

Constrained second-order power corrections in HQET: $R(D^{(*)})$, $|V_{cb}|$, and new physics

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Florian U. Bernlochner¹, Zoltan Ligeti², Michele Papucci³,
Markus T. Prim¹, Dean J. Robinson², and Chenglu Xiong¹

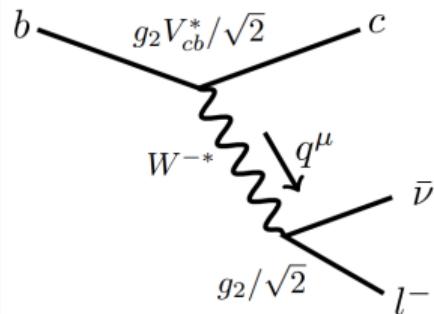
8th July 2022

¹ Physikalisches Institut der Rheinischen Friedrich-Wilhelms-Universität Bonn

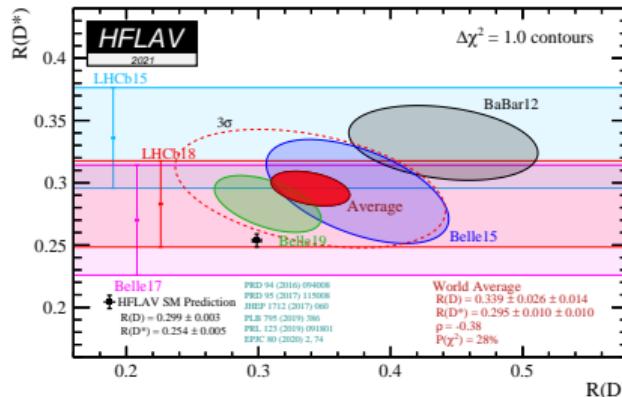
² Ernest Orlando Lawrence Berkeley National Laboratory

³ Walter Burke Institute for Theoretical Physics, California Institute of Technology

The $R(D^{(*)})$ Anomaly



Form factors describe the hadronic interactions with the spectator quark.



Good understanding of the **form factors** is crucial for precise predictions / determinations of:

- Lepton flavor universality ratios $R(D^{(*)})$
- Forward-Backward asymmetries A_{FB}
- τ polarizations $P_\tau(D^{(*)})$
- Longitudinal polarization fractions $F_{L,\tau}(D^{(*)})$
- CKM matrix element $|V_{cb}|$

Many are **sensitive probes for new physics**.

$\bar{B} \rightarrow D^{(*)}$ Form Factors

$\bar{B} \rightarrow D^{(*)}$ transition are described by two (four) form factors in the SM.

Key idea: Exploit expansion of $B \rightarrow D^{(*)}$ form factors into leading $\hat{h}(w) \equiv h(w)/\xi(w)$ and sub-leading $\mathcal{O}(1/m_{b,c}^{(2)})$ and $\mathcal{O}(1/(m_b m_c))$ Isgur-Wise functions.

$$\hat{h}_+ = 1 + \hat{\alpha}_s \left[C_{V1} + \frac{w+1}{2} (C_{V2} + C_{V3}) \right] + \sum_{Q=c,b} \varepsilon_Q \hat{L}_1^{(Q)} - \varepsilon_c \varepsilon_b \hat{M}_8 ,$$

$$\hat{h}_- = \hat{\alpha}_s \frac{w+1}{2} (C_{V2} - C_{V3}) + \varepsilon_c \hat{L}_4^{(c)} - \varepsilon_b \hat{L}_4^{(b)} ,$$

$$\hat{h}_V = 1 + \hat{\alpha}_s C_{V1} + \varepsilon_c [\hat{L}_2^{(c)} - \hat{L}_5^{(c)}] + \varepsilon_b [\hat{L}_1^{(b)} - \hat{L}_4^{(b)}] + \varepsilon_c \varepsilon_b \hat{M}_9 ,$$

$$\hat{h}_{A_1} = 1 + \hat{\alpha}_s C_{A1} + \varepsilon_c \left(\hat{L}_2^{(c)} - \hat{L}_5^{(c)} \frac{w-1}{w+1} \right) + \varepsilon_b \left(\hat{L}_1^{(b)} - \hat{L}_4^{(b)} \frac{w-1}{w+1} \right) + \varepsilon_c \varepsilon_b \hat{M}_9 ,$$

$$\hat{h}_{A_2} = \hat{\alpha}_s C_{A2} + \varepsilon_c [\hat{L}_3^{(c)} + \hat{L}_6^{(c)}] - \varepsilon_c \varepsilon_b \hat{M}_{10} ,$$

$$\begin{aligned} \hat{h}_{A_3} = 1 + \hat{\alpha}_s (C_{A1} + C_{A3}) + \varepsilon_c [\hat{L}_2^{(c)} - \hat{L}_3^{(c)} + \hat{L}_6^{(c)} - \hat{L}_5^{(c)}] + \varepsilon_b [\hat{L}_1^{(b)} - \hat{L}_4^{(b)}] \\ + \varepsilon_c \varepsilon_b [\hat{M}_9 + \hat{M}_{10}] , \end{aligned}$$

