Precise measurement of the Decay $K^\pm \rightarrow \pi^0\pi^0\mu^\pm\nu$

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Outline

1. Theoretical framework
2. NA48/2 setup
3. Selection
4. Acceptance
5. Residual background
6. Signal extraction
7. Result
8. Conclusion
$$K^\pm \rightarrow \pi^0\pi^0\mu^\pm\nu$$ ($K_{\mu4}^{00}$) state of the art

$K \rightarrow \pi\pi\mu\nu$ ($K_{I4}$) depends on $F, G, R, H$ form-factors.

Cabibbo-Maksymowicz variables: $S_\pi$ (dipion mass squared), $S_l$ (dilepton mass squared) and angles $\theta_\pi$ (in the dipion frame), $\theta_l$ (in the dilepton frame), $\phi$.

- For $K_{\mu4}^{00}$, s-wave for $\pi^0\pi^0$, there are no dependence on $\cos \theta_\pi, \phi$, and only $F$ and $R$ contribute.
- Unlike $K_{e4}^{00}$ case, $R$ plays some role due to $\mu$ mass.

<table>
<thead>
<tr>
<th>$K_{I4}$ mode</th>
<th>BR [10^{-5}]</th>
<th>$N_{cand}$</th>
<th>$N_{cand}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{e4}^{\pm}$</td>
<td>$4.26 \pm 0.04$</td>
<td>1108941</td>
<td>NA48/2 (2012)</td>
</tr>
<tr>
<td>$K_{e4}^{00}$</td>
<td>$2.55 \pm 0.04$</td>
<td>65210</td>
<td>NA48/2 (2014)</td>
</tr>
<tr>
<td>$K_{\mu4}^{\pm}$</td>
<td>$1.4 \pm 0.9$</td>
<td>7</td>
<td>Bisi et al. (1967)</td>
</tr>
<tr>
<td>$K_{\mu4}^{00}$</td>
<td>?</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

According to lepton universality, experimental $F(S_\pi, S_l)$ parameterization from $K_{e4}^{00}$ [NA48/2 JHEP 08 (2014) 159] may be used for $K_{\mu4}^{00}$.

The only available source of $R(S_\pi, S_l)$ is ChPT calculation [J.Bijnens, G.Colangelo, J.Gasser, Nucl.Phys.B 427 (1994) 427].
NA48/2 main goal were $K_{3\pi}$ charge asymmetry studies; additional rare decays program.

- Two charged beams:
  - 6% of $K^\pm$
  - $\langle P_K \rangle \approx 60$ GeV/c
  - $\Delta P_K/\langle P_K \rangle \approx \pm 3.8\%$

- KABES (Kaon Beam Spectrometer) resolutions:
  - $\sigma(X, Y) \sim 800$ $\mu$m
  - $\sigma(P_K)/P_K \sim 1\%$
  - $\sigma(T) \sim 600$ ps
NA48/2 setup (CERN SPS, 2003-2004)

- Magnetic spectrometer (drift chambers DCH1–DCH4):
  - $\sigma(X, Y) \sim 90 \mu m$ per chamber
  - $\sigma(P_{DCH})/P_{DCH} = (1.02 \oplus 0.044 \cdot P_{DCH})\%$ ($P_{DCH}$ in GeV/c)
- Scintillator hodoscope (HOD):
  - $\sigma(T) \sim 150$ ps
- Liquid Krypton EM calorimeter (LKr):
  - $\sigma_x = \sigma_y = (0.42/\sqrt{E_\gamma} \oplus 0.06) \text{ cm}$
  - $\sigma(E_\gamma)/E_\gamma = (3.2/\sqrt{E_\gamma} \oplus 9.0/E_\gamma \oplus 0.42)\%$ ($E_\gamma$ in GeV)
- Hadronic calorimeter, muon system MUV.
Events selection

- Signal $K_{\mu4}$ is $K^\pm \rightarrow \pi^0\pi^0\mu^\pm\nu$
- Normalization $K_{3\pi}$ is $K^\pm \rightarrow \pi^\pm\pi^0\pi^0$
- Trigger chain: L1 trigger using HOD and LKr, followed by L2 trigger using DCH for online momentum calculation.
- Event selection: 4 isolated photons consistent with $2\pi^0$ in time-spatial matching with a KABES beam track and a DCH track.

