Cabibbo angle anomalies: indication of new physics at TeV scale?

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B. B. and Z. Berezhiani JHEP 10, 079 (2021), 2103.05549
B. B., R. Beradze and Z. Berezhiani, Eur. Phys. J. C 80, no.2, 149 (2020), 1906.02714

The Standard Model

$$-\mathcal{L}_{\rm Yuk} = Y_u^{ij}\tilde{\phi}\overline{q_{Li}}u_{Rj} + Y_d^{ij}\phi\overline{q_{Li}}d_{Rj} + Y_e^{ij}\phi\overline{\ell_{Li}}e_{Rj} + \text{h.c.}$$

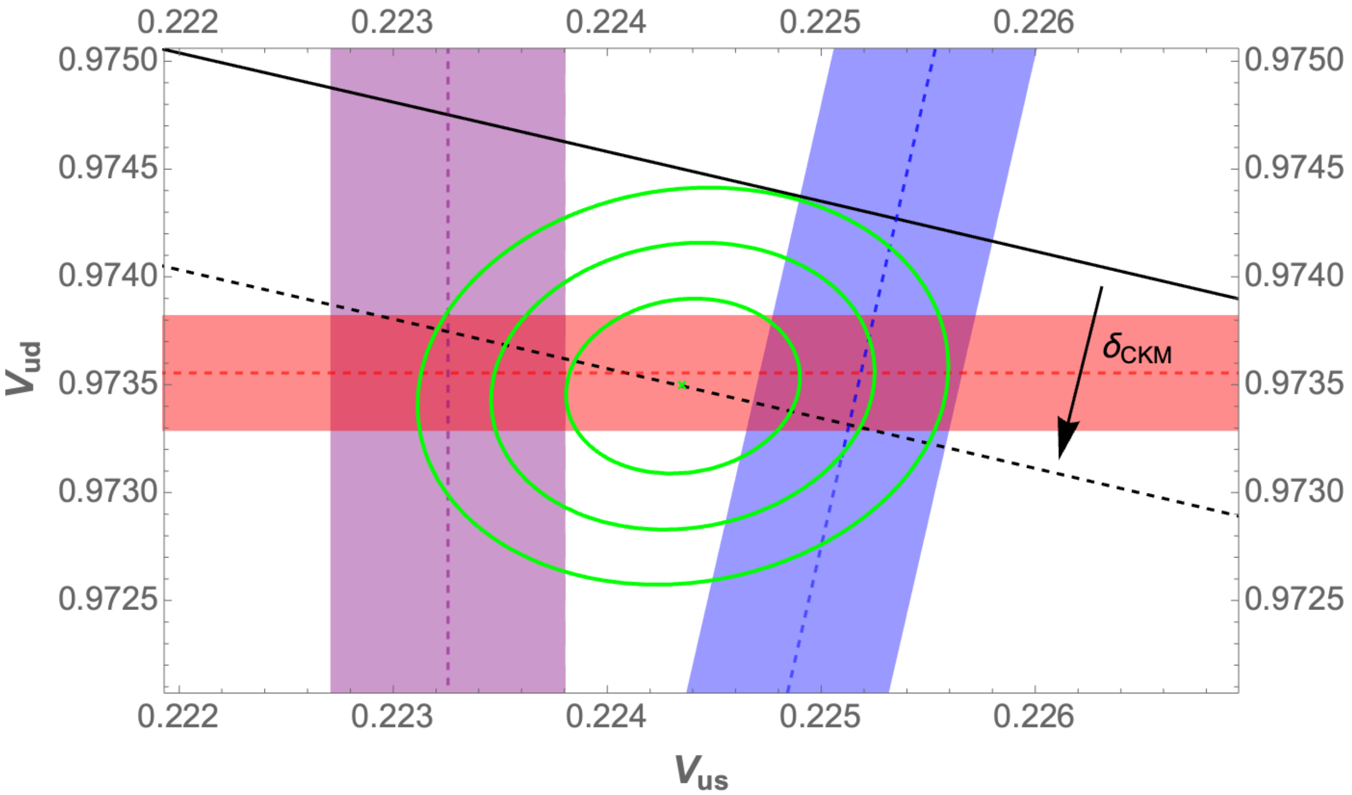
- Three fermion families in identical representations of gauge symmetry; a single Higgs doublet
- Yukawa matrices are not diagonal in weak basis:

$$m^{(u,d,e)ij} = Y^{ij}_{u,d,e} v_w \,, \qquad \mathbf{m}^{u,d,e}_{\rm diag} = V_L^{(u,d,e)\dagger}\mathbf{m}^{(u,d,e)}V_R^{(u,d,e)}$$

- Quark charged currents in mass basis: $\frac{g}{\sqrt{2}}\begin{pmatrix} u & c & t \end{pmatrix}_L \gamma^\mu V_L^{(u)\dagger}V_L^{(d)}\begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\dagger_\mu$
- $V_{\rm CKM} = V_L^{(u)\dagger}V_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$
- **$\mathbf{V}_{\rm CKM}$ is unitary**.
- The unitarity condition for the first row is: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$
- Yukawa couplings and photon/Z couplings (for **unitarity** of $V_L^{(u)}$ and $V_L^{(d)}$) are diagonal in mass basis.
- **NO flavour changing neutral currents** at tree level.

Cabibbo angle anomalies

3.1σ away from unitarity



C: $|V_{ud}| = 0.97355(27)$ β decays

A: $|V_{us}| = 0.22326(55)$ semileptonic $K \to \pi \ell \nu$ decays

B: $\frac{|V_{us}|}{|V_{ud}|} = 0.23130(49)$ ratio of leptonic decay rates $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$

— Unitarity

- CKM unitarity problem: $\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta_{\rm CKM} \ \to \ \delta_{\rm CKM} \approx 2 \times 10^{-3}}$
- Discrepancy between determinations A and B.

Cabibbo angle anomalies

A: $|V_{us}| = 0.22326(55)$

B: $\frac{|V_{us}|}{|V_{ud}|} = 0.23130(49)$

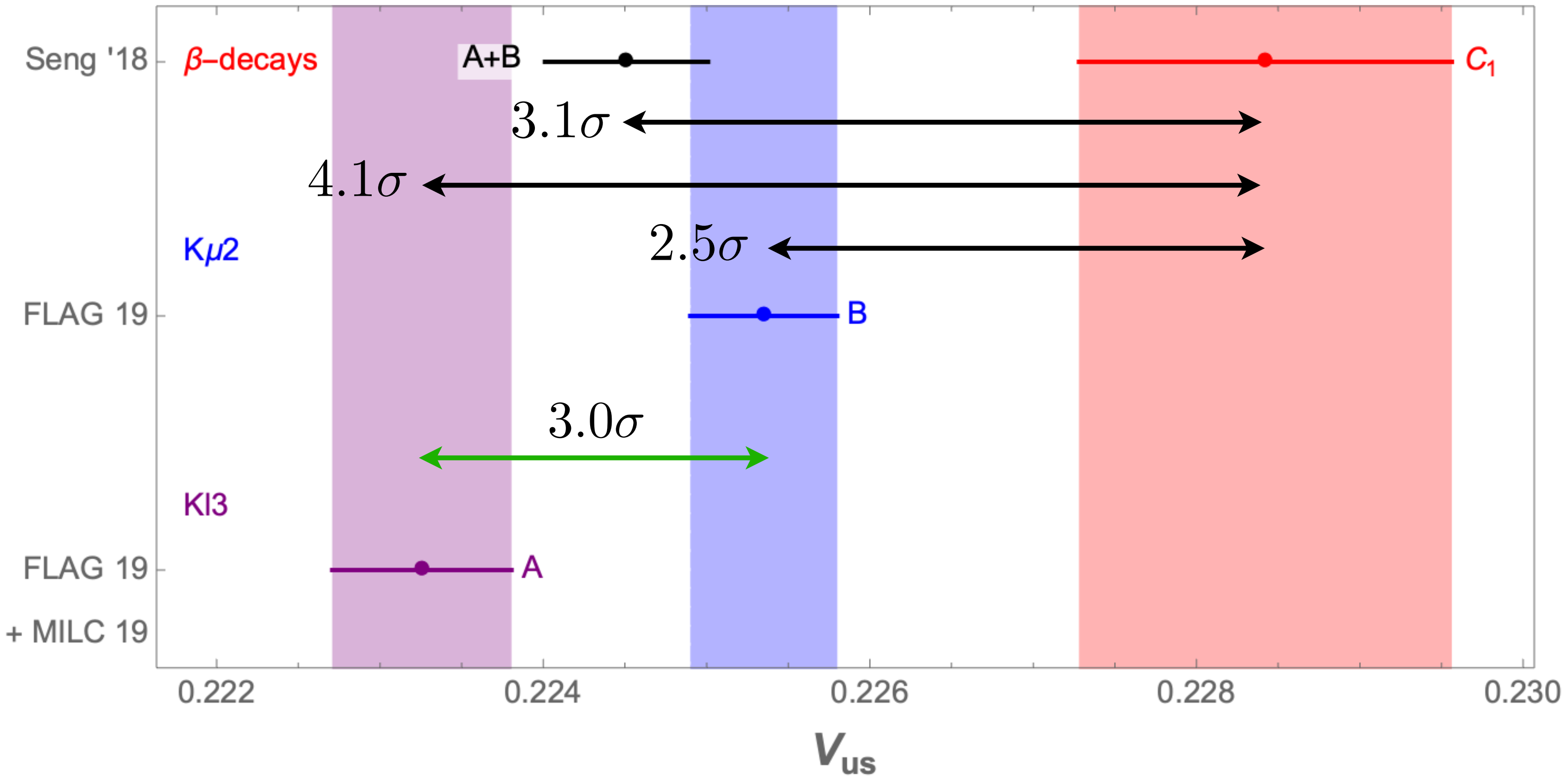
C: $|V_{ud}| = 0.97355(27)$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{us}|_{\mathrm{A}} = 0.22326(55)$

$|V_{us}|_{\mathrm{B}} = 0.22535(45)$

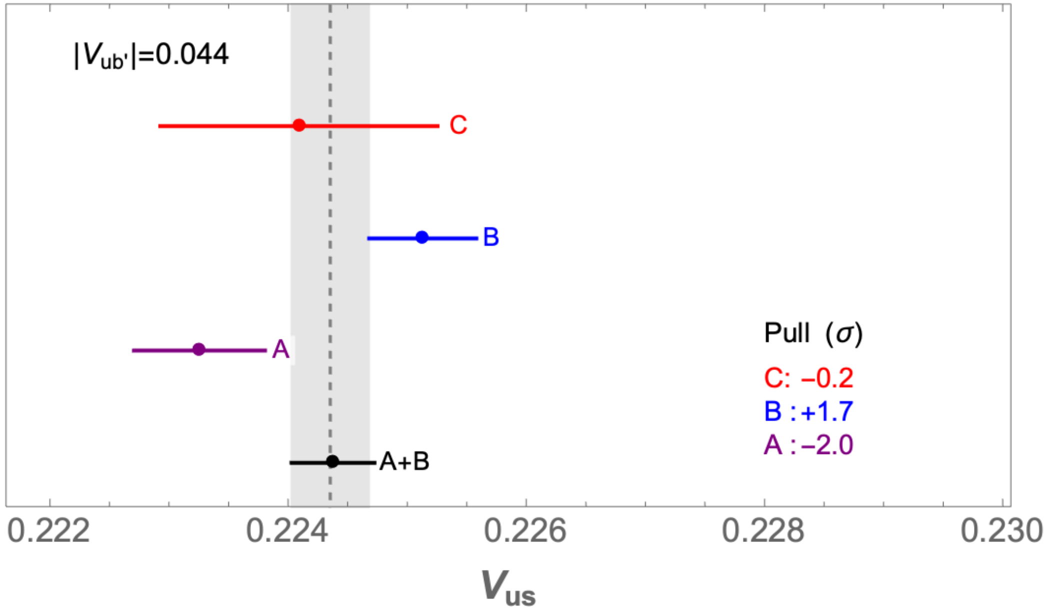
$|V_{us}|_{\mathrm{C}} = 0.2284(11)$

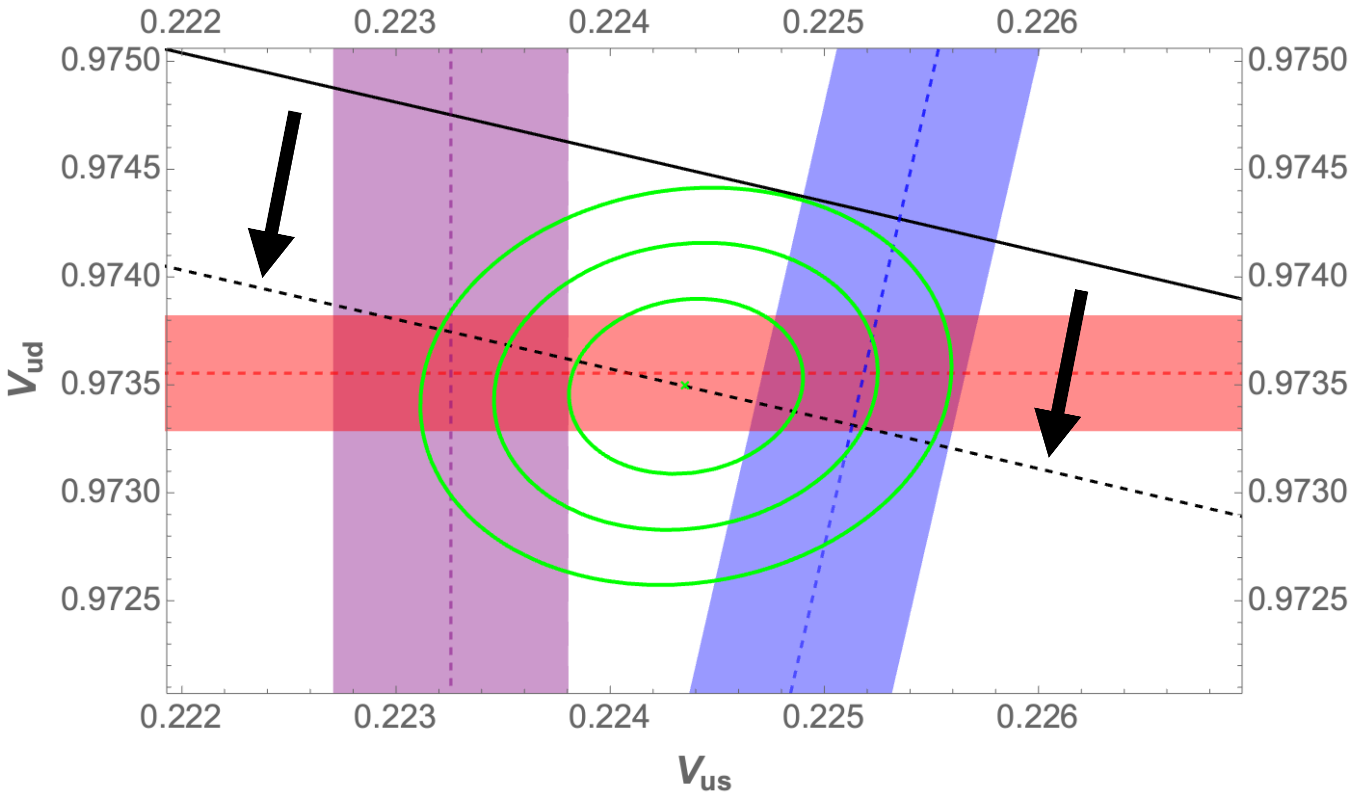


Vector-like weak singlets

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 \,, \qquad |V_{ub'}| \approx 0.044 \qquad (|V_{ub'}| \gg |V_{ub}|)$$

- Extra down-type weak singlets
- Extra up-type weak singlets





4th sequential family is excluded by SM precision tests, LHC mass limits, Higgs production via gluon fusion and its 2γ decay. However additional vector-like fermions can be introduced.