BLPR: $\mathcal{O}(1/m_{c,b}, \alpha_s/m_{c,b})$



BLPR^{XP}: $\mathcal{O}(1/m_{c,b}^2, \alpha_s/m_{c,b})$

HQET order	Isgur-Wise functions		
	All	RC Expansion	VC Limit
$1/m_{c,b}^0$	1	1	1
$1/m_{c,b}^1$	3	3	2
$1/m_c^2$	20	1	2
$1/m_{c,b}^2$	32	3	3

Form factors for $\bar{B} \rightarrow D^{(*)}$ are not independent, but from HQET via \hat{L} and \hat{M} .

Including $1/m_c^2$ Corrections

Full $\mathcal{O}(1/m_{c,b}^2)$ matching of QCD to HQET:

$$\frac{\langle H_c | \bar{c} \Gamma b | H_b \rangle}{\sqrt{m_{H_c} m_{H_b}}} \sim 1 + \underbrace{\frac{1}{2m_c}}_{+3} + \underbrace{\frac{1}{2m_b}}_{+20} + \overbrace{\frac{1}{4m_c^2} + \frac{1}{2m_b^2} + \frac{1}{4m_c m_b}}^{+32}$$

Increasing number of combinations of Isgur-Wise functions can be reduced:

- by supplemental power counting in θ for HQET based on the transverse residual momentum \not{D}_\perp : the **residual chiral expansion**.
- in the **vanishing chromomagnetic limit**: $G_{\alpha\beta} \rightarrow 0$

HQET order	Isgur-Wise functions		
	All	RC Expansion	VC Limit
$1/m_{c,b}^0$	1	1	1
$1/m_{c,b}^1$	3	3	2
$1/m_c^2$	20	1	2
$1/m_{c,b}^2$	32	3	3

Zero-Recoil Predictions [RC Expansion]

Inputs

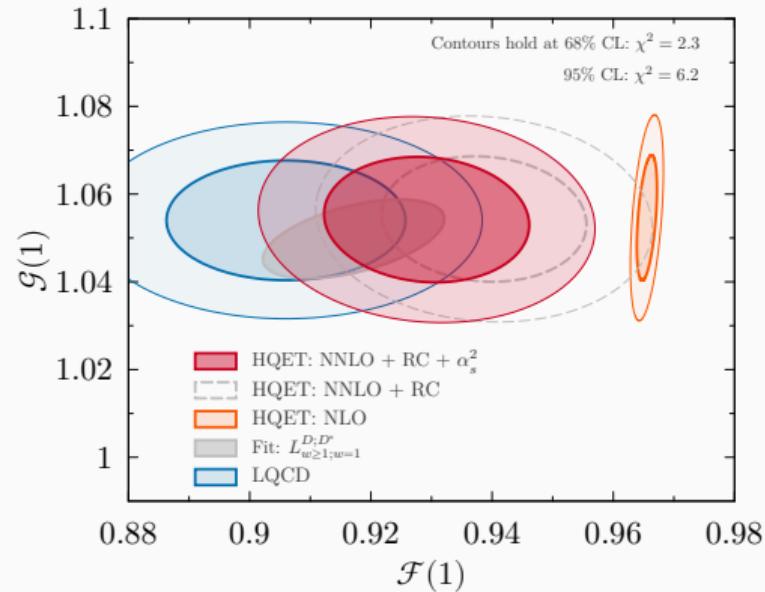
- $m_b^{1S} = (4.71 \pm 0.05) \text{ GeV}$
- $\delta m_{bc} \equiv m_b - m_c = (3.40 \pm 0.02) \text{ GeV}$
- $\rho_1 = (-0.1 \pm 0.2) \text{ GeV}^3$
- $\lambda_2(\mu_{bc}) \simeq 0.11 \pm 0.02 \text{ GeV}^2$

short-distance scheme, pre-fit uncertainties

- $\hat{\eta}(1) = 0.3 \pm 0.05$

from prior fits

Tension at $\mathcal{O}(1/m)$, because the corrections for $\mathcal{F}(1)$ vanish.