Normalization $K_{3\pi}$ kinematic selection ellipse:
- center:
  - $M(K_{3\pi}) = M_{K}^{PDG}$
  - $P_t = 5\text{ MeV/c}$
- semi-axes:
  - $\Delta M(K_{3\pi}) = 10\text{ MeV/c}^2$
  - $\Delta P_t = 20\text{ MeV/c}$
- $72.99 \times 10^6 K_{3\pi}$ selected data events.
$K^\pm \rightarrow \pi^0\pi^0\mu^\pm\nu$ signal events selection

- Off the $K_{3\pi}$ kinematic ellipse
- DCH track has associated MUV response

$$M^2_{\text{miss}} = (P_K - P(\pi^0_1) - P(\pi^0_2) - P(\mu^\pm))^2$$

$$M^2_{\text{miss}}(\pi^\pm) = (P_K - P(\pi^0_1) - P(\pi^0_2) - P(\pi^\pm))^2$$

- $M^2_{\text{miss}}(\pi^\pm) < 0.5M^2_{\text{miss}} - 0.0008 \text{ GeV}^2/c^4$
$K^\pm \to \pi^0\pi^0\mu^\pm\nu$ signal events selection

- $\cos(\Theta_l) < 0.6$
$K^\pm \rightarrow \pi^0\pi^0\mu^\pm\nu$ signal events selection

- $S_i = m(\mu\nu)^2 > 0.03 \text{ GeV}^2/c^4$ (to reject $\pi^\pm \rightarrow \mu^\pm\nu$).

- 3718 $K_{\mu 4}$ data candidates selected

- 2437 data candidates in $M_{\text{miss}}^2$ signal region $[-0.002,0.002] \text{ GeV}^2/c^4$

- The MC $M_{\text{miss}}^2$ signal region contains 98.2% of all selected MC events
**Acceptances**

- $K_{\mu 4}^{00}$ signal acceptance is

  \[ A_S = \frac{N_{MC}^{Selected \ in \ signal \ region}}{N_{MC}^{Generated (all \ S_{\text{true}})}} = (0.651 \pm 0.001)\% , \]

- However, for the restricted phase space region $S_{\text{true}} > 0.03$ GeV$^2$/c$^4$, the signal acceptance is

  \[ A'_S = \frac{N_{MC}^{Selected \ in \ signal \ region \ (S_{\text{true}} > 0.03)}}{N_{MC}^{Generated \ (S_{\text{true}} > 0.03)}} = (3.453 \pm 0.007)\% . \]

- $K_{3\pi}$ normalization channel acceptance is

  \[ A_N = \frac{N_{MC}^{Selected}}{N_{MC}^{Generated}} = (4.477 \pm 0.002)\% . \]
Residual background

- $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$, followed by $\pi^\pm \rightarrow \mu \nu$ before MUV with a probability $\approx 10\%$ for $P(\pi^\pm) \approx 10 \text{ GeV}/c$.

- $K_{3\pi}$ background with $\pi^\pm$ decay before LKr: from MC.

- $K_{3\pi}$ background with late $\pi^\pm$ decay or muon emission in a late hadron shower:
  - Can not be easily simulated
  - Data-driven method of estimation
  - Background-enhanced control sample, selected using $E_{LKr}$ and $P_{DCH}$
$K^{00}_{\mu4}$ signal extraction fit

- 2437 candidates in the signal region.
- Fit in the $M^2_{miss}$ interval [-0.003, 0.006] GeV$^2/c^4$, ignoring the signal region to decrease sensitivity to the imperfect MC resolution.
- Data fit by a linear combination of background and MC signal tails.
- 354 ± 33$_{stat}$ ± 62$_{syst}$ background events.
- The background-related systematics are determined by varying the way the background is estimated.
Signal versus $S_\pi, S_l$

The branching ratio is measured for the restricted phase space $S_l^{true} > 0.03$ GeV$^2$/c$^4$.

