

$$
B F\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-}\right) \stackrel{S M}{=}(0.9-1.0) \times 10^{-10}
$$ $B F\left(B^{0} \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-}\right) \stackrel{S M}{=}(0.4-4.0) \times 10^{-12}$

- Strategy:

- Six Signal modes: non resonant, BSM scalar resonance ( $m_{a}=1 \mathrm{GeV}$ ), resonant $b \rightarrow c$
- B $\rightarrow J / \psi(\rightarrow \mu \mu) \phi(\rightarrow \mu \mu)$ as normalisation channel
- Negligible background from misID
- Main systematic from simulation model (no theoretical description of the decay's dynamic)
@95\% CL


| $\mathcal{B}\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-}\right)$ | $<8.6 \times 10^{-10}$, |
| :--- | :--- |
| $\mathcal{B}\left(B^{0} \rightarrow \mu^{+} \mu^{-} \mu^{+} \mu^{-}\right)$ | $<1.8 \times 10^{-10}$, |
| $\mathcal{B}\left(B_{s}^{0} \rightarrow a\left(\mu^{+} \mu^{-}\right) a\left(\mu^{+} \mu^{-}\right)\right)<5.8 \times 10^{-10}$ |  |
| $\mathcal{B}\left(B^{0} \rightarrow a\left(\mu^{+} \mu^{-}\right) a\left(\mu^{+} \mu^{-}\right)\right)<2.3 \times 10^{-10}$, |  |
| $\mathcal{B}\left(B_{s}^{0} \rightarrow J / \psi\left(\mu^{+} \mu^{-}\right) \mu^{+} \mu^{-}\right)$ | $<2.6 \times 10^{-9}$, |
| $\mathcal{B}\left(B^{0} \rightarrow J / \psi\left(\mu^{+} \mu^{-}\right) \mu^{+} \mu^{-}\right)$ | $<1.0 \times 10^{-9}$. |

- No signal observed, most stringent limits up to date!
- $B F \sim \mathcal{O}\left(10^{-11}\right)$, sensitive to new mediators
- Strategy:

$$
\begin{aligned}
& \mathcal{R}<4.4 \times 10^{-3} \text { at a } 90 \% \\
& \mathcal{B}\left(B^{0} \rightarrow \phi \mu^{+} \mu^{-}\right)<3.2 \times 10^{-9} \text { at a } 90 \% \mathrm{CL}
\end{aligned}
$$

- Measure the ratio $R=B F\left(B^{0} \rightarrow \phi \mu \mu\right) / B F\left(B_{s}^{0} \rightarrow \phi \mu \mu\right)$

First limit

- $B_{s}^{0} \rightarrow J / \psi \phi$ employed for BDT training and mass modelling
- No signal observed



## Conclusion \& outlook

- Electroweak penguin decays are ideal probes for New Physics
- LHCb intensively studied these processes over the years
- Several measurements to be updated with the full dataset
- Run3 is starting!

Reduce statistical + data-driven models uncertainties

- $\sim 3$ times Run1+2 dataset collected in 3 years
- LHCb detector undergoing staged upgrades
* Replaced vertex, tracking detectors: Better vertex resolution
* Removed hardware trigger: Better efficiency

Reduce background from charged and neutral tracks Electron modes more accessible

## Thank you

## Backup

## The flavour puzzle

$$
\begin{aligned}
& \psi=Q_{L}, u_{r}, d_{r}, L_{L}, e_{r} \\
& Q_{L}=\binom{u_{L}}{d_{L}} \quad L_{L}=\binom{v_{L}}{e_{L}}
\end{aligned}
$$

$$
\mathscr{L}_{\mathrm{SM}}=\mathscr{L}_{\text {gauge }}\left(\psi_{i}, A_{a}\right)+\mathscr{L}_{\mathrm{Higgs}}\left(\psi_{i}, A_{a}, H\right)
$$

$$
\mathscr{L}_{\text {gauge }}=\sum_{a} \frac{-1}{4 g_{a}^{2}}\left(F_{\mu \nu}\right)^{2}+\sum_{i=1}^{3} \bar{\psi}_{i} i \nabla \psi_{i}
$$

$$
\mathscr{L}_{\text {Higgs }}=\mathscr{L}_{H}+\mathscr{L}_{\text {Yukawa }}
$$

Only Yukawa interaction distinguishes the families

3 identical replica ( $i=1,2,3$ ) of the same family differing only in mass
Gauge interactions have the same amplitude for all the families: Lepton Flavour Universality (LFU)
Flavour is conserved: stringent limits on
Lepton Flavour Violating (LFV) decays

- Why 3 generations?
- What is the origin of their different mass?