Down-type weak singlets

Down-type vector-like quark $D_{L,R}$, with left and right components both $SU(2)$ singlets, mixing with SM quarks:

$$\ldots + h_{Dj}\,\overline{q_{Lj}}\phi D_R + M_D \overline{D_L} D_R + \text{h.c.}$$

- $V_L^{(d)\dagger}\mathbf{m}^{(d)}V_R^{(d)} = \mathbf{m}^{(d)}_{\text{diag}} = \text{diag}(m_d, m_s, m_b, M_{b'})$ $\qquad |V_{LD\alpha}| \approx |h_{D\alpha}| v_w / M_D$

- Charged weak interactions:

$$\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_{L1}} & \overline{u_{L2}} & \overline{u_{L3}} \end{pmatrix} \gamma^\mu \begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \end{pmatrix} W^+_\mu + \text{h.c.} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{pmatrix} \gamma^\mu \tilde{V}_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W^+_\mu + \text{h.c.}$$

$$\tilde{V}_{CKM} = V_L^{(u)\dagger}\tilde{V}_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix}$$

- $|V_{ub'}| \approx |V_{LDd}| \approx h_{d1} v_w / M_{b'}$

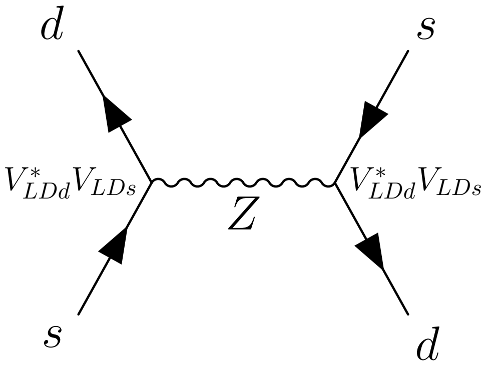
- Weak neutral currents: $\mathcal{L}_{\text{nc}} = \dfrac{g}{\cos\theta_W} Z_\mu \left[\boxed{T_3(f_{L,R})} - Q(f)\sin^2\theta_W\right] \overline{f_{L,R}}\gamma^\mu f_{L,R}$

(not the same quantum numbers)

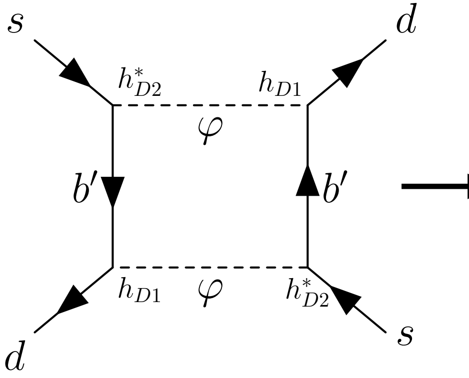
- Tree and loop level **flavour changing** couplings with the Higgs boson and with Z-boson.

Down-type weak singlets

Neutral K mesons mixing:

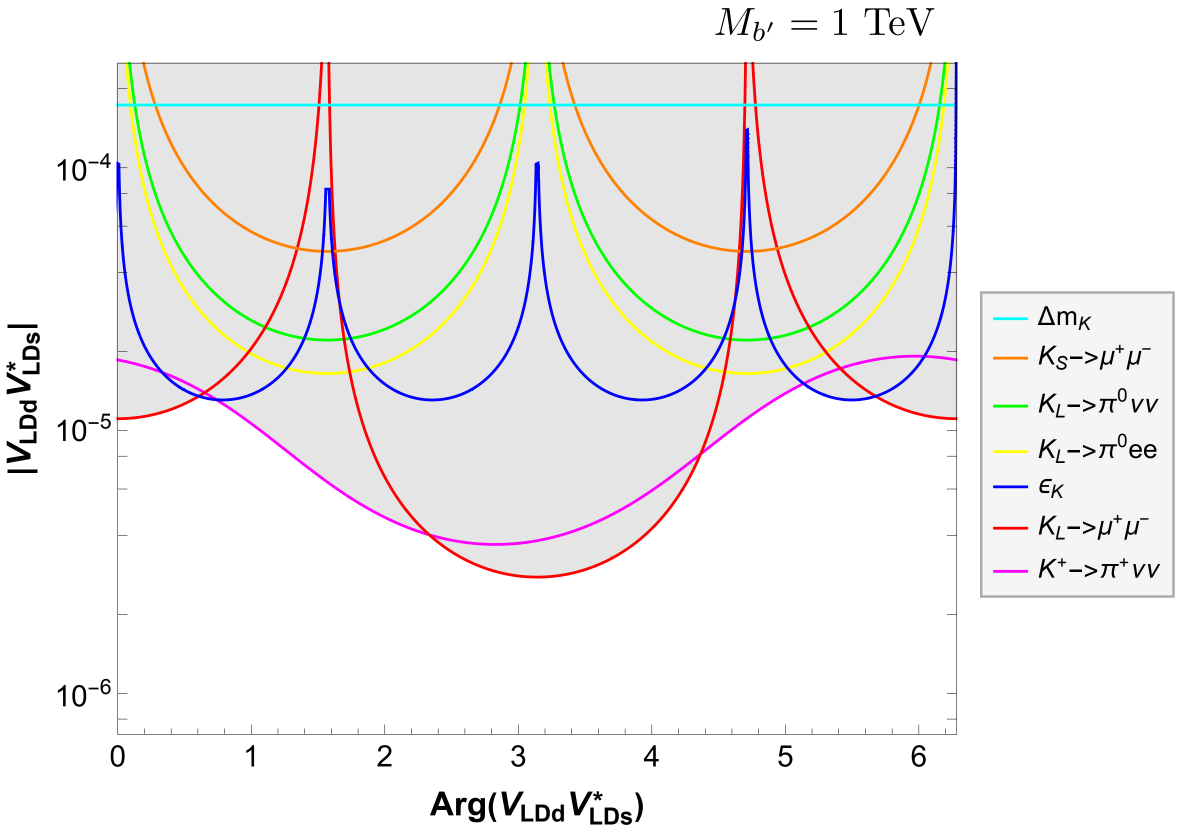


$\propto (V^*_{LDd}V_{LDs})^2$

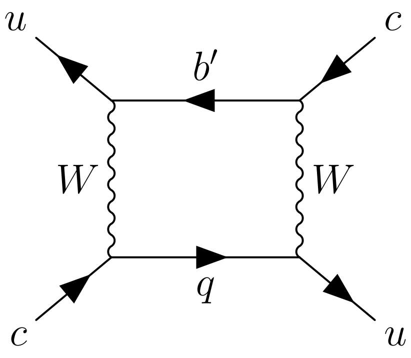


$\propto (V^*_{LDd}V_{LDs})^2 M^2_{b'}$

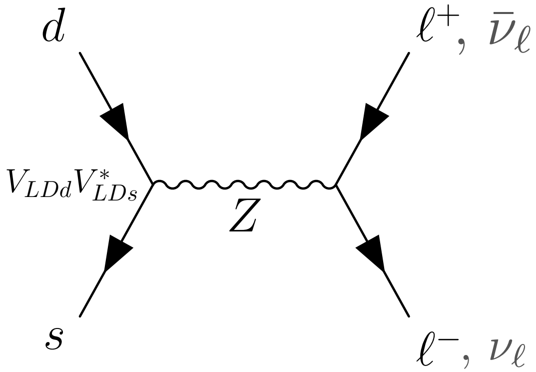
$\longrightarrow$ Relevant for $M_{b'} > 3$ TeV



Neutral D mesons mixing:

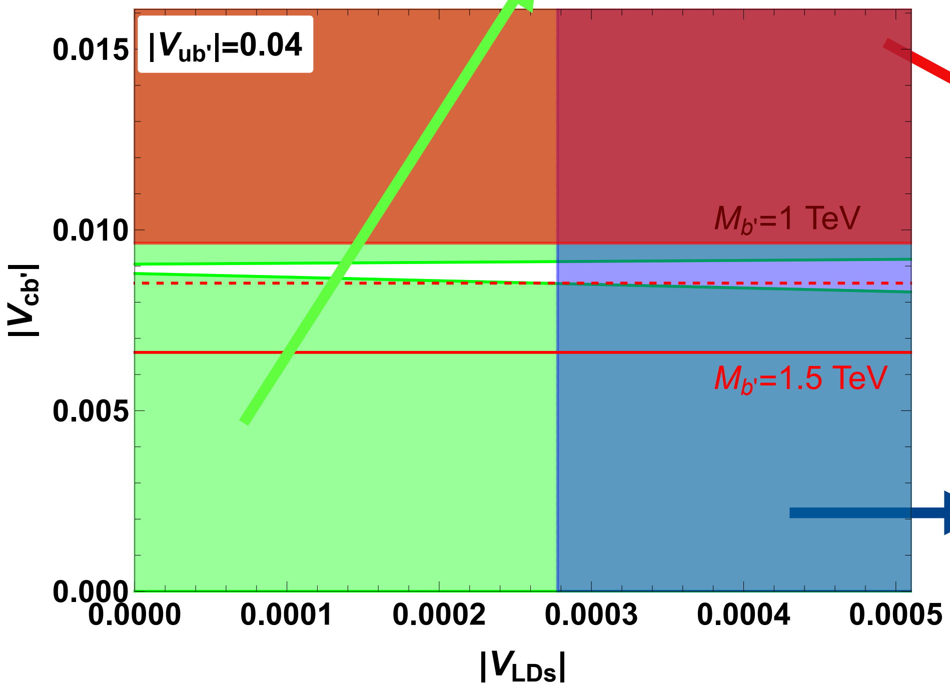


Kaon decays:



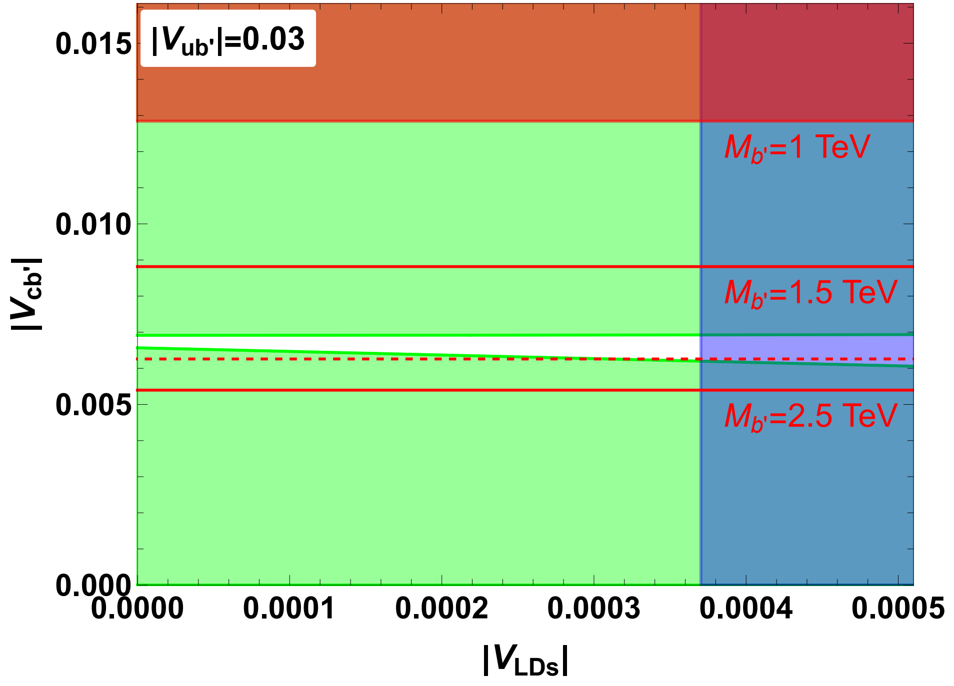
Down-type weak singlets

$$|V_{cb'}| = |-V^*_{LDd}V_{cd} - V^*_{LDs}V_{cs} - V^*_{LDb}V_{cb}|$$



Excluded by D-mesons mixing

Excluded by K-mesons FC processes



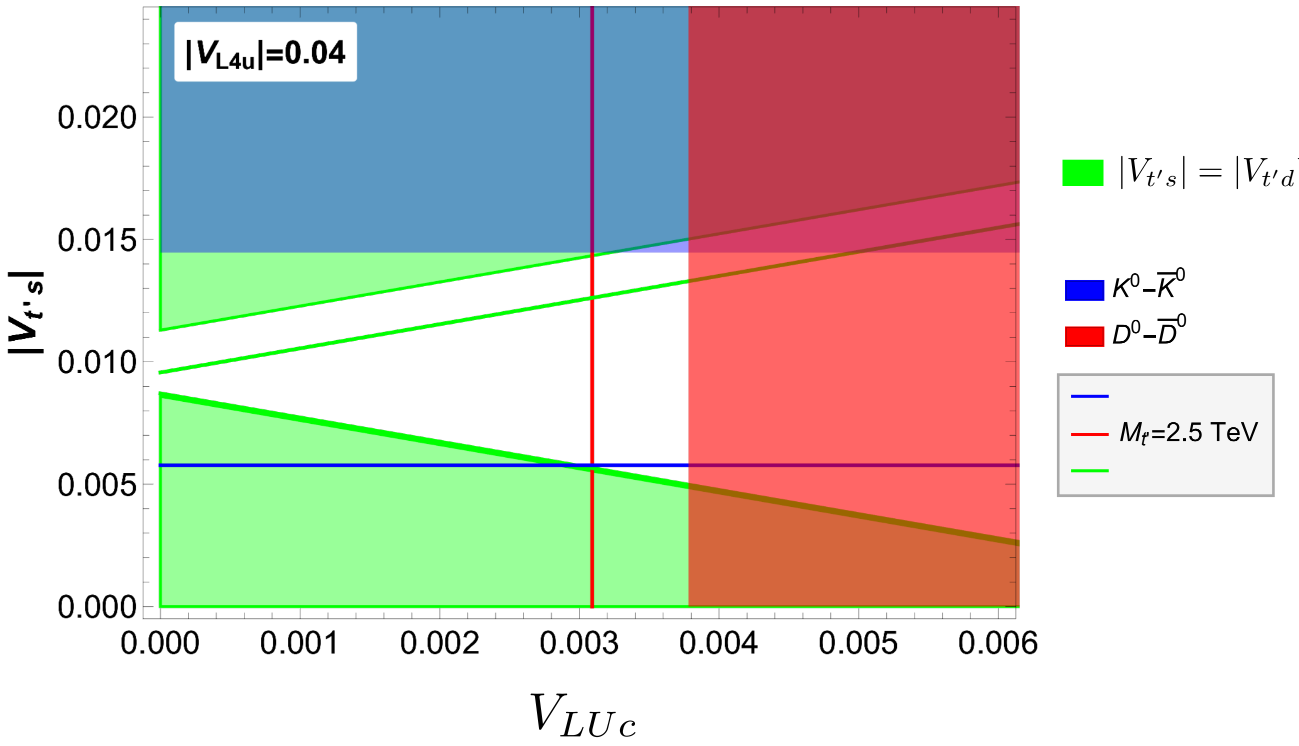
- Mass of the extra quark cannot exceed **few TeV**, also in the most conservative case.
- $|V_{ub'}| < 0.042$ with $M_{b'} = 1$ TeV.
- $|V_{ub'}| < 0.050$ from Z boson decay rate.
- $|V_{cb'}| \lesssim 10^{-2}$, $|V_{tb'}| < 7.4 \cdot 10^{-3}$ (from B-mesons physics).
- Yukawa couplings h_{Ds}, h_{Db} should be respectively 50 and 4 times smaller than h_{Dd}.