Extrapolation to the full phase space depends on the theory.

**Figure:** 1D projections comparison for $S_l > 0.03$ GeV$^2$/c$^4$
Preliminary result: Central values and errors budget

\[ \text{BR}(K_{\mu 4}^{00}) = \frac{N_S}{N_N} \cdot \frac{A_N}{A_S} \cdot K_{\text{trig}} \cdot \text{BR}(K_{3\pi}^{00}). \]

<table>
<thead>
<tr>
<th></th>
<th>Full phase space</th>
<th>(S_l &gt; 0.03) GeV(^2/c^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{BR}(K_{\mu 4})) central value ([10^{-6}])</td>
<td>3.45</td>
<td>0.651</td>
</tr>
<tr>
<td>(\delta \text{BR}[10^{-6}])</td>
<td>(\delta \text{BR}/\text{BR})</td>
<td>(\delta \text{BR}[10^{-6}])</td>
</tr>
<tr>
<td>Data stat. error</td>
<td>0.10</td>
<td>2.85%</td>
</tr>
<tr>
<td>MC stat. error</td>
<td>0.01</td>
<td>0.21%</td>
</tr>
<tr>
<td>Trigger</td>
<td>0.01</td>
<td>0.18%</td>
</tr>
<tr>
<td>Background</td>
<td>0.10</td>
<td>2.96%</td>
</tr>
<tr>
<td>Accidentals</td>
<td>0.01</td>
<td>0.32%</td>
</tr>
<tr>
<td>MUV inefficiency</td>
<td>0.06</td>
<td>1.65%</td>
</tr>
<tr>
<td>Form Factor modelling</td>
<td>0.05</td>
<td>1.37%</td>
</tr>
<tr>
<td>(\text{BR}(K_{3\pi})) error (external)</td>
<td>0.05</td>
<td>1.31%</td>
</tr>
<tr>
<td>Total error</td>
<td>0.17</td>
<td>4.83%</td>
</tr>
</tbody>
</table>

- Accidentals obtained from side bands of time distributions;
- MUV inefficiency uncertainty taken as full inefficiency effect.
Preliminary result: Comparison to theory

Theory:
  - Tree approximation;
  - 1-loop;
- Re-calculated now:
  - $F(K_{e4})$ from NA48/2 (2015);
  - $R_1 = R(1\text{loop})$;
  - 1-loop (F,R) phase;
  - 2020 PDG constants.
Conclusion

A first observation and branching fraction precise measurement of $K^\pm \rightarrow \pi^0 \pi^0 \mu^\pm \nu$ decay is performed by NA48/2 experiment at SPS in CERN.

- We observe 2437 signal candidates with an estimated background of $354 \pm 33_{\text{stat}} \pm 62_{\text{syst}}$ events. Signal/Background ratio is $5.9 \pm 1.4_{\text{tot}}$.
- Preliminary result for restricted phase space ($S_l > 0.03$) is

$$BR(K_{\mu4}^{00}, S_l > 0.03) = (0.65 \pm 0.019_{\text{stat}} \pm 0.024_{\text{syst}}) \times 10^{-6} = (0.65 \pm 0.03) \times 10^{-6};$$

- Preliminary full phase space result is

$$BR(K_{\mu4}^{00}) = (3.4 \pm 0.10_{\text{stat}} \pm 0.13_{\text{syst}}) \times 10^{-6} = (3.4 \pm 0.2) \times 10^{-6};$$

- The results are consistent with a contribution of the R form factor, as computed at 1-loop ChPT.