Quark

$$
\bar{Q}_{L}{ }^{i} Y_{D}{ }^{i k} d_{R}{ }^{k} \mathrm{H}+\text { h.c. } \rightarrow \bar{d}_{L}{ }^{i} M_{D}{ }^{i k} d_{R}{ }^{k}+\ldots
$$

$$
\text { sector: } \quad \bar{Q}_{L}{ }^{i} Y_{U}{ }^{i k} u_{R}{ }^{k} H_{\mathrm{c}}+\text { h.c. } \rightarrow \bar{u}_{L}{ }^{i} M_{U}{ }^{i k} u_{R}{ }^{k}+\ldots
$$

Only one mass matrix at time can
be diagonalised (for gauge flavour invariance)

$M_{D}=\operatorname{diag}\left(\mathrm{m}_{\mathrm{d}}, \mathrm{m}_{\mathrm{s}}, \mathrm{m}_{\mathrm{b}}\right)$ $M_{U}=\mathrm{V}^{+} \times \operatorname{diag}\left(\mathrm{m}_{\mathrm{u}}, \mathrm{m}_{\mathrm{c}}, \mathrm{m}_{\mathrm{t}}\right)$

V_CKM appears in chargedcurrent gauge interaction (mixing $u$ and $d$ )

Saracelafhat is the origin of the hierarchy in quatk ${ }^{2}$ mizixing?

## LHCb upgrade - Phase I



## LFU ratios: Electron vs Muon

- Relative rates are measured as double ratios

$$
R_{X}=\frac{\mathcal{N}_{B \rightarrow X \mu^{+} \mu^{-}}}{\mathcal{N}_{B \rightarrow X\left(J / \psi \rightarrow \mu^{+} \mu^{-}\right)}} \cdot \frac{\mathcal{N}_{B \rightarrow X\left(J / \psi \rightarrow e^{+} e^{-}\right)}}{\mathcal{N}_{B \rightarrow X e^{+} e^{-}}} \cdot \frac{\epsilon_{B \rightarrow X\left(J / \psi \rightarrow \mu^{+} \mu^{-}\right)}}{\epsilon_{B \rightarrow X \mu^{+} \mu^{-}}} \cdot \frac{\epsilon_{B \rightarrow X e^{+} e^{-}}}{\epsilon_{B \rightarrow X\left(J / \psi \rightarrow e^{+} e^{-}\right)}}
$$

$$
X=K, K^{*}, \Lambda_{b} \ldots
$$

- J/ $\psi \rightarrow \ell \ell$ satisfies lepton universality at $0.4 \%$ precision (PDG)
- Reduced systematics due to leptons reconstruction differences:
- Most electrons emit bremsstrahlung photons
- Electrons has worse $p$ resolution
- Electrons has lower trigger, PID and tracking efficiencies





## Cross-checks: $r_{J / \psi}$ and $R_{\psi(2 S)}$

- Extensive use of $B \rightarrow X_{s}\left(J / \psi \rightarrow \ell^{+} \ell^{-}\right)$and $B \rightarrow X_{s}\left(\psi(2 S) \rightarrow \ell^{+} \ell^{-}\right)$ to check that efficiencies are under control
- Check: $r_{J / \psi} \equiv \frac{\mathcal{B}\left(B \rightarrow X J / \psi\left(\rightarrow \mu^{+} \mu^{-}\right)\right)^{\mathrm{SM}}}{\mathcal{B}\left(B \rightarrow X J / \psi\left(\rightarrow e^{+} e^{-}\right)\right)}=1$ [0.4\% precision (PDG)]

+ absence of trends on any kinematics variables

- Check: $R_{\psi(2 S)}=\frac{\mathcal{B}(B \rightarrow X(\psi(2 S) \rightarrow \mu \mu))}{\mathcal{B}(B \rightarrow X(J / \psi \rightarrow \mu \mu))} \cdot \frac{\mathcal{B}(B \rightarrow X(J / \psi \rightarrow e e))}{\mathcal{B}(B \rightarrow X(\psi(2 S) \rightarrow e e))} \stackrel{\text { SM }}{=} 1 \quad[\sim 1 \%$ precision (PDG)]

Validation of the double ratio procedure (effective cancelation of syst uncertainties)