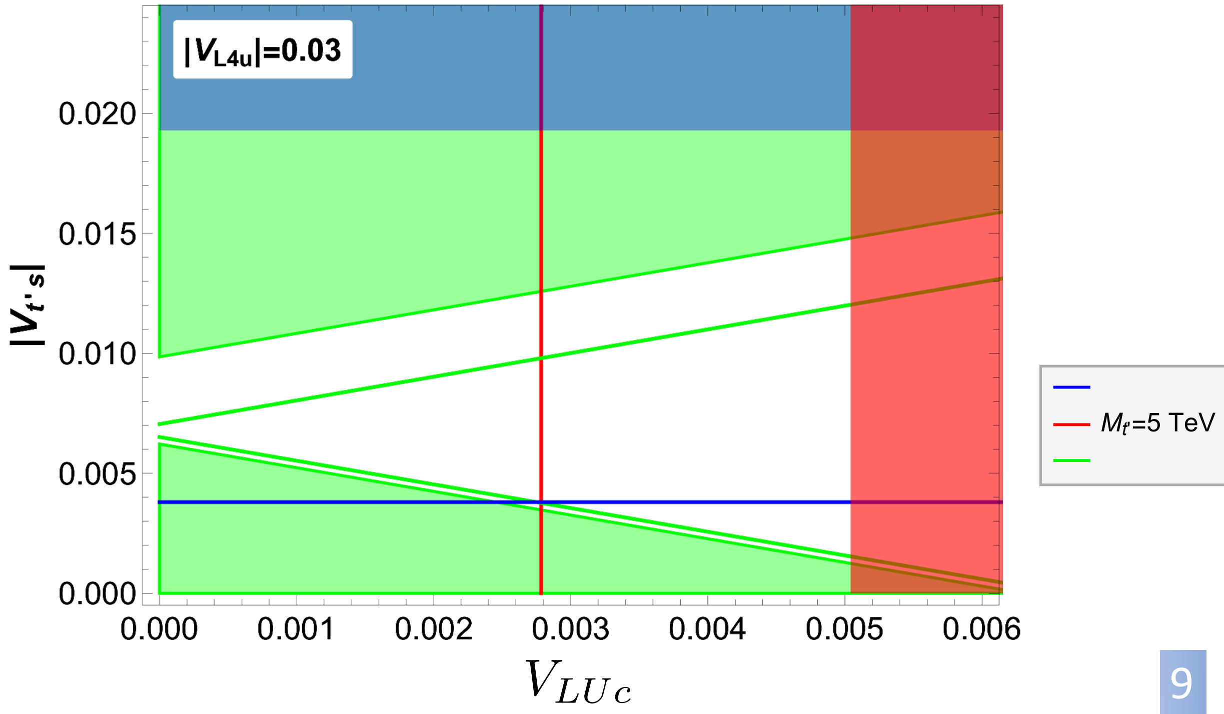
Up-type weak singlets

Vector-like up-type quark $U_{L,R}$ with left and right components both $SU(2)$ singlets, mixing with SM quarks:

$$\ldots + h_{Uj}\,\overline{q_{Lj}}\tilde{\phi}\,U_R + M_{t'}\overline{U_L}U_R + \text{h.c.}$$

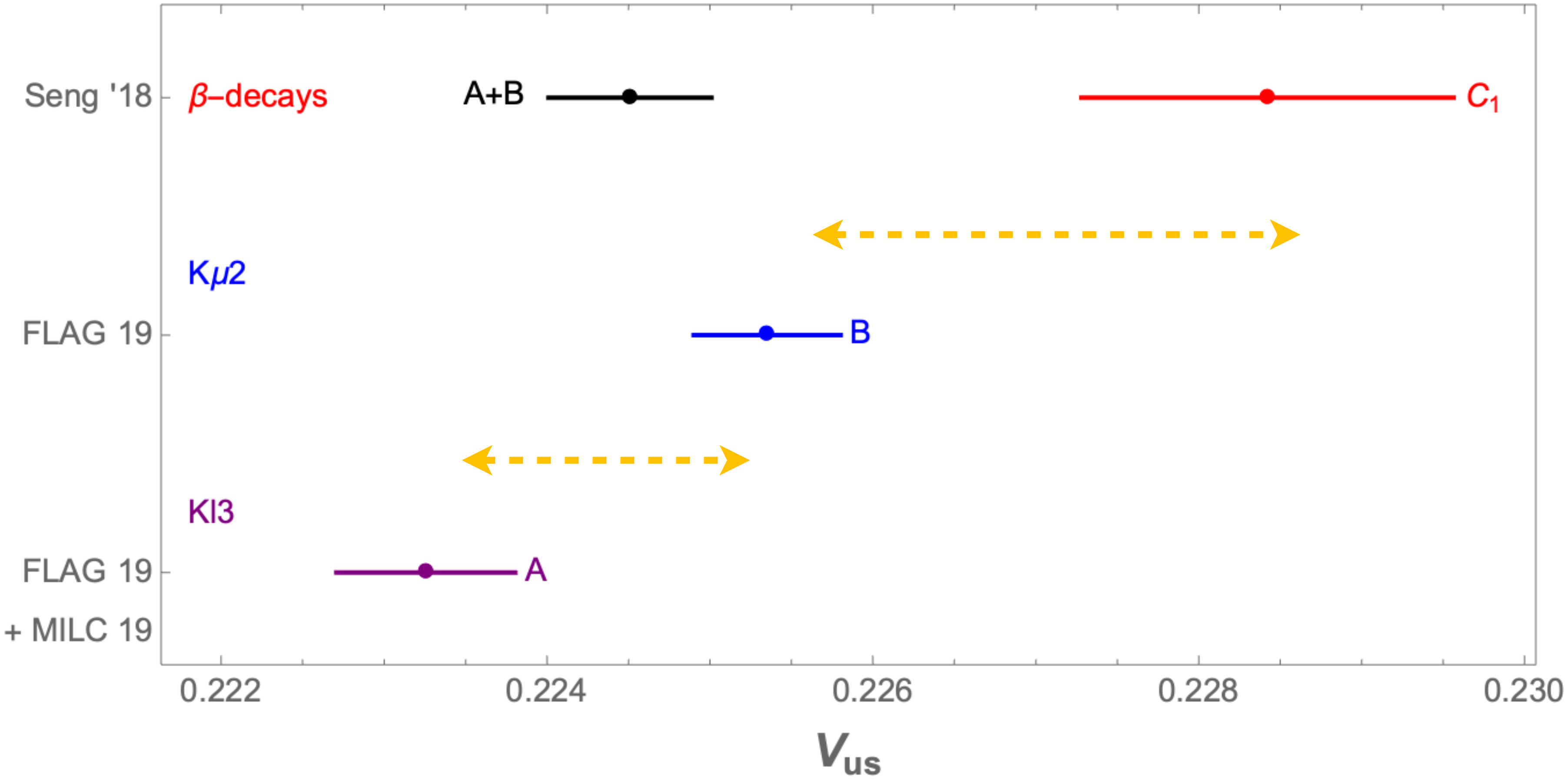
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{LUu}|^2\,, \qquad |V_{LUu}| \approx |h_{Uu}|v_w/M_{t'} \approx 0.04$$





Vector-like weak doublet

- Vectorlike extra $SU(2)$-doublet: $q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R}$



trying to explain both the gaps…

Vector-like weak doublet

- Vectorlike extra $SU(2)$-doublet: $q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R}$

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \sum_{i=1}^{4} (\overline{u_{Li}} \gamma^\mu d_{Li}) W^+_\mu + \frac{g}{\sqrt{2}} \overline{u_{R4}} \gamma^\mu d_{R4} W_\mu + \text{h.c.}$$

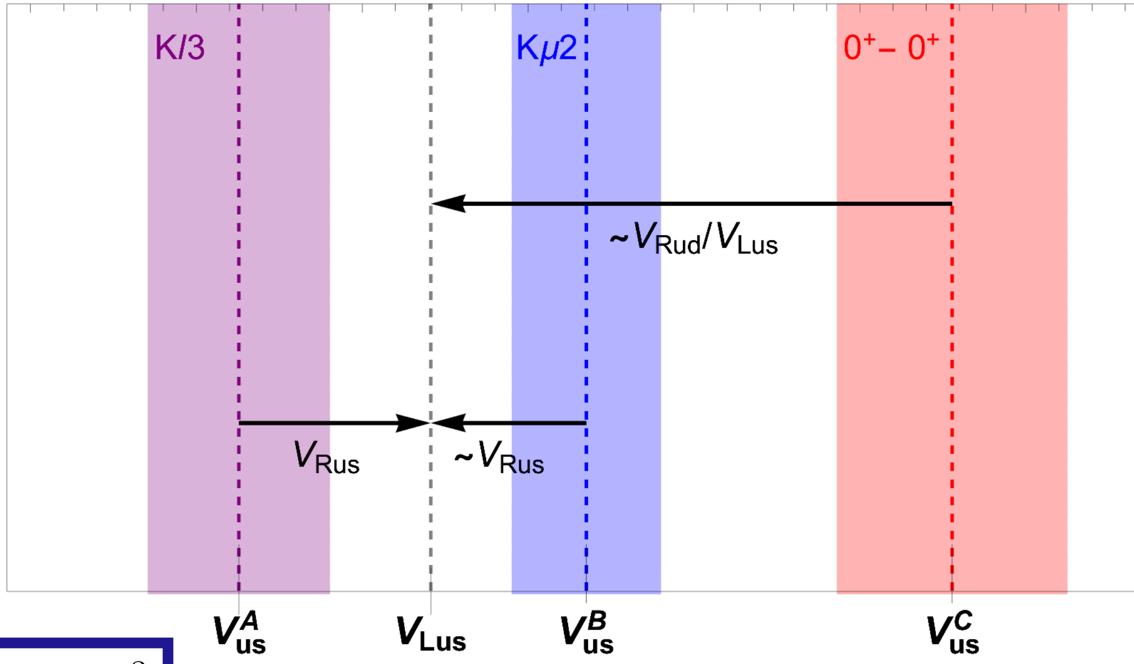
$$\frac{g}{\sqrt{2}} (\overline{u_L} \gamma^\mu V_{L\,ud} d_L + \overline{u_R} V_{R\,ud} \gamma^\mu d_R)\, W_\mu + \frac{g}{\sqrt{2}} (\overline{u_L} \gamma^\mu V_{L\,us} s_L + \overline{u_R} V_{R\,us} \gamma^\mu s_R)\, W_\mu + \text{h.c.}$$

- $V_{\text{CKM,L}} = V_L^{(u)\dagger} V_L^{(d)}$ is a 4×4 unitary matrix.
- Weak **charged RH currents** with mixing $\mathbf{V}_{\text{CKM,R}}$.
- In this scenario, we are determining vector and axial couplings:

semileptonic decays $K\ell 3$: $|V_{L\,us} + V_{R\,us}| = 0.22326(55)$

leptonic decays $K\mu 2/\pi\mu 2$: $\dfrac{|V_{L\,us} - V_{R\,us}|}{|V_{L\,ud} - V_{R\,ud}|} = 0.23130(49)$

superallowed beta decays : $|V_{L\,ud} + V_{R\,ud}| = 0.97355(27)$



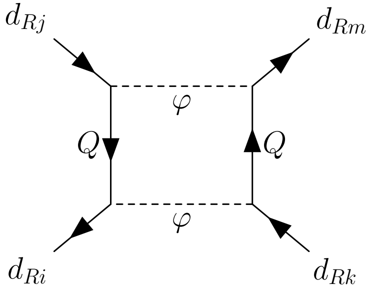
$$\boxed{V_{Rus} = V^*_{R4u} V_{R4s} = -1.17(37) \times 10^{-3} \qquad V_{Rud} = V^*_{R4u} V_{R4d} = -0.87(27) \times 10^{-3}}$$

Vector-like weak doublet

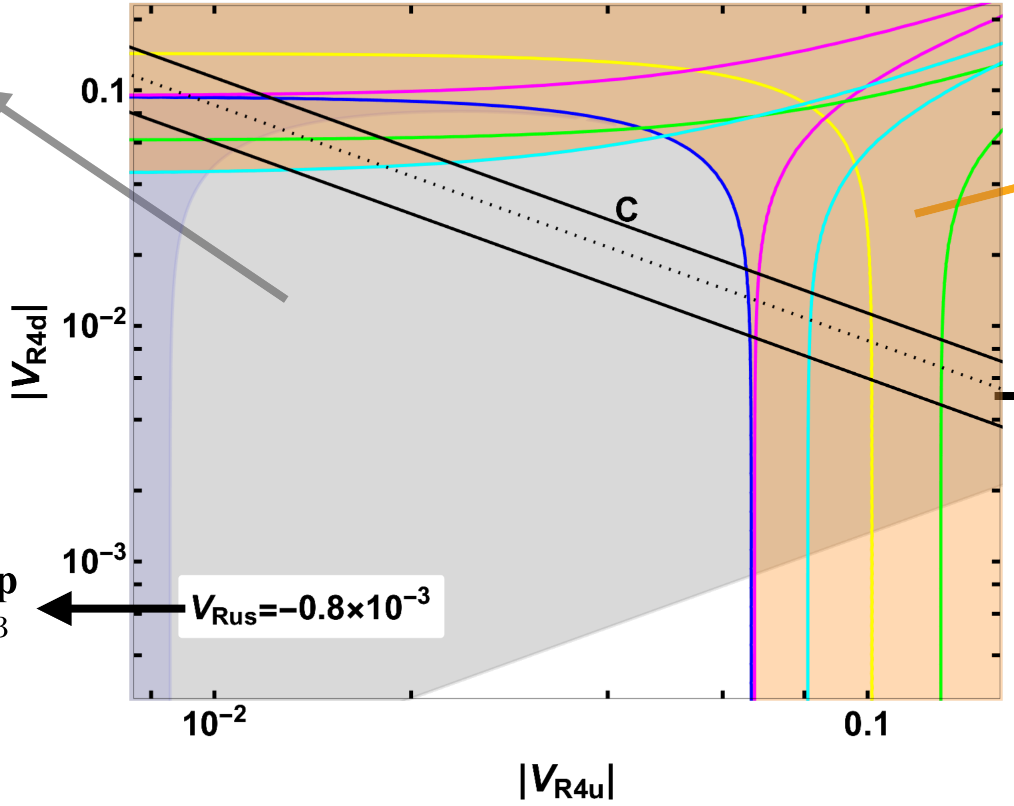
However, also in this scenario FCNC at tree level

$$V_{Rus} = V^*_{R4u} V_{R4s} = -1.17(37) \times 10^{-3}\,, \qquad V_{Rud} = V^*_{R4u} V_{R4d} = -0.87(27) \times 10^{-3}$$

Excluded by kaon flavor changing processes:
$|V_{R4d} V^*_{R4s}| < 1.0 \times 10^{-5}$



needed value to explain the gap
$|V^*_{R4u} V_{R4s}| = 1.17(37) \times 10^{-3}$



Excluded by low energies EW quantities and Z physics (Z decay into hadrons,..)