Input Data

Experimental Data

- $\bar{B} \rightarrow D\ell\nu$ tagged ('Belle 16')
- $\bar{B}^0 \rightarrow D^{*+}\ell\nu$ tagged ('Belle 17')
- $\bar{B}^0 \rightarrow D^{*+}\ell\nu$ untagged ('Belle 19')

Lattice Input at zero-recoil

- $\bar{B} \rightarrow D\ell\nu$:

$$\mathcal{G}(1)_{\text{LQCD}} = 1.054(9)$$

- $\bar{B} \rightarrow D^{*}\ell\nu$

$$\mathcal{F}(1)_{\text{LQCD}} = 0.906(13) = h_{A_1}(1)$$

Lattice Input beyond zero-recoil

Form factor	$w = 1.0$	$w = 1.08$	$w = 1.16$
f_+	1.1994(95)	1.0941(104)	1.0047(123)
f_0	0.9026(72)	0.8609(77)	0.8254(94)
Form factor	$w = 1.03$	$w = 1.10$	$w = 1.17$
h_{A_1}	0.877(16)	0.807(15)	0.745(22)
h_{A_2}	-0.624(84)	-0.586(81)	-0.492(82)
h_{A_3}	-0.391(95)	1.259(78)	1.213(75)
h_V	1.103(74)	0.989(86)	1.270(46)

Baseline fit scenario: $L_{w \geq 1;=1}^{D;D^*}$

Fit Results [RC Expansion]

The free parameters in our model are

- entering at zero-recoil $|V_{cb}|$, m_b^{1S} , δm_{bc} , ρ_1 , λ_2 ; ρ_*^2 , c_* ; $\hat{\eta}(1)$
- and beyond: $\hat{\eta}'(1)$, $\hat{\chi}_2(1)$, $\hat{\chi}'_2(1)$, $\hat{\chi}'_3(1)$, $\hat{\varphi}'_1(1)$, $\hat{\beta}_2(1)$, $\hat{\beta}'_3(1)$

Nested Hypothesis Test (NHT) to determine the optimal set of fit parameters.

- Starting point are the parameters contributing at zero-recoil.
- Subsequently add parameters to the model in all combinations.
- Test alternative fit hypotheses with cut-off $\Delta\chi^2 = \chi_N^2 - \chi_{N+1}^2 < 1$
- Reject combinations with highly correlated parameters.

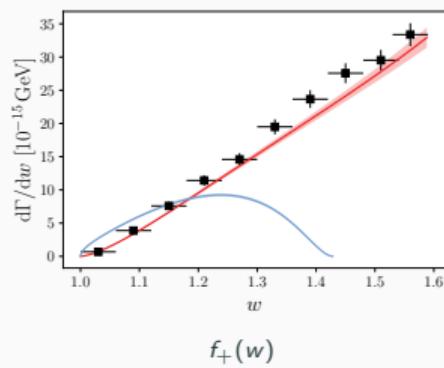
Fit Results [RC Expansion]

Params	<i>S1</i>	<i>S2</i>	<i>S3</i>	<i>S4</i>	<i>S5</i>	<i>S6</i>	<i>S7</i>	<i>S8</i>
$ V_{cb} \times 10^3$	38.70(62)	38.90(64)	38.70(68)	38.70(68)	38.70(69)	38.70(67)	38.80(68)	38.70(69)
ρ_*^2	1.10(4)	1.15(4)	1.19(5)	1.15(5)	1.15(4)	1.10(7)	1.12(8)	1.10(4)
c_*	2.39(18)	2.44(19)	2.16(24)	2.25(23)	2.29(29)	2.38(19)	2.41(20)	2.40(29)
$\hat{\chi}_2(1)$	-0.12(2)	-0.14(3)	—	—	-0.12(5)	—	-0.13(4)	-0.12(5)
$\hat{\chi}'_2(1)$	—	—	-0.15(8)	-0.08(7)	-0.07(11)	—	—	0.00(10)
$\hat{\chi}'_3(1)$	—	—	0.04(1)	0.04(1)	—	0.04(1)	—	—
$\hat{\eta}(1)$	0.34(4)	0.33(4)	0.34(4)	0.34(4)	0.34(4)	0.34(4)	0.34(4)	0.34(4)
$\hat{\eta}'(1)$	—	0.12(10)	0.14(11)	—	0.15(11)	-0.15(14)	0.05(19)	—
$m_b^{1S} [\text{GeV}]$	4.71(5)	4.71(5)	4.70(5)	4.70(5)	4.71(5)	4.71(5)	4.71(5)	4.71(5)
$\delta m_{bc} [\text{GeV}]$	3.41(2)	3.41(2)	3.41(2)	3.41(2)	3.41(2)	3.41(2)	3.41(2)	3.41(2)
$\hat{\beta}_2(1)$	—	—	—	—	—	—	—	—
$\hat{\beta}'_3(1)$	—	—	—	—	—	—	—	—
$\hat{\varphi}'_1(1)$	0.25(21)	—	—	0.24(21)	—	0.53(31)	0.17(40)	0.25(21)
$\lambda_2 [\text{GeV}^2]$	0.12(2)	0.12(2)	0.12(2)	0.12(2)	0.12(2)	0.12(2)	0.12(2)	0.12(2)
$\rho_1 [\text{GeV}^3]$	-0.36(24)	-0.35(24)	-0.37(24)	-0.36(24)	-0.37(24)	-0.36(24)	-0.36(24)	-0.36(24)
χ^2	29.8	30.0	28.9	29.3	29.5	29.6	29.8	29.8
ndf	31	31	30	30	30	30	30	30
ρ^2	1.35(5)	1.37(5)	1.34(6)	1.34(6)	1.34(6)	1.34(6)	1.36(6)	1.35(6)
c	2.41(17)	2.43(17)	2.14(22)	2.26(21)	2.29(28)	2.40(17)	2.42(17)	2.42(27)