## $B \rightarrow K^{*} \mu^{+} \mu^{-}$angular distributions

$$
\frac{1}{\mathrm{~d}(\Gamma+\bar{\Gamma}) / \mathrm{d} q^{2}} \frac{\mathrm{~d}^{4}(\Gamma+\bar{\Gamma})}{\mathrm{d} q^{2} \mathrm{~d} \vec{\Omega}}=\frac{9}{32 \pi}\left[\frac{3}{4}\left(1-F_{\mathrm{L}}\right) \sin ^{2} \theta_{K}+F_{\mathrm{L}} \cos ^{2} \theta_{K}\right.
$$



$$
+\frac{1}{4}\left(1-F_{\mathrm{L}}\right) \sin ^{2} \theta_{K} \cos 2 \theta_{l}
$$

$$
-F_{\mathrm{L}} \cos ^{2} \theta_{K} \cos 2 \theta_{l}+S_{3} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \cos 2 \phi
$$

$$
+S_{4} \sin 2 \theta_{K} \sin 2 \theta_{l} \cos \phi+S_{5} \sin 2 \theta_{K} \sin \theta_{l} \cos \phi
$$

$$
+\frac{4}{3} A_{\mathrm{FB}} \sin ^{2} \theta_{K} \cos \theta_{l}+S_{7} \sin 2 \theta_{K} \sin \theta_{l} \sin \phi
$$

$\left.+S_{8} \sin 2 \theta_{K} \sin 2 \theta_{l} \sin \phi+S_{9} \sin ^{2} \theta_{K} \sin ^{2} \theta_{l} \sin 2 \phi\right]$

$$
B_{s, d}^{0} \rightarrow \mu^{+} \mu^{-}(\gamma)
$$

- Helicity suppressed, very rare decays $B F \sim \mathcal{O}\left(10^{-9}, 10^{-10}\right)$
- Precise SM predictions $\sim \mathcal{O}(4 \%)$
- Sensitive to axial-vector coupling $C_{10}$
- Strategy:
- Two opposite charged tracks from a displaced vertex
- BDT vs combinatorial, stringent PID vs $\mu \leftrightarrow K, \pi$ misID
- Yields normalised to $B^{0} \rightarrow K^{+} \pi^{-}$and $B^{+} \rightarrow J / \psi K^{+}$



- The $B_{s}^{0}$ lifetime is sensitive to NP

$$
\tau_{\mu^{+} \mu^{-}}=\frac{\tau_{B_{s}^{0}}}{1-y_{s}^{2}}\left[\frac{1+2 A_{\Delta \Gamma_{s}}^{\mu \mu} y_{s}+y_{s}^{2}}{1+A_{\Delta \Gamma_{s}}^{\mu \mu} y_{s}}\right]
$$

- $A_{\Delta \Gamma_{s}}^{\mu \mu}=1$ for the SM
* $B_{s}^{0} \rightarrow \mu \mu$ only from heavy mass eigenstate
* Access to the CP structure of the interaction
- Strategy:
- Dataset split into two BDT bins
- Fit to background-subtracted $\tau_{\mu \mu}$ distribution via the sPlot technique

$$
\tau_{\mu^{+} \mu^{-}}=2.07 \pm 0.29 \pm 0.03 \mathrm{ps}
$$

Consistent with the SM at $1.5 \sigma$

- Oscillations between flavour eigenstates $B_{s}^{0}, \bar{B}_{s}^{0}$
- Two mass eigenstates $B_{H}, B_{L}$
- For SM $B_{s}^{0} \rightarrow \mu^{+} \mu^{-}$only from heavy eigenstates
- $A_{\Delta \Gamma_{s}}^{\mu \mu}$ sensitive to scalar or pseudo scalar NP contribution: $C_{10}^{\left({ }^{(1)}\right.}, C_{S}^{\left({ }^{()}\right.}, C_{P}^{(1)} \mathrm{WCs}$

$$
\begin{aligned}
& \tau_{\mu^{+} \mu^{-}}=\frac{\tau_{B_{s}^{0}}}{1-y_{s}^{2}}\left[\frac{1+2 A_{\Delta \Gamma_{s}}^{\mu \mu} y_{s}+y_{s}^{2}}{1+A_{\Delta \Gamma_{s}}^{\mu \mu} y_{s}}\right] \\
& y_{s} \equiv \Delta \Gamma_{s} /\left(2 \Gamma_{s}\right) \quad A_{\Delta \Gamma_{s}}^{\mu \mu} \equiv-2 \Re(\lambda) /\left(1+|\lambda|^{2}\right),
\end{aligned}
$$

$$
\text { with } \lambda=(q / p)\left(A\left(\bar{B}_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right) / A\left(B_{s}^{0} \rightarrow \mu^{+} \mu^{-}\right)\right)
$$

$$
\mathcal{A}_{\Delta \Gamma}=\frac{R_{H}-R_{L}}{R_{H}+R_{L}} \stackrel{S M}{=} 1
$$