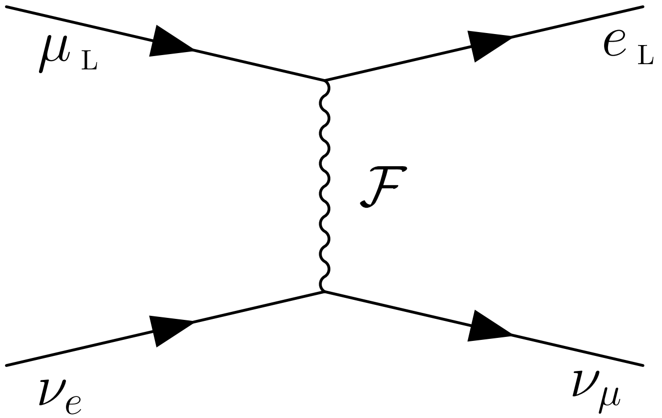
needed value to explain the gap
$|V^*_{R4u} V_{R4d}| = 0.87(27) \times 10^{-3}$

- Two weak doublets or one isodoublet with isosinglet (up or down type) can alleviate FC phenomena and explain all discrepancies.

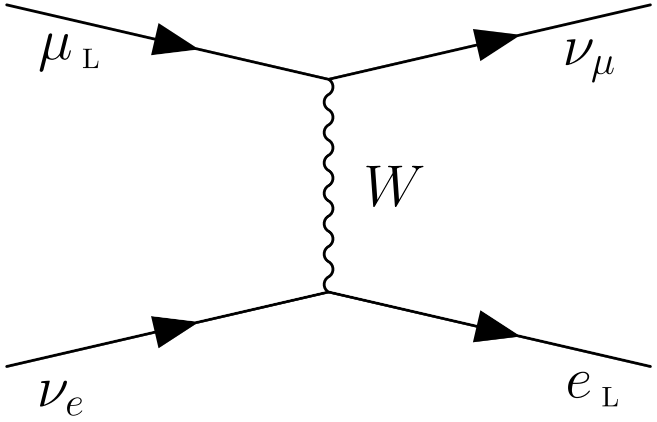
Solution #2

- Suppose the existence of **flavor changing bosons**.

$$-\frac{4G_{\mathcal{F}}}{\sqrt{2}}(\overline{e_L}\gamma^\alpha\mu_L)(\overline{\nu_\mu}\gamma_\alpha\nu_e)$$



$$-\frac{4G_{\rm F}}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e)$$



- Horizontal interactions have positive interference with SM;
- After Fierz transformation, the sum of the diagrams gives the operator:

$$-\frac{4G_\mu}{\sqrt{2}}(\overline{\nu_\mu}\gamma^\alpha\mu_L)(\overline{e_L}\gamma_\alpha\nu_e) \qquad G_\mu = G_F + G_{\mathcal{F}} = G_F(1+\delta_\mu) \qquad \boxed{G_\mu \neq G_F}$$

Flavour bosons for CKM

- Different $\mathbf{G}_\mu = \mathbf{G_F} + \mathbf{G}_\mathcal{F} = \mathbf{G_F}(\mathbf{1} + \delta_\mu) = 1 + \frac{v_\mathrm{w}^2}{v_\mathcal{F}^2}$
- The values of V_us, V_ud (and corresponding errorbars) should be rescaled:

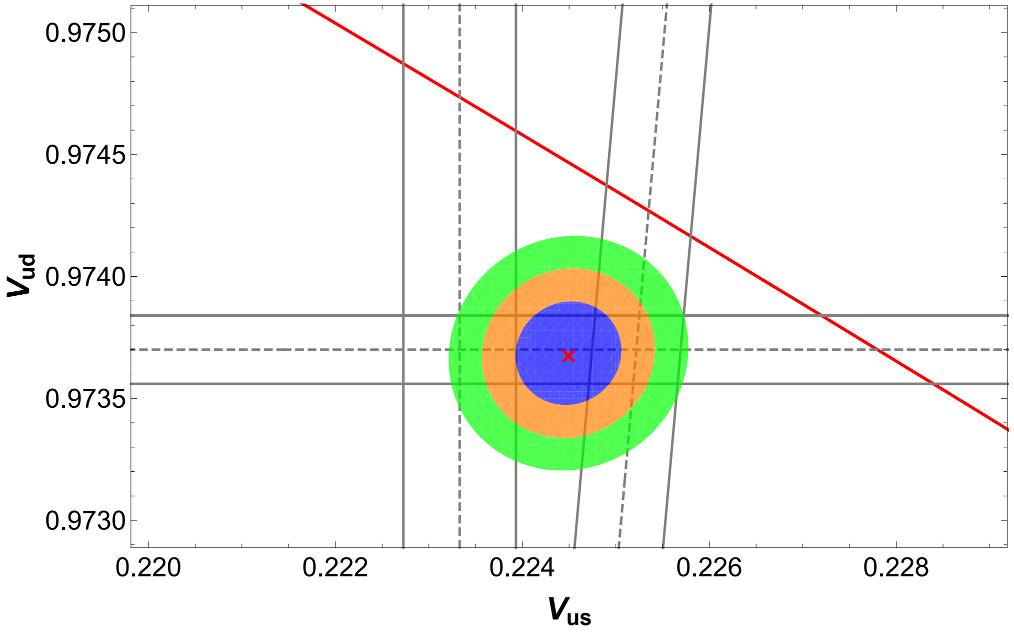
$$|V_\mathrm{us}| = 0.22333(60) \times (1 + \delta_\mu)\,, \qquad |V_\mathrm{ud}| = 0.97370(14) \times (1 + \delta_\mu)$$

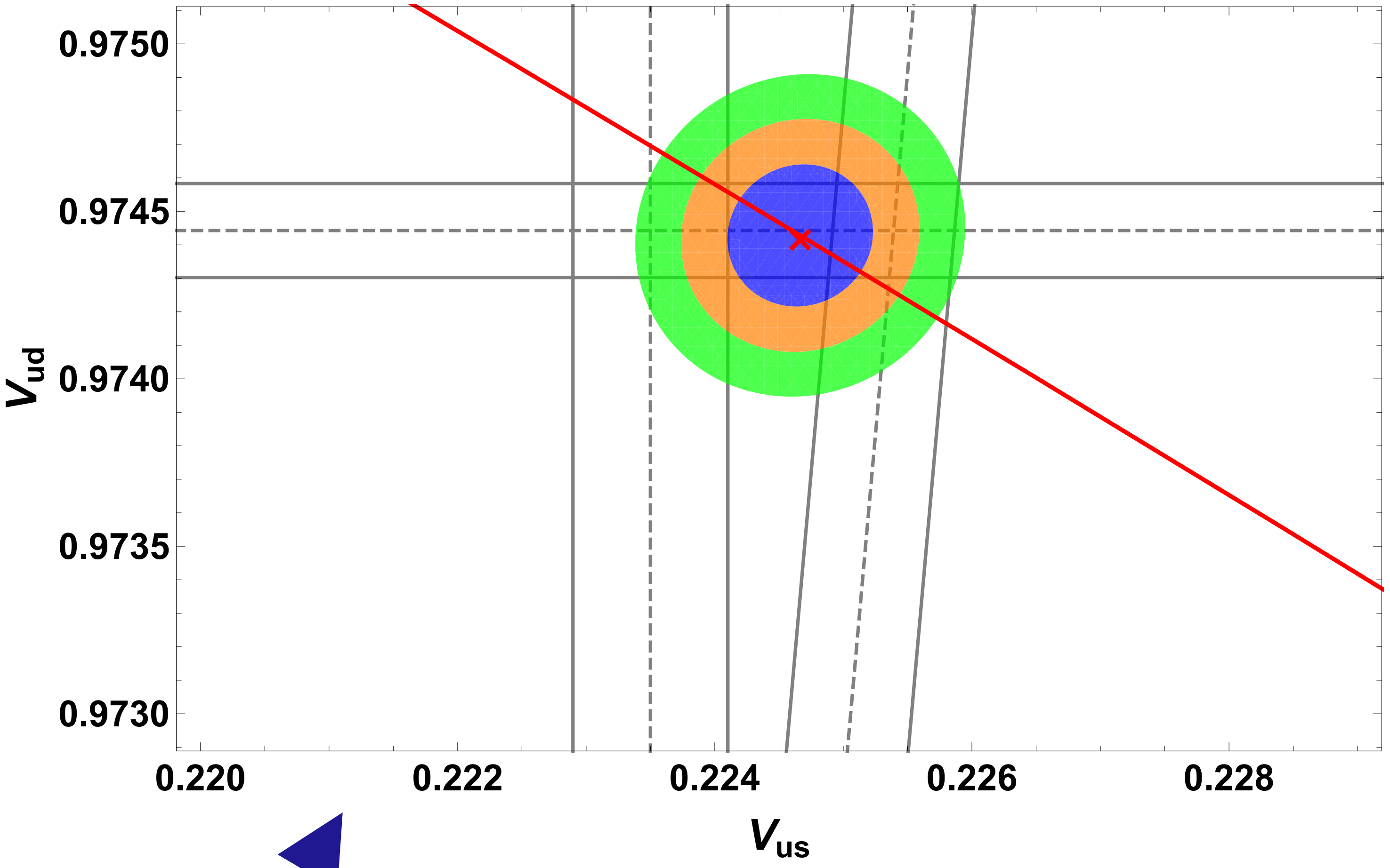
  while the ratio is not affected.
- Unitarity recovered: $\left(\frac{G_F}{G_\mu}\right)^2 (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 1 - \frac{2G_\mathcal{F}}{G_\mathrm{F}}$
- CKM unitarity is recovered ($\chi^2_{dof} = 3.0$) with $\delta_\mu = \mathbf{7.6 \cdot 10^{-4}}$, or

$$\boxed{v_\mathcal{F} = 6\text{-}7~\mathrm{TeV}}$$

Flavour bosons for CKM

$$\delta_\mu = 7.6 \cdot 10^{-4}$$
$$v_{\mathcal{F}} = 6.3\,\mathrm{TeV}$$





How light flavour can changing gauge bosons be?

Family symmetries

- In the limit of vanishing Yukawa couplings, $Y_f \to 0$, the SM acquires maximal global symmetry:
$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

- Fermions transform as triplets, Yukawa interactions break the $SU(3)^5$ symmetry.

- Gauge the symmetry $SU(3)^5$ (the $U(1)$ factors remain as global symmetries).

- Fermions cannot get mass if the symmetry is unbroken: LH and RH particles transform in different representations.

- Yukawa couplings are induced by non-zero VEVs of scalars. The fermion mass hierarchy can be related to the breaking pattern.

$$\frac{y_{ij}\bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri} + \frac{h_{ij}\bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3}\varphi\varphi\ell_{L\alpha}^T C\ell_\beta + \text{h.c.}$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing} \qquad w_2^2 + w_1^2 = v_{\mathcal{F}}^2 \qquad \left(\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4}\right)$$

- Masses of gauge bosons $M_{\ell 1,2}^2 = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2} v_{\mathcal{F}}^2$.
- If $w_3 = w_2 = w_1$ (e. g. symmetry between ηs), gauge bosons have equal masses, $\lambda_a \to V^\dagger \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices. From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2} (\overline{\mathrm{e_L}}\, \lambda^a\, \gamma^\mu \mathrm{e_L})(\overline{\mathrm{e_L}}\, \lambda^a\, \gamma_\mu \mathrm{e_L}) = -\frac{1}{3v_\ell^2} (\overline{\mathrm{e_L}}\, \mathbb{I}\, \gamma_\mu \mathrm{e_L})^2$$

No FCNC, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

- In the general case, FC ($\mu \to 3e$, $\tau \to 3\mu$,...) under control.
- $\mathbf{v}_{\mathcal{F}} \simeq \mathbf{6}$ **TeV** is not contradicting experimental constraints.

Conclusions

- There is **tension** between independent determinations of the **CKM matrix** elements of the first row.
- A new effective operator in positive interference with the SM muon decay as the one generated by flavour changing gauge bosons can solve CKM unitarity problem. **G_F can be different** from muon decay constant without contradicting experimental data.
- Natural explanation for masses and mixings of fermions from the spontaneous breaking pattern of the symmetry.
- CKM unitarity is restored with a breaking scale of the symmetry (and gauge boson mass) of few TeV. Flavour gauge bosons can be as light as **TeV** without contradicting experimental constraints .
- Extra vector-like quarks can be possible explanation for CKM anomalies. A quite large mixing with SM fermions is needed to restore unitarity.
- Their mass should be no more than few **TeV,** since experimental constraints on flavor changing phenomena become more stringent with larger masses.
- Only one type of extra multiplet cannot entirely explain all the discrepancies, and some their combination is required, e.g. two species of isodoublet, or one isodoublet and one (up or down type) isosinglet.
- These scenarios are testable with future experiments (Z boson decay, mass few TeV).

Backup

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- $|V_{us}|$ can be directly determined from **semileptonic** $K \to \pi \ell \nu$ ($K\ell 3$) decays

$$\mathbf{A}: \qquad f_+(0)|V_{us}| = 0.21654 \pm 0.00041$$

- The ratio of **leptonic** decay rates $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ determines:

$$\mathbf{B}: \qquad \frac{f_{K^\pm}}{f_{\pi^\pm}} \frac{|V_{us}|}{|V_{ud}|} = 0.27599 \pm 0.00038$$

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

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$$\mathbf{B}: \quad \boxed{\frac{f_{K^\pm}}{f_{\pi^\pm}}} \frac{|V_{us}|}{|V_{ud}|} = 0.27599 \pm 0.00038$$

- Form factor $f_+(0)$ and decay constant ratio f_K/f_π from lattice QCD.
- New $f_{K^\pm}/f_{\pi^\pm}$ (FLAG 2019) and $f_+(0)$ (FLAG+Fermilab Lattice & MILC)

$$\mathbf{A}: \quad |\mathbf{V_{us}}| = \mathbf{0.22326(55)} \qquad \mathbf{B}: \quad \frac{|\mathbf{V_{us}}|}{|\mathbf{V_{ud}}|} = \mathbf{0.23130(49)}$$

- PDG 2018 (referring to FLAG 2017): $|V_{us}| = 0.2238(8)$, $\left|\frac{V_{us}}{V_{ud}}\right| = 0.2315(10)$.

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Superallowed nuclear beta-decays (0^+–0^+ Fermi transitions) determine vector coupling $\mathbf{G_V = G_F|V_{ud}|}$:

$$|V_{ud}|^2 = \frac{K}{2G_F^2 \mathcal{F}t\,(1+\Delta_R^V)} = \frac{0.97142(58)}{1+\Delta_R^V}$$

- $\mathcal{F}t = ft(1+\delta'_R)(1+\delta_{NS}-\delta_C)$ nucleus-independent value obtained from ft-values by absorbing nucleus-dependent radiative (δ'_R, δ_{NS}) and isospin-breaking (δ_C) corrections, averaging values for 15 transitions, recently updated: $\boldsymbol{\mathcal{F}t} = 3072.24(1.85)$ s (Hardy & Towner 2020);

- $\mathbf{K} = 2\pi^3 \log 2/m_e^5 = 8120.2765(3)\times10^{-10}\,\mathrm{s/GeV^4}$, $\mathbf{G_F} = \mathbf{G}_\mu = 1.1663787(6)\times10^{-5}\,\mathrm{GeV^{-2}}$;

- $\mathbf{\Delta_R^V}$ are short-distance (transition independent) radiative corrections.