Our considered nominal scenario: S1: Smallest χ^2 , fewest model parameters

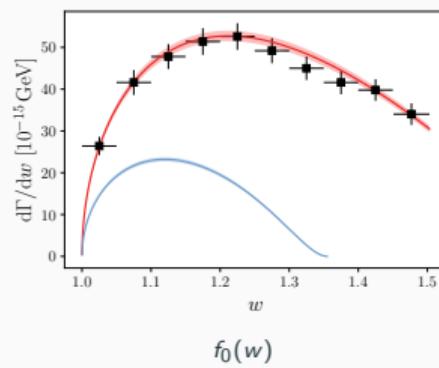
Fit Results [RC Expansion]

$d\Gamma(\bar{B} \rightarrow D\ell\nu)/dw$ (Belle '16)



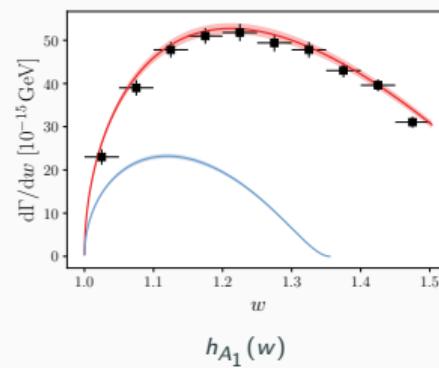
$f_+(w)$

$d\Gamma(\bar{B} \rightarrow D^*\ell\nu)/dw$ (Belle '17)

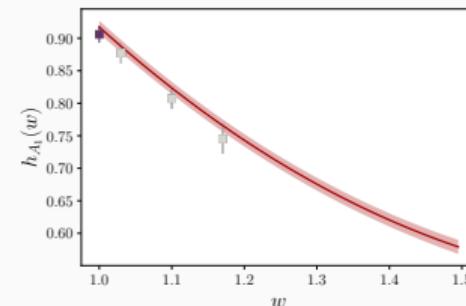
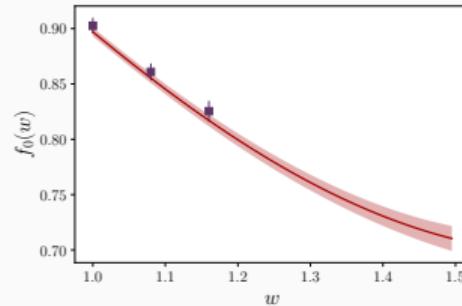
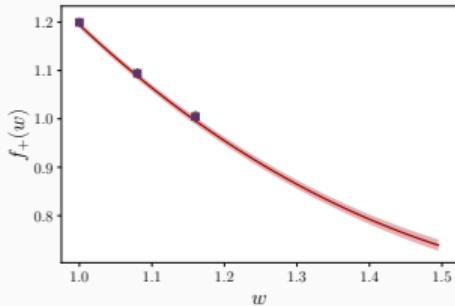


$f_0(w)$

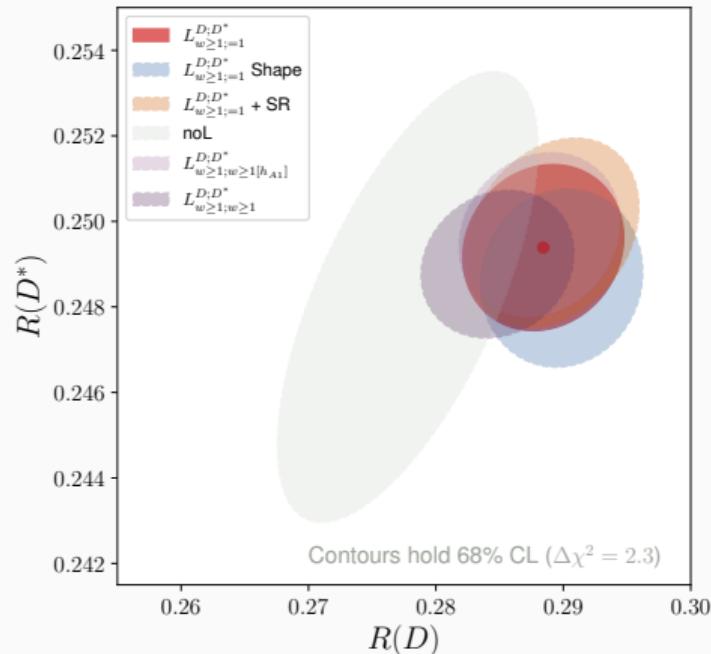
$d\Gamma(\bar{B} \rightarrow D^*\ell\nu)/dw$ (Belle '19)