Present situation of CKM first row

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- Superallowed nuclear beta-decays and free neutron decay:

$$|V_{ud}|^2 = \frac{K}{2G_F^2 \mathcal{F}t\,(1+\Delta_R^V)}\,, \qquad |V_{ud}|^2 = \frac{K/\ln 2}{G_F^2 \mathcal{F}_n \tau_n (1+3g_A^2)(1+\Delta_R^V)}$$

- Marciano & Sirlin 2006 $\Delta_R^V = 0.02361(38)$: $|V_{ud}| = 0.97420(21)$ adopted by PDG 2018.

- Seng et al. 2018 $\Delta_R^V = 0.02467(22)$:

$$C_1 \;:\quad \mathbf{|V_{ud}| = \frac{G_V^{exp}}{G_F(=G_\mu)} = 0.97355(27)}$$

- Other more recent studies confirmed the shift of Δ_R^V.

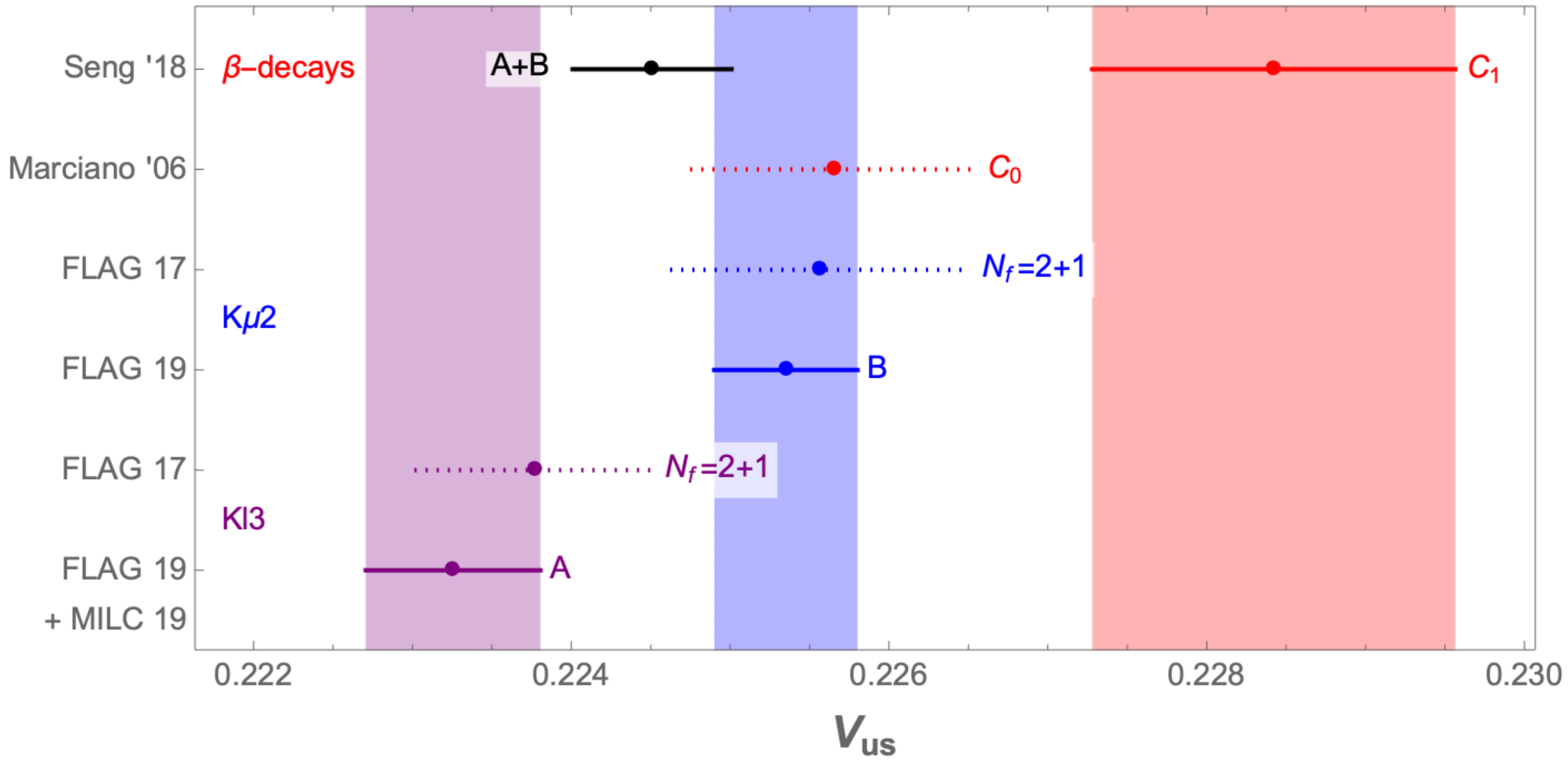
Cabibbo angle anomalies

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

A: $|V_{us}| = 0.22326(55)$ → $|V_{us}|_{\mathrm{A}} = 0.22326(55)$

B: $\frac{|V_{us}|}{|V_{ud}|} = 0.23130(49)$ → $|V_{us}|_{\mathrm{B}} = 0.22535(45)$

C: $|V_{ud}| = 0.97355(27)$ → $|V_{us}|_{\mathrm{C}} = 0.2284(11)$



The CKM unitarity problem

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta_{\mathrm{CKM}}$$

	$\lvert V_{ud}\rvert_{\mathrm{C}_1} = 0.97370(14)$	$\lvert V_{ud}\rvert_{\mathrm{C}_2} = 0.97389(18)$	$\lvert V_{ud}\rvert_{\mathrm{C}} = 0.97376(16)$
A: $\lvert V_{us}\rvert = 0.22326(55)$	$2.05(37) \cdot 10^{-3}$	$1.68(43) \cdot 10^{-3}$	$1.94(40) \cdot 10^{-3}$
B: $\lvert V_{us}\rvert = 0.23130(49)\,\lvert V_{ud}\rvert$	$1.17(35) \cdot 10^{-3}$	$0.78(41) \cdot 10^{-3}$	$1.05(38) \cdot 10^{-3}$
Average*	$1.55(36) \cdot 10^{-3}$	$1.17(42) \cdot 10^{-3}$	$1.43(39) \cdot 10^{-3}$

Values of δ_{CKM} obtained for different choices of the values of $|V_{us}|$ and $|V_{ud}|$.

Down-type weak singlets

- Down-type vector-like species of quark $d_{4L,R}$ whose left and right components are both $SU(2)$ singlets involved in quark mixing:

$$...+h_{dj}\phi\overline{q_{Lj}}d_{R4}+M\overline{d_{L4}}d_{R4}+h.c.$$

- $\overline{d_{Li}}\mathbf{m}_{ij}^{(d)}d_{Rj}+\text{h.c.}=(\overline{d_{L1}},\overline{d_{L2}},\overline{d_{L3}},\overline{d_{L4}})\left(\begin{array}{ccc|c} & & & h_{d1}v_w \\ & \mathbf{y}_{3\times3}^{(d)}v_w & & h_{d2}v_w \\ & & & h_{d3}v_w \\ \hline 0 & 0 & 0 & M \end{array}\right)\begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ d_{R4} \end{pmatrix}+\text{h.c.}$

- $V_L^{(d)\dagger}\mathbf{m}^{(d)}V_R^{(d)}=\mathbf{m}_{\text{diag}}^{(d)}=\text{diag}(m_d,m_s,m_b,M_{b'})$

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}_L = V_L^{(d)}\begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L = \begin{pmatrix} V_{1d} & V_{1s} & V_{1b} & V_{1b'} \\ V_{2d} & V_{2s} & V_{2b} & V_{2b'} \\ V_{3d} & V_{3s} & V_{3b} & V_{3b'} \\ V_{4d} & V_{4s} & V_{4b} & V_{4b'} \end{pmatrix}_L \begin{pmatrix} d \\ s \\ b \\ b' \end{pmatrix}_L$$

- $|V_{ub'}|\approx|V_{L4d}|\approx h_{d1}v_w/M_{b'}$

Down-type weak singlets

- Charged weak interactions:

$$\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_{L1}} & \overline{u_{L2}} & \overline{u_{L3}} \end{pmatrix} \gamma^\mu \begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \end{pmatrix} W_\mu^+ + \text{h.c.} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{pmatrix} \gamma^\mu \tilde{V}_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W_\mu^+ + \text{h.c.}$$

- $\tilde{V}_{\text{CKM}}$ is a $\mathbf{3 \times 4}$ **matrix**

- $\tilde{V}^\dagger_{\text{CKM}} \tilde{V}_{\text{CKM}} \neq \mathbf{1}$, $\tilde{V}_{\text{CKM}} \tilde{V}^\dagger_{\text{CKM}} = \mathbf{1}$: $\quad |\mathbf{V_{ud}}|^2 + |\mathbf{V_{us}}|^2 + |\mathbf{V_{ub}}|^2 = \mathbf{1} - |\mathbf{V_{ub'}}|^2$

$$\tilde{V}_{CKM} = V_L^{(u)\dagger} \tilde{V}_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix}$$

- Weak neutral currents: $\quad \mathcal{L}_{\text{nc}} = \frac{g}{\cos\theta_W} Z_\mu \boxed{T_3(f_{L,R})} - Q(f) \sin^2\theta_W]\, \overline{f_{L,R}} \gamma^\mu f_{L,R}$

(not the same quantum numbers)

- Tree level **flavour changing** couplings with the Higgs boson and with Z-boson.

Down-type weak singlets

- Weak neutral currents: $\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W} Z_\mu \left[\boxed{T_3(f_{L,R})} - Q(f)\sin^2\theta_W\right] \overline{f_{L,R}}\gamma^\mu f_{L,R}$

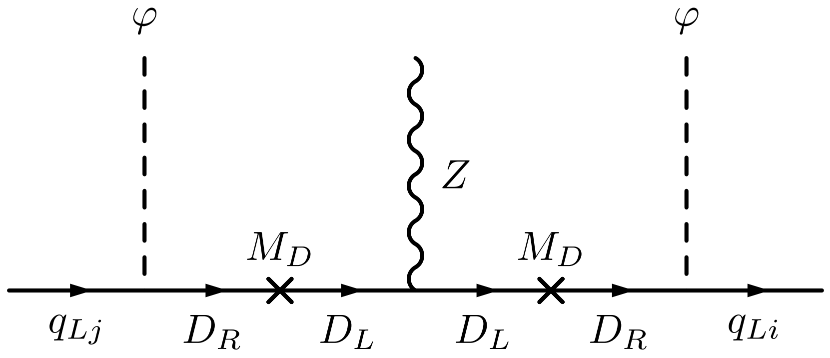
(not the same quantum numbers)

$$-\frac{1}{2}\frac{g}{\cos\theta_W}\begin{pmatrix} \overline{d_{L1}} & \overline{d_{L2}} & \overline{d_{L3}} \end{pmatrix}\gamma^\mu \begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \end{pmatrix} Z_\mu =$$

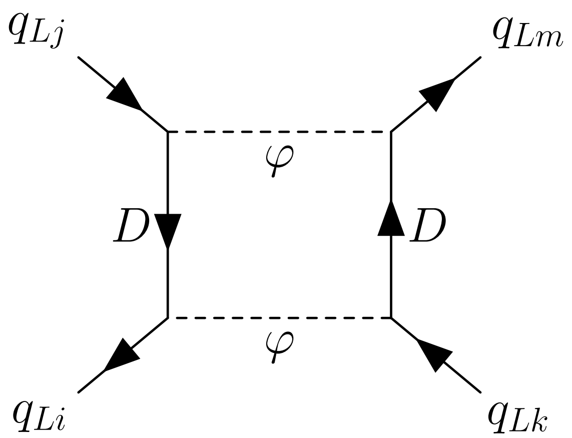
$$= -\frac{1}{2}\frac{g}{\cos\theta_W}\begin{pmatrix} \overline{d_L} & \overline{s_L} & \overline{b_L} & \overline{b'_L} \end{pmatrix}\gamma^\mu V_L^{(d)\dagger}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} V_L^{(d)} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} Z_\mu$$

- Tree level **flavour changing** couplings with the Higgs boson and with Z-boson.

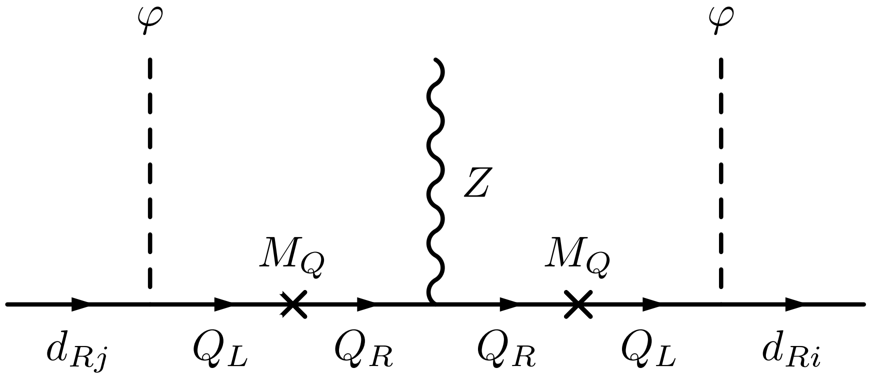
FCNC



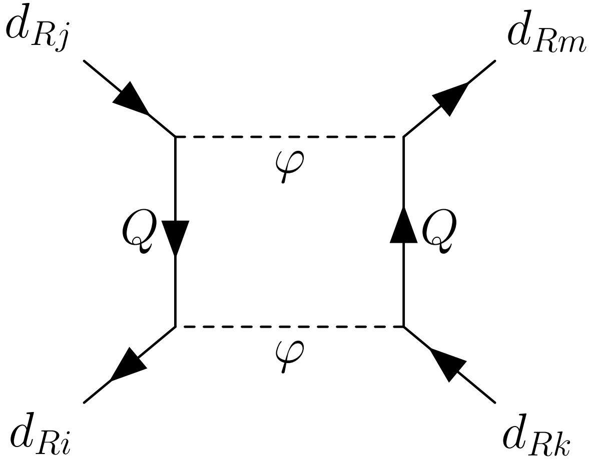
φ
φ
Z
M_D
M_D
q_{Lj}
D_R
D_L
D_L
D_R
q_{Li}



q_{Lj}
q_{Lm}
φ
D
D
φ
q_{Li}
q_{Lk}



φ
φ
Z
M_Q
M_Q
d_{Rj}
Q_L
Q_R
Q_R
Q_L
d_{Ri}



d_{Rj}
d_{Rm}
φ
Q
Q
φ
d_{Ri}
d_{Rk}

Up-type weak singlets

- Vector-like up-type quark $u_{4L,R}$ whose left and right components are both $SU(2)$ singlets involved in quark mixing:

$$... + h_{uj}\tilde{\phi}\overline{q_{Lj}}u_{4R} + M_{t'}\overline{u_{4L}}u_{4R} + h.c.$$

- $\tilde{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{t'd} & V_{t's} & V_{t'b} \end{pmatrix} = \tilde{V}_L^{(u)\dagger} V_L^{(d)}$;

- $\tilde{V}_L^{(u)}$ is the 3×4 submatrix of $V_L^{(u)}$ without the last row, $\tilde{V}_{CKM}$ is 4×3 matrix.

- $\tilde{V}_L^{(u)}$ and $\tilde{V}_{\rm CKM}$ are not unitary $\tilde{V}_{\rm CKM}\tilde{V}_{\rm CKM}^\dagger = \tilde{V}_L^{(u)\dagger}\tilde{V}_L^{(u)} \neq \mathbf{1}$.

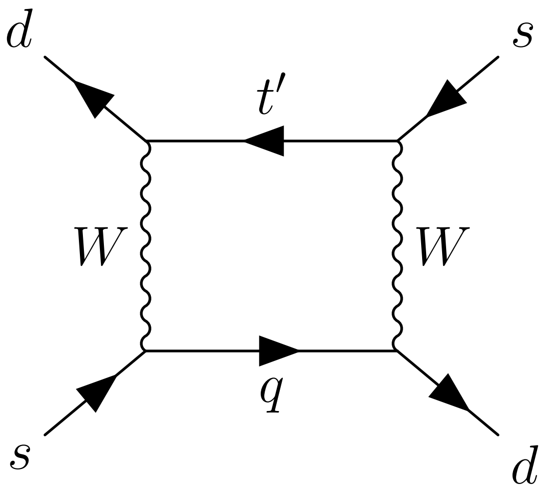
- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{L4u}|^2$, $\qquad |V_{L4u}| \approx 0.04$

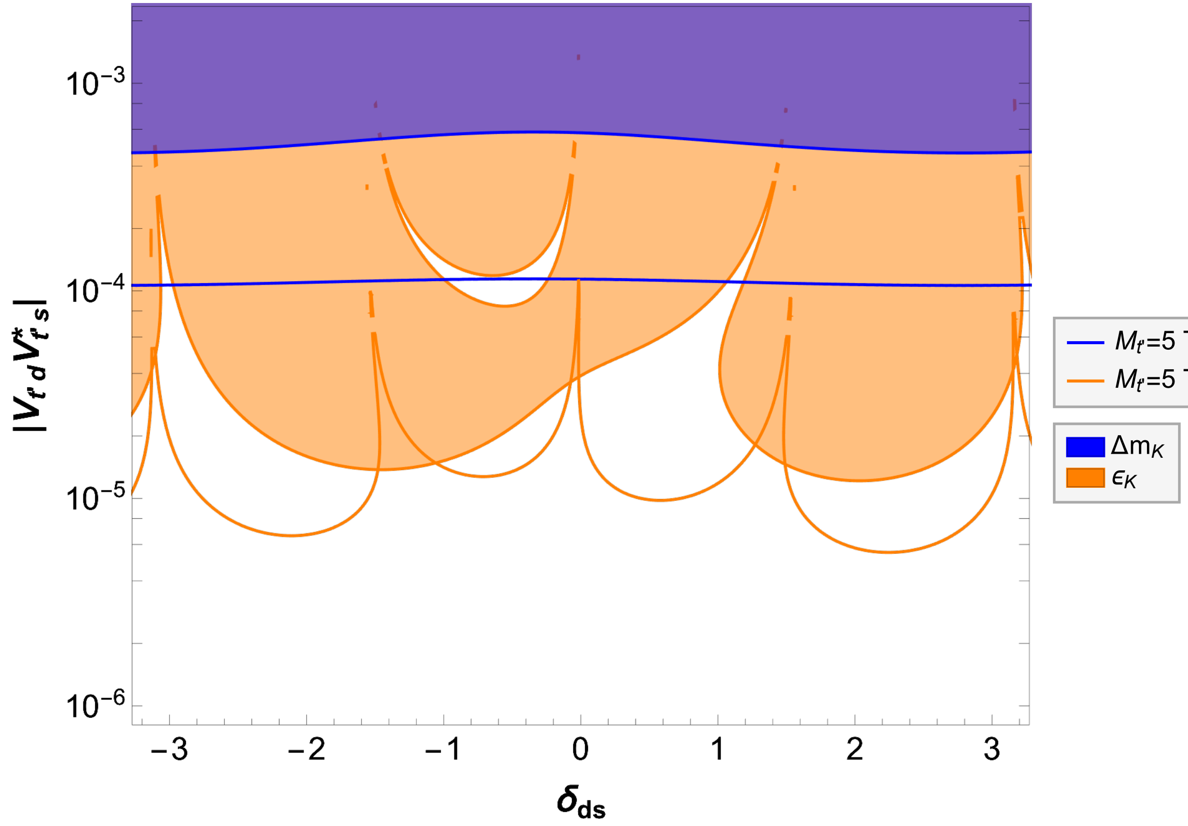
Up-type isosinglet

- The non-unitarity of $\tilde{V}_L^{(u)}$ gives flavour changing couplings of quarks with Z boson (and Higgs). The weak neutral currents Lagrangian is:

$$\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W}\left[\frac{1}{2}\begin{pmatrix}\overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L}\end{pmatrix}\gamma^\mu \tilde{V}_L^{(u)\dagger}\tilde{V}_L^{(u)}\begin{pmatrix}u\\ c\\ t\\ t'\end{pmatrix}_L - \frac{2}{3}\sin^2\theta_W\,(\overline{\mathbf{u}_L}\gamma^\mu\mathbf{u}_L + \overline{\mathbf{u}_R}\gamma^\mu\mathbf{u}_R)\right] Z_\mu$$

Constraints from K mesons mixing:





Vector-like weak doublet

- Vectorlike extra $SU(2)$-doublet: $\quad q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R}$

$$\overline{d_{Li}}\mathbf{m}^{(d)}_{ij} d_{Rj} + \text{h.c.} = \begin{pmatrix} \overline{d_{L1}} & \overline{d_{L2}} & \overline{d_{L3}} & \overline{d_{L4}} \end{pmatrix} \begin{pmatrix} & \mathbf{y}^{(d)}_{3\times 3} v_w & & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ y^d_{41} v_w & y^d_{42} v_w & y^d_{43} v_w & M_q \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ q_{R4} \end{pmatrix} + \text{h.c.}$$

$$V_L^{(d)\dagger} \mathbf{m}^{(d)} V_R^{(d)} = \mathbf{m}^{(d)}_{\text{diag}}\,, \quad V_L^{(u)\dagger} \mathbf{m}^{(u)} V_R^{(u)} = \mathbf{m}^{(u)}_{\text{diag}}$$

- $V^{(d,u)}_{L,R}$ are unitary 4×4 matrices:

$$\begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ d_{R4} \end{pmatrix} = V_R^{(d)} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix}_{L,R} , \qquad \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ u_{R4} \end{pmatrix} = V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \end{pmatrix}$$

Vector-like weak doublet

- Vectorlike extra $SU(2)$-doublet: $q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R}$

- The charged-current Lagrangian is:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \sum_{i=1}^{4} (\overline{u_{Li}} \gamma^\mu d_{Li}) W_\mu^+ + \frac{g}{\sqrt{2}} \overline{u_{R4}} \gamma^\mu d_{R4} W_\mu + \text{h.c.} =$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix} \gamma^\mu V_{\text{CKM,L}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W_\mu^+ + \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_{\text{CKM,R}} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix} W_\mu^+ + \text{h.c.}$$

- $V_{\text{CKM,L}} = V_L^{(u)\dagger} V_L^{(d)}$ is a 4×4 unitary matrix.
- Weak charged currents involve also right currents with mixing matrix $\mathbf{V}_{\text{CKM,R}}$

Vector-like weak doublet

- Couplings of u-quark with down quarks (first row of SM CKM matrix) become:

$$\frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,ud} d_L + \overline{u_R} V_{R\,ud}\gamma^\mu d_R)\,W_\mu + \frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,us} s_L + \overline{u_R} V_{R\,us}\gamma^\mu s_R)\,W_\mu + \text{h.c.}$$

- Then, in this scenario, we are determining vector and axial couplings:

$$\begin{aligned} \text{semileptonic K decay} \quad A &: \quad |V_{L\,us} + V_{R\,us}| = 0.22326(55) \\ \text{leptonic K decay} \quad B &: \quad \frac{|V_{L\,us} - V_{R\,us}|}{|V_{L\,ud} - V_{R\,ud}|} = 0.23130(49) \\ \text{superallowed beta decays} \quad C &: \quad |V_{L\,ud} + V_{R\,ud}| = 0.97355(27) \end{aligned}$$

- $|V_{Lud}|^2 + |V_{Lus}|^2 + |V_{Lub}|^2 = 1 - |V_{Lub'}|^2 = 1$

$V_{Lus} = 0.22443(35)$

Vector-like weak doublet

- The charged-current Lagrangian is:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \sum_{i=1}^{4} (\overline{u_{Li}} \gamma^\mu d_{Li}) W_\mu^+ + \frac{g}{\sqrt{2}} \overline{u_{R4}} \gamma^\mu d_{R4} W_\mu + \text{h.c.} =$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix} \gamma^\mu V_{\text{CKM,L}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W_\mu^+ + \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_{\text{CKM,R}} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix} W_\mu^+ + \text{h.c.}$$

- $V_{\text{CKM,L}} = V_L^{(u)\dagger} V_L^{(d)}$ is a 4×4 unitary matrix.
- Weak charged currents involve also right currents with mixing matrix $\mathbf{V}_{\text{CKM,R}}$:

$$V_{\text{CKM,R}} = V_R^{(u)\dagger} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} V_R^{(d)} = \begin{pmatrix} V^*_{R\,4u} V_{R\,4d} & V^*_{R\,4u} V_{R\,4s} & V^*_{R\,4u} V_{R\,4b} & V^*_{R\,4u} V_{R\,4b'} \\ V^*_{R\,4c} V_{R\,4d} & V^*_{R\,4c} V_{R\,4s} & V^*_{R\,4c} V_{R\,4b} & V^*_{R\,4c} V_{R\,4b'} \\ V^*_{R\,4t} V_{R\,4d} & V^*_{R\,4t} V_{R\,4s} & V^*_{R\,4t} V_{R\,4b} & V^*_{R\,4t} V_{R\,4b'} \\ V^*_{R\,4t'} V_{R\,4d} & V^*_{R\,4t'} V_{R\,4s} & V^*_{R\,4t'} V_{R\,4b} & V^*_{R\,4t'} V_{R\,4b'} \end{pmatrix}$$

Vector-like weak doublet

- However also in this scenario flavour changing neutral currents appear at tree level.

$$\mathcal{L}_{\rm fcnc} = \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_R^{(u)\dagger} {\rm diag}(0,0,0,1) V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \end{pmatrix} +$$

$$- \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} & \overline{b'_R} \end{pmatrix} \gamma^\mu V_R^{(d)\dagger} {\rm diag}(0,0,0,1) V_R^{(d)} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix}$$

where

$$V_R^{(u)\dagger} {\rm diag}(0,0,0,1) V_R^{(u)} = \begin{pmatrix} |V_{R\,4u}|^2 & V^*_{R\,4u} V_{R\,4c} & V^*_{R\,4u} V_{R\,4t} & V^*_{R\,4u} V_{R\,4t'} \\ V^*_{R\,4c} V_{R\,4u} & |V_{R\,4c}|^2 & V^*_{R\,4c} V_{R\,4t} & V^*_{R\,4c} V_{R\,4t'} \\ V^*_{R\,4t} V_{R\,4u} & V^*_{R\,4t} V_{R\,4c} & |V_{R\,4t}|^2 & V^*_{R\,4t} V_{R\,4t'} \\ V^*_{R\,4t'} V_{R\,4u} & V^*_{R\,4t'} V_{R\,4c} & V^*_{R\,4t'} V_{R\,4t} & |V_{R\,4t'}|^2 \end{pmatrix}$$

$$V_R^{(d)\dagger} {\rm diag}(0,0,0,1) V_R^{(d)} = \begin{pmatrix} |V_{R\,4d}|^2 & V^*_{R\,4d} V_{R\,4s} & V^*_{R\,4d} V_{R\,4b} & V^*_{R\,4d} V_{R\,4b'} \\ V^*_{R\,4s} V_{R\,4d} & |V_{R\,4s}|^2 & V^*_{R\,4s} V_{R\,4b} & V^*_{R\,4s} V_{R\,4b'} \\ V^*_{R\,4b} V_{R\,4d} & V^*_{R\,4b} V_{R\,4s} & |V_{R\,4b}|^2 & V^*_{R\,4b} V_{R\,4b'} \\ V^*_{R\,4b'} V_{R\,4d} & V^*_{R\,4b'} V_{R\,4s} & V^*_{R\,4b'} V_{R\,4b} & |V_{R\,4b'}|^2 \end{pmatrix}$$

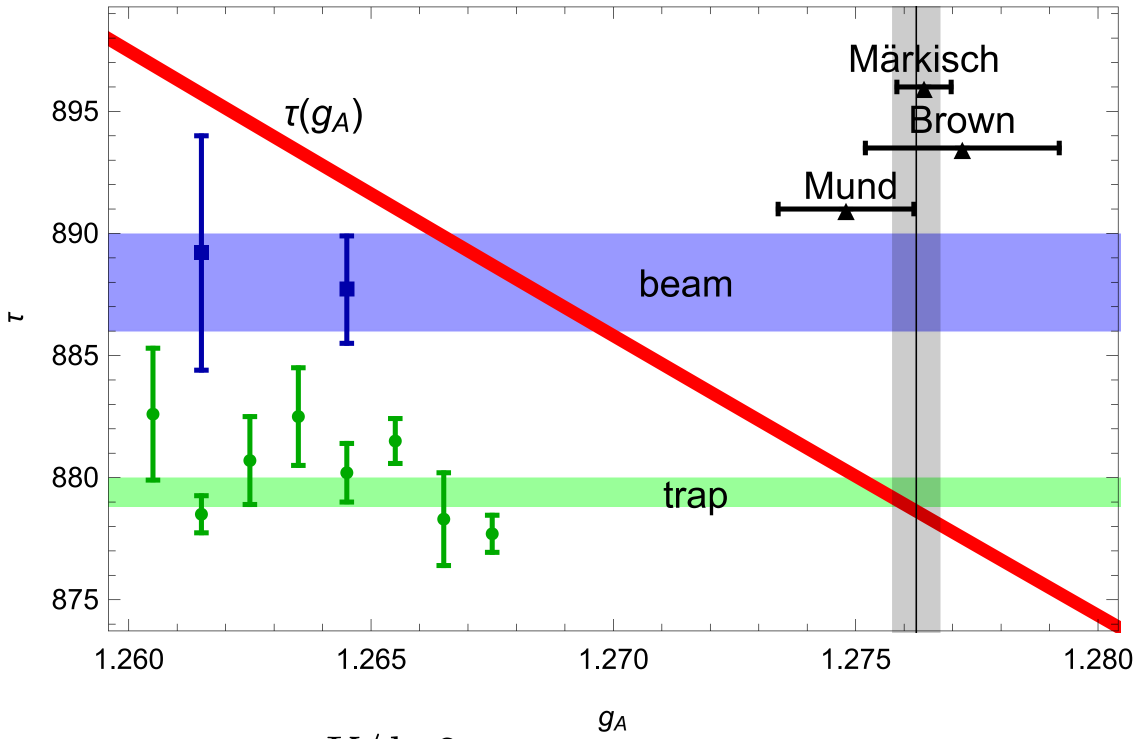
Possible solutions

- There can be two or more vector-like doublets or a vector-like isodoublet with a down-type or up-type isosinglet:

$$\overline{q_{Li}}\mathbf{m}^{(d)}_{ij} d_{Rj} = \left(\begin{array}{ccccc} \overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} & \overline{d_{5L}} \end{array} \right) \left(\begin{array}{ccccc} & & & 0 & y^d_{15} v_w \\ & \mathbf{y}^{(d)}_{3\times 3} v_w & & 0 & 0 \\ & & & 0 & 0 \\ 0 & y^d_{42} v_w & 0 & M_4 & 0 \\ 0 & 0 & 0 & 0 & M^d_5 \end{array} \right) \left(\begin{array}{c} d_{R1} \\ d_{R2} \\ d_{R3} \\ q_{R4} \\ d_{R5} \end{array} \right)$$

$$\overline{q_{Li}}\mathbf{m}^{(u)}_{ij} u_{Rj} = \left(\begin{array}{cccc} \overline{q_{1L}} & \overline{q_{2L}} & \overline{q_{3L}} & \overline{q_{4L}} \end{array} \right) \left(\begin{array}{cccc} & & & 0 \\ & \mathbf{y}^{(u)}_{3\times 3} v_w & & 0 \\ & & & 0 \\ y^u_{41} v_w & 0 & 0 & M_4 \end{array} \right) \left(\begin{array}{c} u_{R1} \\ u_{R2} \\ u_{R3} \\ q_{R4} \end{array} \right)$$