$h_{A_1}(w)$



Predicting $R(D^{(*)})$ [RC Expansion]



Predictions are robust when including

- additional lattice input for $B \rightarrow D^* \ell \nu$
- QCD sum rule input

No lattice input results in shift for $R(D)$, with larger uncertainties.

Biases and the Major Axis of Doom [RC Expansion]

- BLPR $R(D) = 0.298(3)$, $R(D^*) = 0.261(4)$, $\rho = 0.19$
↓
 2.7σ shift
- BLPR^{XP} $R(D) = 0.288(4)$, $R(D^*) = 0.249(1)$, $\rho = 0.12$

We identify two sources of external biases:

- The tension using the 'Belle '17' vs 'Belle '17 + '19' $B \rightarrow D^* \ell \nu$ data
- The CLN major-axis approximation

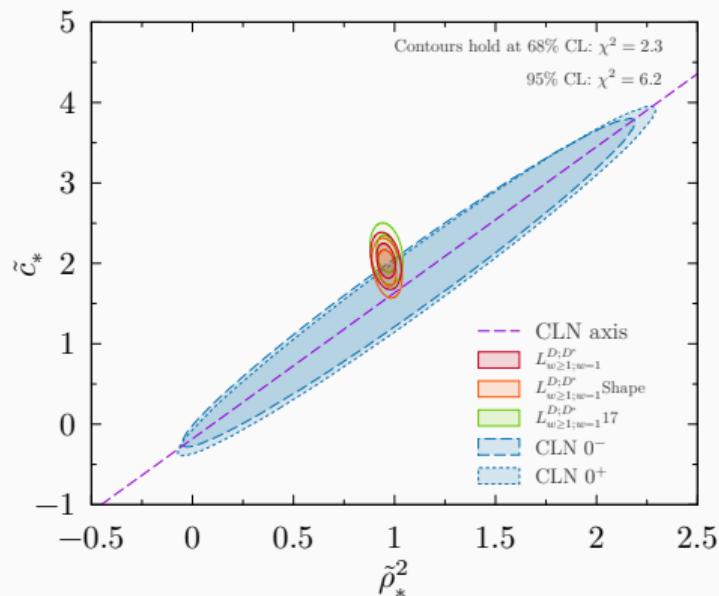
Biases and the Major Axis of Doom [RC Expansion]

CLN parametrization:

- Application of dispersive bounds from unitarity constraints to $\mathcal{G}(w)$
- constrain the allowed parameter space in the slope-curvature $(\tilde{\rho}_*^2 - \tilde{c}_*)$ plane

Approximated by the **major axis**, enforcing a linear relationship between $\tilde{\rho}_*^2$ and \tilde{c}_*

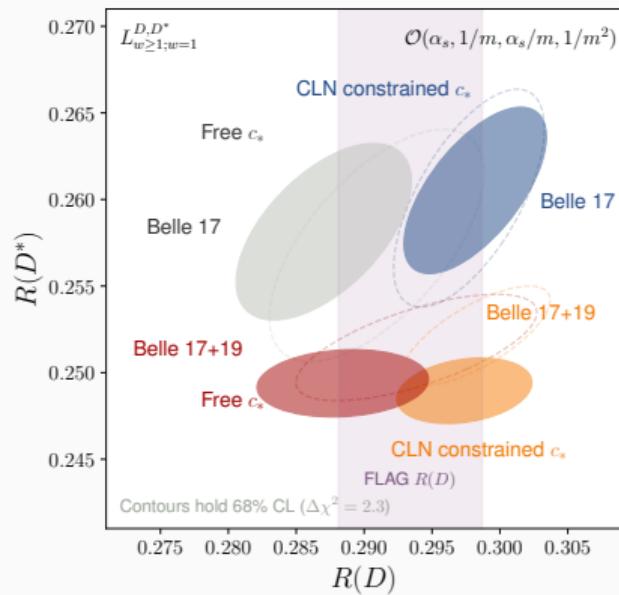
The precision of the available experimental and LQCD data is high enough to resolve the **minor axis**.