CKM and neutron lifetime problem



$$|V_{ud}|^2 = \frac{K/\ln 2}{G_F^2 \mathcal{F}_n \tau_n (1+3g_A^2)(1+\Delta_R^V)} = \frac{5024.46(30)\,\mathrm{s}}{\tau_n (1+3g_A^2)(1+\Delta_R^V)}$$

- $\mathcal{F}_n = f_n(1+\delta_R')$ f-value corrected by LD QED correction.
- $g_A = 1.27625(50)$ axial current coupling from β-asymmetry.
- $\tau_{\text{beam}} = 888.0(2.0)$ s (4.4σ away from SM prediction)
- $\tau_{\text{trap}} = 879.4(6)$ s
- τ_{trap} & $\Delta_R^V = 0.02454(27)$ & $g_A \to |V_{ud}| = 0.97333(47)$

$$G_V^2 = \frac{K/\ln 2}{\mathcal{F}_n \tau_n (1+3g_A^2)(1+\Delta_R^V)}$$

$$G_V^2 = \frac{K}{2\mathcal{F}t(1+\Delta_R^V)}$$

$\longrightarrow$

$$\tau_n = \frac{2\mathcal{F}t}{\ln 2 \mathcal{F}_n (1+3g_A^2)} = \frac{5172.0(1.1)\,\mathrm{s}}{(1+3g_A^2)}$$

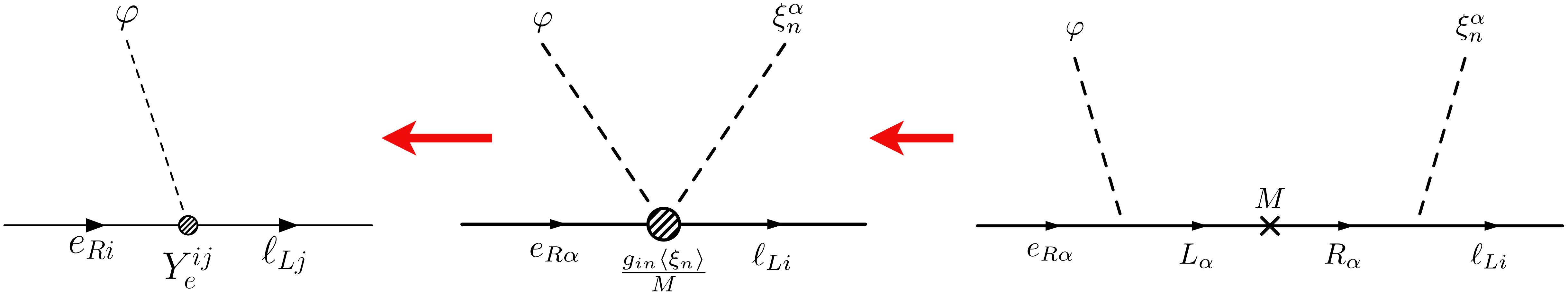
- G_V and Δ_R^V cancel out even in BSM $G_V \neq G_F |V_{ud}|$, $g_A = -G_A/G_V$
- new Δ_R^V calculations have no influence on τ_n determination.
- $g_A = 1.27625(50) \to \tau_n^{\text{SM}} = 878.7(6)$ s $\approx \tau_{\text{trap}}$

Fermion masses

$$U(3)_e\,, \qquad e_R \sim 3_e$$

- Fermions cannot get mass if the symmetry is unbroken.
- Yukawa couplings are induced by non-zero VEVs of scalars.

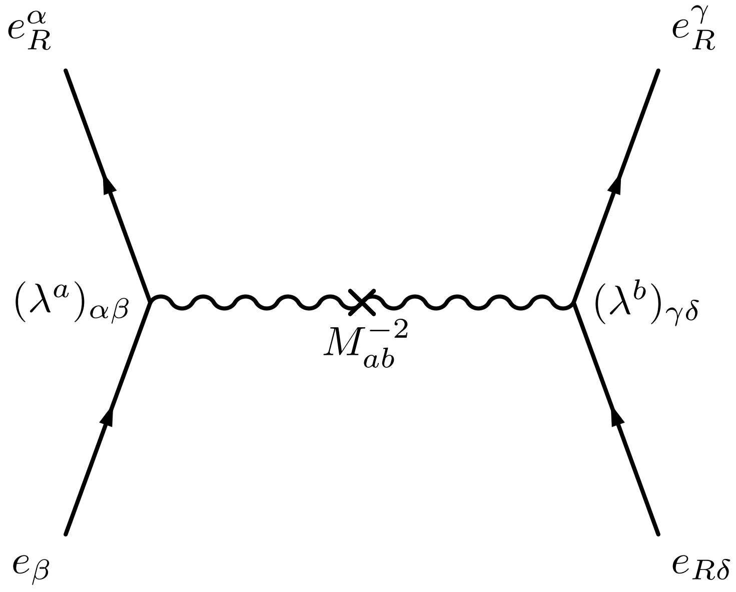
$\xi \sim 3_e$



$$Y_e^{ij}\varphi\overline{l_{Lj}}e_{Ri} \qquad \frac{g_{in}\xi_n^\alpha}{M_L}\varphi\overline{\ell_{Li}}e_{R\alpha}$$

Gauge bosons & RH charged leptons

- Generically, FCNC:

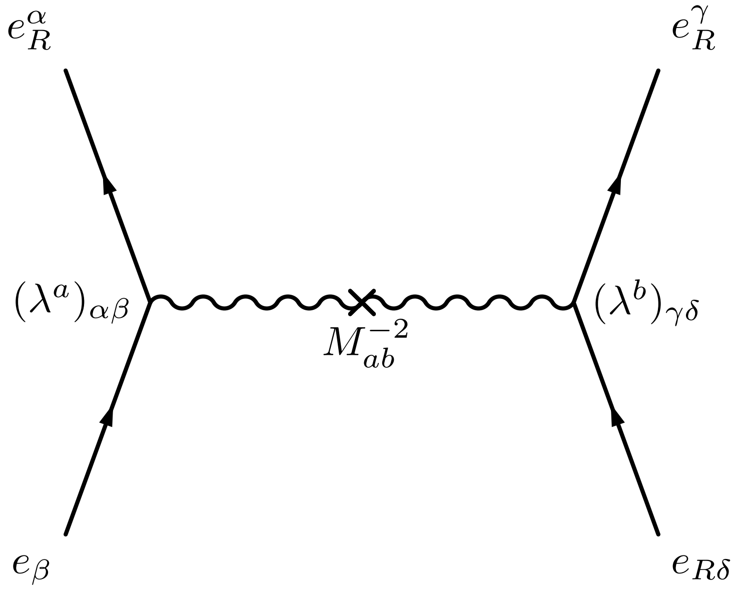


$$e_{Ri} = V^{(e)}_{R1e} e_R + V^{(e)}_{R1\mu} \mu_R + V^{(e)}_{R1\tau} \tau_R$$

Gauge bosons & RH charged leptons

- Generically, FCNC:

$$SU(3)_e$$



$$e_{Ri} = V^{(e)}_{R1e} e_R + V^{(e)}_{R1\mu} \mu_R + V^{(e)}_{R1\tau} \tau_R$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \quad m_\tau : m_\mu : m_e \approx v_3 : v_2 : v_1$$

$$v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1$$

$$-\frac{1}{4v_2^2} \sum_{a=1}^{3} (\overline{\mathbf{e}_R}\, \lambda_a \gamma^\mu \mathbf{e}_R)^2 =$$

$$= -\frac{1}{4v_2^2} \left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2$$

Experimental constraint

- **Flavour conserving operators** constrained by compositeness limits:

$$-\frac{1}{4v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)^2-\frac{1}{2v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)\left(\overline{\mu_R}\gamma_\nu \mu_R\right) \qquad \boxed{v_2 > 2\ \text{TeV}} \quad \to v_3 > 10\ \text{TeV}$$

- From FCNC:

LFV mode	Exp. $\Gamma_i/\Gamma_\mu(\Gamma_\tau)$	Main contribution to $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$	Predicted value of $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$
$\mu \to eee$	$< 1.0 \cdot 10^{-12}$	$\frac{1}{8}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu} + V_{2e}^* V_{2\mu}\epsilon^2\right\rvert^2$	$\leq 1.1 \cdot 10^{-13}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau^- \to \mu^- e^+ e^-$	$< 1.8 \cdot 10^{-8}$	$\frac{1}{4}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$= 6.2 \cdot 10^{-9}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\tau \to \mu\mu\mu$	$< 2.1 \cdot 10^{-8}$	$\frac{1}{8}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$= 3.1 \cdot 10^{-9}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\mu \to e\gamma$	$< 4.2 \cdot 10^{-13}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu}\right\rvert^2$	$= 3.1 \cdot 10^{-15}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau \to \mu\gamma$	$< 4.4 \cdot 10^{-8}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\rm EW}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\rm w}}{\Gamma_\tau}$	$= 8.7 \cdot 10^{-11}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$

New interactions in left-handed sector

$$\mathcal{L}_{\rm eff}^{e\nu} = -\frac{2G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma^{\mu} \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right) \left(\overline{\boldsymbol{\nu}_L}\, \gamma_{\mu} \frac{\lambda_a}{x_a}\, \boldsymbol{\nu}_L \right)$$

muon decay, tau decays,
non-standard neutrino interactions with leptons

$$\mathcal{L}_{\rm eff}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_{\mu} \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

Charged leptons flavour conserving interactions (compositeness limits):

$$v_{\mathcal{F}} > 3 \text{ TeV}$$

lepton flavour violating interactions

$$\mathcal{L}_{\rm eff}^{\nu\nu} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{\nu}_L}\, \gamma_{\mu} \frac{\lambda_a}{x_a}\, \boldsymbol{\nu}_L \right)^2$$

Non-standard interactions between neutrinos

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

If $w_3 = w_2 = w_1$ (e. g. symmetry between ηs) then

- Gauge bosons have equal masses and do not mix.
- $\lambda_a \to V^\dagger \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices
- From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2}(\overline{\mathrm{e_L}}\, \lambda^a\, \gamma^\mu \mathrm{e_L})(\overline{\mathrm{e_L}}\, \lambda^a\, \gamma_\mu \mathrm{e_L}) = -\frac{1}{3v_\ell^2}(\overline{\mathrm{e_L}}\, \mathbb{I}\, \gamma_\mu \mathrm{e_L})^2$$

- **no FCNC**, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$ $\qquad w_2^2 + w_1^2 = v_{\mathcal{F}}^2$ $\qquad (\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4})$

- In general case e.g. $\mu \to 3e$ decay:

$$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to e\nu_\mu\bar{\nu}_e)} \simeq \frac{1}{8} \left(C(r)|U_{3e}^* U_{3\mu}|\right)^2 \delta_\mu^2$$

  $r = 2u_3^2/v_\ell^2$, $|C(r)| < 1$. $|U_{3\mu}|$ and $|U_{3e}|$ can be as large as $\sin\theta_C = V_{us}$.

- The experimental limits on other LFV effects as $\tau \to 3\mu$ are much weaker.

- $\mathbf{v}_{\mathcal{F}} \simeq$ **6 TeV** is not contradicting experimental constraints.

- If $w_3 = w_2 = w_1$ (e. g. symmetry between ηs), $r = 1$, then **no FCNC**, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

Leptons and family symmetry

$$SU(3)_\ell \times SU(3)_e$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

$$\frac{y_{ij}\bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri} + \frac{h_{ij}\bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3}\varphi\varphi\ell_{L\alpha}^T C\ell_\beta + h.c.$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\langle\eta_1\rangle = \begin{pmatrix} w_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\eta_2\rangle = \begin{pmatrix} 0 \\ w_2 \\ 0 \end{pmatrix}, \quad \langle\eta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ w_3 \end{pmatrix}$$

$$m_\nu^{ij} = \frac{h_{ij}w_i w_j v_w^2}{M_\nu^3}, \qquad U_{\mathrm{PMNS}}$$

Leptons and family symmetry

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$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\langle\xi_1\rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\xi_2\rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \quad \langle\xi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}$$

$$v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1 \longrightarrow Y_e = y_\tau \begin{pmatrix} \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \end{pmatrix} \longrightarrow m_e\,, m_\mu\,, m_\tau$$

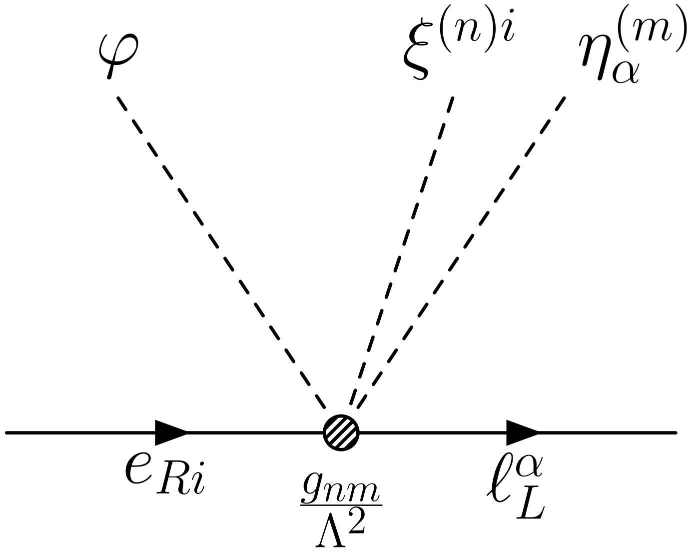
$$Y_e^{ij} = \frac{y_{ij}w_i v_j}{M_L^2}$$

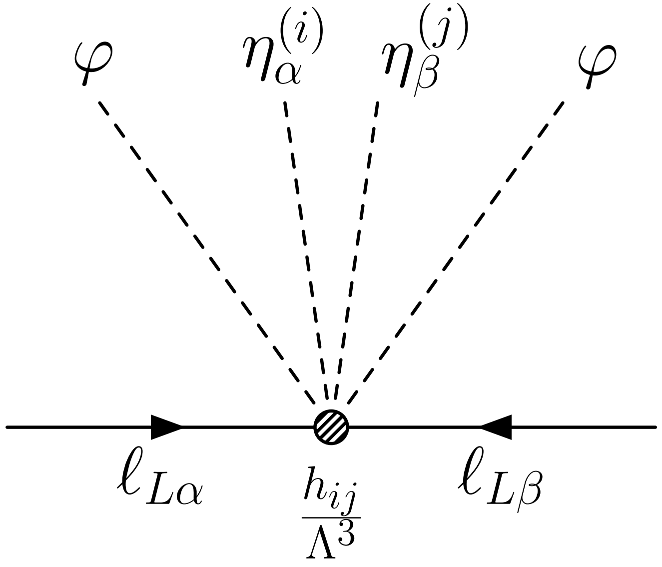
Leptons and family symmetry

$$SU(3)_\ell \times SU(3)_e$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

$$\frac{y_{ij}\bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri} + \frac{h_{ij}\bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3}\varphi\varphi\ell_{L\alpha}^T C\ell_\beta + h.c.$$





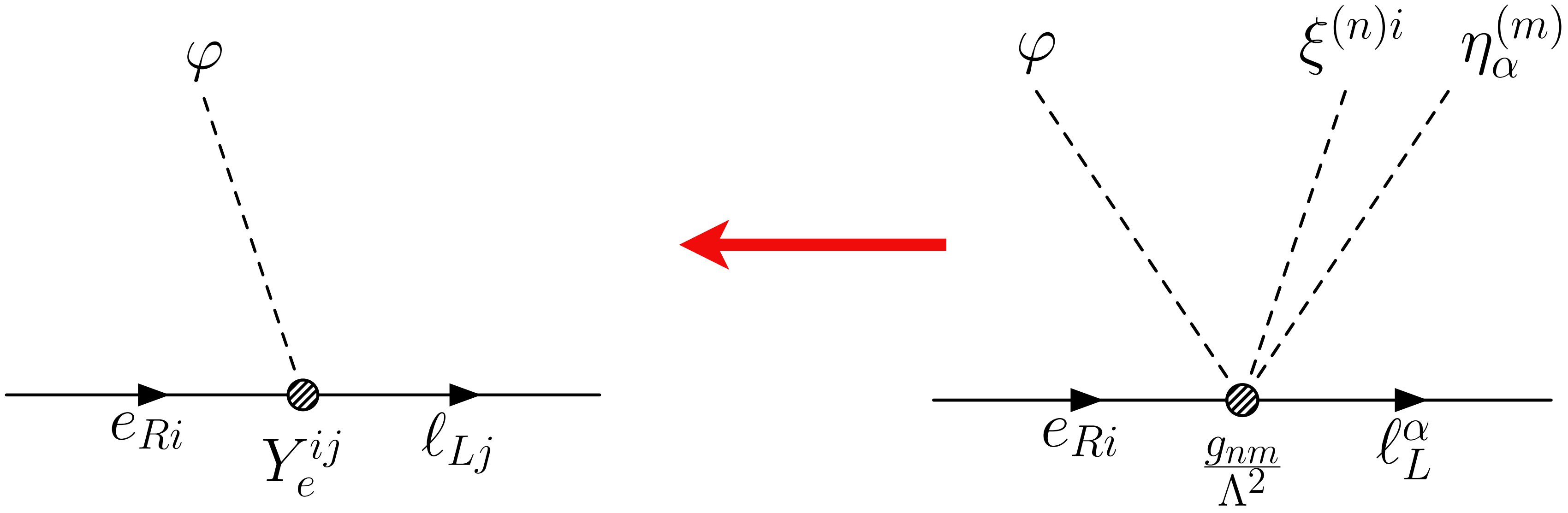
Leptons and family symmetry

$$SU(3)_\ell \times SU(3)_e$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$
- Fermions cannot get mass if the symmetry is unbroken.
- Yukawa couplings are induced by non-zero VEVs of scalars.

$$Y_e^{ij}\varphi\overline{l_{Lj}}e_{Ri}$$

$$\frac{y_{ij}\bar{\xi}_j^{\gamma}\eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri}$$



Gauge bosons, $SU(3)_e$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1$$

$$\frac{1}{2}\begin{pmatrix} \boxed{\begin{matrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 \end{matrix}} & \begin{matrix} \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_6 - i\mathcal{F}_7 \end{matrix} \\ \begin{matrix} \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 \end{matrix} & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M^2_{4,5,6,7} = \frac{g^2 v_3^2}{2} \qquad M^2_{1,2} = \frac{g^2 v_2^2}{2} \qquad M^2_{38} = \frac{g^2}{2}\begin{pmatrix} v_2^2 & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & \frac{1}{3}(4v_3^2 + v_2^2) \end{pmatrix}$$

$$v_2 \gtrsim ?$$

Flavour changing neutral currents

In $\mathbf{SU(2)_e}$ gauge symmetry limit ($v_3 \gg v_2$):

- $SU(2)_e$ gauge bosons have equal masses;
- **no FCNC** for CUSTODIAL SYMMETRY, no matter if two families are mixed:

$$-\frac{1}{v_2^2}\sum_{a=1}^{3}(J_a^\mu)^2 = -\frac{1}{4v_2^2}\sum_{a=1}^{3}(\overline{\mathbf{e}_R}\,\lambda_a\gamma^\mu\mathbf{e}_R)^2 =$$

$$= -\frac{1}{4v_2^2}\left[\begin{pmatrix}\overline{e_R} & \overline{\mu_R} & \overline{\tau_R}\end{pmatrix}\gamma_\mu V_R^{(e)\dagger}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}V_R^{(e)}\begin{pmatrix}e_R\\ \mu_R\\ \tau_R\end{pmatrix}\right]^2$$

- no mixing with 3rd family $\rightarrow$ NO FCNC.
- Constraints on masses are proportional to violation of custodial symmetry $(\epsilon,\,\tilde{\epsilon})$.

Flavour changing neutral currents

- Constraints on masses are proportional to violation of custodial symmetry:

$$\mathcal{L}_{\rm fcnc} = -\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 +$$

$$-\frac{\epsilon^2}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 =$$

$$-\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu \begin{pmatrix} 1 & \sim \tilde{\epsilon}\epsilon^2 & \sim \tilde{\epsilon}\epsilon \\ \sim \tilde{\epsilon}\epsilon^2 & 1 & \sim \epsilon \\ \sim \tilde{\epsilon}\epsilon & \sim \epsilon & \sim \epsilon^2 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 +$$

$$-\frac{\epsilon^2}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu \begin{pmatrix} 1 & \sim \tilde{\epsilon} & \sim \tilde{\epsilon}\epsilon \\ \sim \tilde{\epsilon} & \sim \epsilon^2 & \sim \epsilon \\ \sim \tilde{\epsilon}\epsilon & \sim \epsilon & 1 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2$$

Gauge bosons in left-handed sector, $SU(3)_\ell$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\frac{1}{2}\begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M^2_{\ell 4,5} = \frac{g^2}{2}(w_3^2 + w_1^2) \qquad M^2_{\ell 6,7} = \frac{g^2}{2}(w_3^2 + w_2^2) \qquad M^2_{\ell 38} = \frac{g^2}{2}\begin{pmatrix} w_2^2 + w_1^2 & \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) \\ \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) & \frac{1}{3}(4w_3^2 + w_1^2 + w_2^2) \end{pmatrix}$$

$$M^2_{\ell 1,2} = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2}v_{\mathcal{F}}^2$$

VEVS

Can the hierarchy of the VEVs of ξs be natural? The generic potential is:

$$V(\xi) = \lambda_n(|\xi_n|^2 - \frac{\mu_n^2}{2\lambda_n})^2 + \lambda_{klnm}\xi_k^\dagger\xi_l\xi_n^\dagger\xi_m + (\mu\xi_1\xi_2\xi_3 + \text{h.c.})$$

- The dimensional constant μ can be arbitrarily small since if $\mu \to 0$ the Lagrangian acquires global $U(1)_e$ symmetry.
- $v_2/v_3 \sim m_\mu/m_\tau$ (one order of magnitude) can emerge from a natural fluctuation of mass terms μ_n^2 and coupling constants λ.
- Small v_1 is naturally obtained when the third flavon ξ_1 has positive mass squared. Then for $\mu \neq 0$ non-zero VEV $\langle\xi_1\rangle$ is induced:

$$v_1 = \frac{\mu v_2 v_3}{\mu_1^2}$$

- Taking μ small enough, say $\mu < v_2$, one can naturally get $v_1 \ll v_2$. **The hierarchy of the VEVs of ξs can be natural**.

Flavons and LFV

- Also flavons can mediate the LFV processes.
- Lepton Yukawa couplings with the flavon fields ξ_n:

$$h_{in}\, \xi_n^\alpha \overline{\ell_{Li}} e_{R\alpha} \qquad h_{in} = \frac{g_{in} v_{\rm w}}{M}$$

  which are generically flavor-changing.

- ξ_2, with mass $\mu_2 \sim v_2$, induces the effective operator:

$$-\frac{h_{32}h_{22}}{\mu_2^2} (\overline{\tau}\mu)(\overline{\mu}\mu)\,, \qquad \frac{h_{32}h_{22}}{\mu_2^2} \simeq \frac{m_\mu^2}{v_2^4}$$

  For $v_2 > 2$ TeV, the width of $\tau \to 3\mu$ decay induced by this operator is more than 12 orders of magnitude below the experimental limit.

- The width of $\mu \to 3e$ decay induced by analogous operator mediated by flavon ξ_1 is also suppressed by orders of magnitude.

Triangle anomalies

$$SU(3)_\ell \times SU(3)_e \times SU(3)_Q \times SU(3)_u \times SU(3)_d$$

$$\ell_L \sim 3_\ell,\ e_R \sim 3_e,\ Q_L \sim 3_Q,\ u_R \sim 3_u,\ d_R \sim 3_d$$

- In order to cancel $SU(3)^3$ anomalies for each triplet another triplet (SM singlet) with opposite chirality is needed.
- An interesting possibility is to introduce the mirror twins with opposite chirality and analogous representation of mirror SM gauge symmetry $SU(3)' \times SU(2)' \times U(1)'$:

$$\ell'_R \sim 3_\ell,\ e'_L \sim 3_e,\ Q'_R \sim 3_Q,\ u'_L \sim 3_u,\ d'_L \sim 3_d$$

- Couplings with flavons:

$$\frac{g_{in}\xi_n^\alpha}{M}(\phi\overline{\ell_{Li}}e_{R\alpha} + \phi'\overline{\ell'_{Ri}}e'_{L\alpha}) + h.c.$$

Triangle anomalies 2

- As an example, for $SU(3)_e$, mixed triangle anomaly $U(1) \times SU(3)_e^2$ must be cancelled. New leptons

$$\mathcal{E}_{L\alpha} \sim (1, -2, 3_e; X), \qquad \mathcal{E}_{Ri} \sim (1, -2, 1; X)$$

  and for mirror parity

$$\mathcal{E}'_{R\alpha} \sim (1, -2', 3_e; X), \qquad \mathcal{E}'_{Li} \sim (1, -2', 1; X)$$

  cancel the mixed triangle

$$U(1) \times SU(3)_e^2,\ U(1)_X \times SU(3)_e^2,\ U(1) \times U(1)_X^2,\ U(1)_X \times U(1)^2$$

- Masses from Yukawa couplings

$$y_{in}\xi_n^\alpha \overline{\mathcal{E}_{Ri}} \mathcal{E}_{L\alpha} + y_{in}\xi_n^\alpha \overline{\mathcal{E}_{Li}}' \mathcal{E}'_{R\alpha} + h.c.$$

- The lightest has mass $O(100)$ GeV. If $U(1)_X$ is unbroken, then it is stable. Current experimental lower limit on charged new leptons is 102.6 GeV.

Cosmological implications

- Mirror matter is a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms.
- The flavor gauge bosons are messengers between the two sectors and so a portal for direct detection.
- $T'/T < 0.2 \div 0.3$ from CMB and large scale structures.
- Freeze-out temperature of horizontal interactions between the two sectors should not exceed $T_d \simeq (v_2/2)^{\frac{4}{3}} \times 130$ MeV. Or $v'_{\rm EW} \gg v_{\rm EW}$.
- For neutrinos

$$\frac{Y_\nu^{ij}}{\mathcal{M}}(\phi\phi l_{Li}^T C l_{Lj} + \phi'\phi' l'^T_{Ri} C l'_{Rj}) + \frac{\tilde{Y}_\nu^{ij}}{\mathcal{M}}\phi\phi' \overline{l_{Li}} l'_{Rj} + h.c.$$

the last operator gives COLEPTOGENESIS.

Quarks and family symmetries

$U(3)_q \times U(3)_d \times U(3)_u$ with gauge factors $SU(3)_q \times SU(3)_d \times SU(3)_u$

- Quark masses:

$$\frac{y^d_{ij}}{M_d^2}\,\eta^q_{i\alpha}\overline{\xi}^{d\gamma}_j\,\phi\,\overline{q_{L\alpha}}d_{R\gamma} + \frac{y^u_{ij}}{M_u^2}\,\eta^q_{i\alpha}\overline{\xi}^{u\gamma}_j\,\tilde{\phi}\,\overline{q_{L\alpha}}u_{R\gamma} + \text{h.c.}$$

- mass hierarchy is related with hierarchies in breaking of $SU(3)_q \times SU(3)_d \times SU(3)_u$ gauge symmetry:

$$m_b : m_s : m_d = 1 : \epsilon_d\epsilon_q : \epsilon_d\tilde{\epsilon}_d\epsilon_q\tilde{\epsilon}_q$$
$$m_t : m_c : m_u = 1 : \epsilon_u\epsilon_q : \epsilon_u\tilde{\epsilon}_u\epsilon_q\tilde{\epsilon}_q$$

Quarks and family symmetries

- $K^0 - \bar{K}^0$ oscillation is induced by:

$$-\frac{1}{4v_{d2}^2}\left[(V_{3d}V_{3s}^*)^2 + \epsilon_d^2(V_{2d}V_{2s}^*)^2\right](\overline{s_R}\gamma^\mu d_R)^2$$

- Since $|V_{2d}V_{2s}^*| \sim \tilde{\epsilon}_d$, $|V_{3d}V_{3s}^*| \sim \epsilon_d^2\tilde{\epsilon}_d$, K^0 mixing is suppressed by $\epsilon_d^2\tilde{\epsilon}_d^2 \ll 1$.
- New contribution can be constrained to be less than the SM contribution. By taking $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, the mass scale $v_{d2} \sim 7\,\mathrm{TeV}$ is compatible with the constraint from the neutral kaons mass difference.
- As regards the imaginary part contributing to ϵ_K, with the same choice $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, $v_{d2} \sim 7\,\mathrm{TeV}$ is still allowed if the phase of $V_{2d}V_{2s}^*$ is $O(0.1)$.