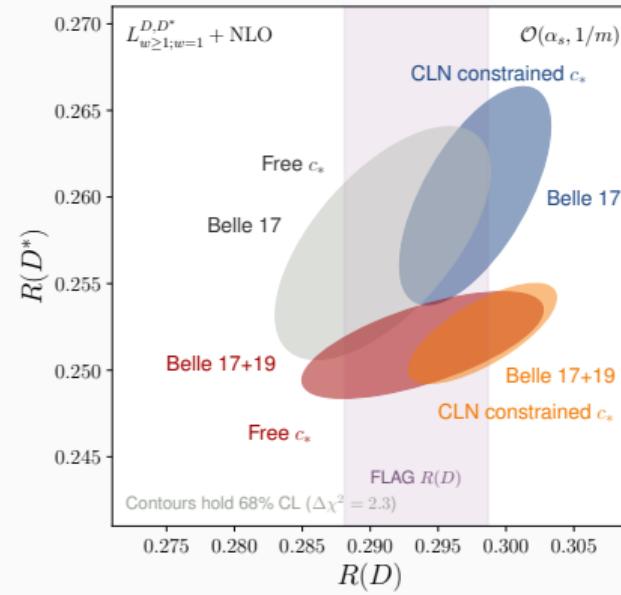
Origin of Fit Bias

Biases and the Major Axis of Doom [RC Expansion]

BLPR^{XP} (NNLO)

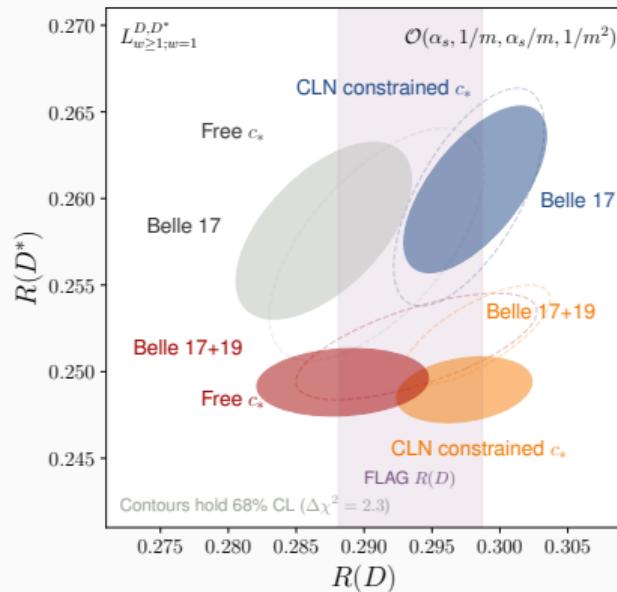


BLPR (NLO)



Biases and the Major Axis of Doom [RC Expansion]

BLPR^{XP} (NNLO)



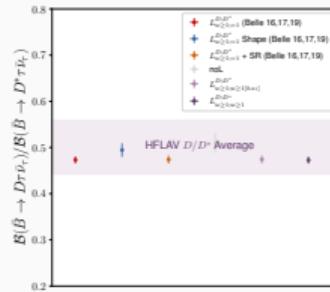
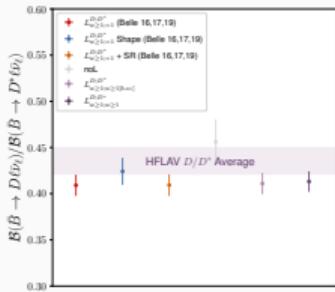
Biases

- Downward shift for $R(D) \rightarrow$ not a feature of the higher order corrections but due to major axis of doom.
- Downward shift for $R(D^*)$ from tension between the Belle '17 and '19 datasets:
Addressed with scaled uncertainties $\sqrt{\chi^2}$ for our prediction.

Branching Ratios, FB-Asymmetries, Polarizations

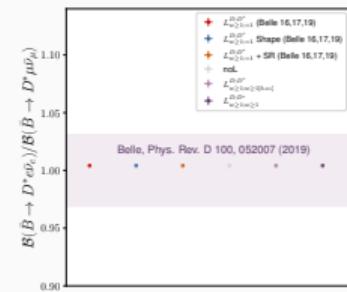
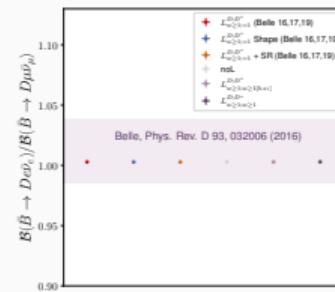
$D-D^*$ Ratio:

$$R_{D/D^*}^I \equiv \frac{\mathcal{B}[\bar{B} \rightarrow D l \nu]}{\mathcal{B}[\bar{B} \rightarrow D^* l \nu]}$$



$e-\mu$ -Universality

$$R_{e/\mu}(D^{(*)}) \equiv \frac{\Gamma[\bar{B} \rightarrow D^{(*)} e \nu]}{\Gamma[\bar{B} \rightarrow D^{(*)} \mu \nu]}$$



We also predict A_{FB} , $F_L(D^{(*)})$, $P_\tau(D^{(*)})$.

Summary & Conclusion

NNLO $\mathcal{O}(1/m_{c,b}, 1/m_{c,b}^2, 1/(m_c m_b), \alpha_s/m_{c,b})$ results for BLPR form factors: BLPR^{XP}

Key Idea: Supplemental power counting in θ for HQET based on the transverse residual momentum \not{D}_\perp : The residual chiral (RC) expansion.

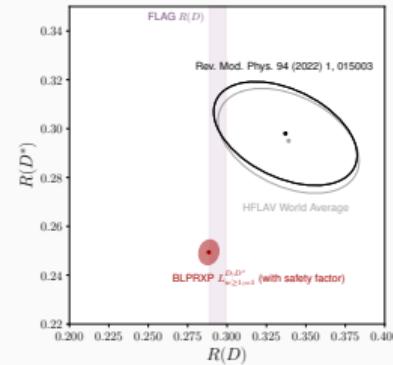
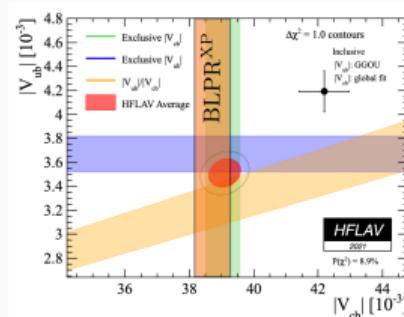
⇒ dramatic simplification in HQET, when truncating the RC expansion at $\mathcal{O}(\theta^2)$.

⇒ Reduction of the number of subleading Isgur-Wise functions.

Available in [HAMMER](#)

Comprehensive analysis of the experimental and lattice data.

- Consistent results with different inputs.
- Origin of a bias, the major axis of doom.



Backup

Parametric form of the Leading Order Isgur-Wise function

Optimized conformal variable

$$z_*(w) = \frac{\sqrt{w+1} - \sqrt{2}a}{\sqrt{w+1} + \sqrt{2}a}, \quad \text{with} \quad a^2 \equiv \frac{w_0 + 1}{2} = \frac{1 + r_D}{2\sqrt{r_D}}. \quad (1)$$

Leading order Isgur-Wise function parametrized as polynomial in z_* .

$$\frac{\xi(w)}{\xi(w_0)} = 1 - 8a^2\rho_*^2 z_* + 16(2c_*a^4 - \rho_*^2 a^2)z_*^2 + \dots \quad (2)$$

No sensitivity to cubic terms given the current experimental and lattice data.

Parametric form of $\mathcal{G}(1)$

$$\frac{\mathcal{G}(w)}{\mathcal{G}(w_0)} = 1 - 8a^2\tilde{\rho}_*^2z_* + 16(2\tilde{c}_*a^4 - \tilde{\rho}_*^2a^2)z_*^2 + \dots , \quad (3)$$

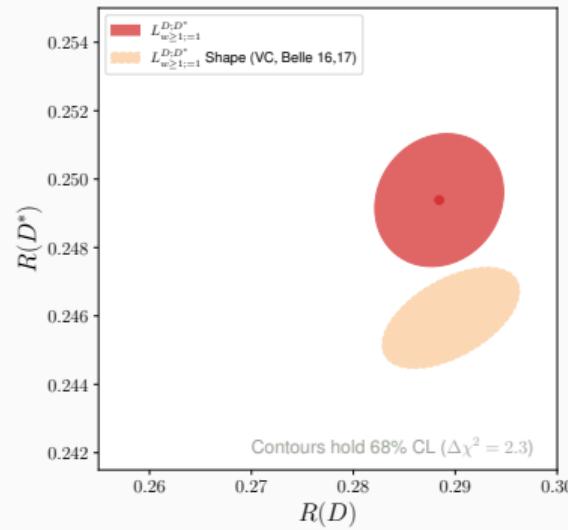
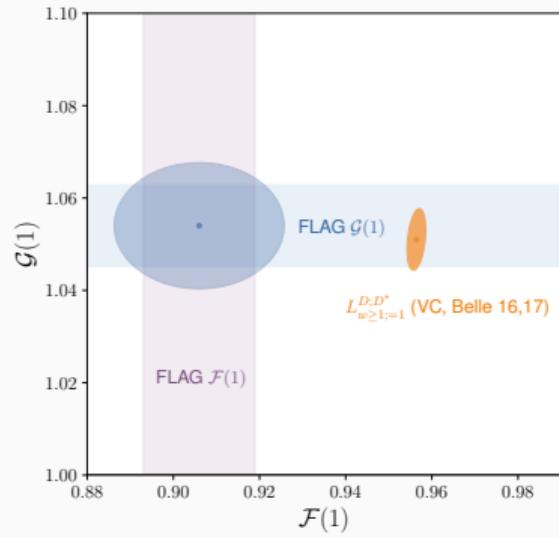
Major axis of doom:

$$\tilde{c}_* \simeq [(V_{21} + 16a^2)\tilde{\rho}_*^2 - V_{20}]/32a^4 , \quad (4)$$

$$\rho_*^2 = \tilde{\rho}_*^2 + \frac{\hat{h}'_+(w_0) - \rho_D \hat{h}'_-(w_0)}{\hat{h}_+(w_0) - \rho_D \hat{h}_-(w_0)} , \quad (5)$$

$$c_* = \tilde{c}_* + 2\rho_*^2(\rho_*^2 - \tilde{\rho}_*^2) - \frac{\hat{h}''_+(w_0) - \rho_D \hat{h}''_-(w_0)}{\hat{h}_+(w_0) - \rho_D \hat{h}_-(w_0)} ,$$

Fit Results [VC Limit]



- Strong tension between predicted $\mathcal{F}(1)$ and LQCD.

- In the shape-only scenario, the VC limit exhibits a tension for $R(D^*)$

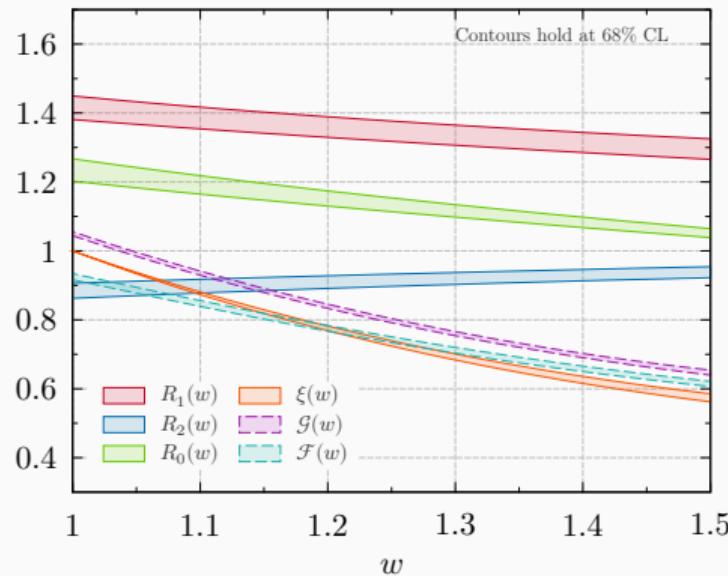
Shape and normalization cannot be describe simultaneously.

Fit Scenarios

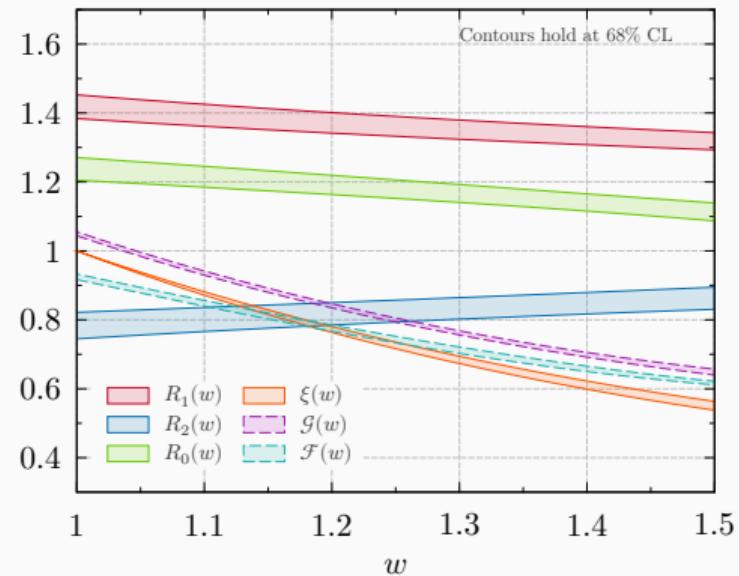
Fit	Order	Floating norm.	Lattice QCD			QCDSR	Belle Data		
			$f_{+,0}(w \geq 1)$	$\mathcal{F}(1)$	$h_{A_1}(w > 1)$		'15	'17	'19
$L_{w \geq 1;=1}^{D;D^*}$	$\alpha_s/m_Q, 1/m_Q^2$	—	✓	✓	—	—	✓	✓	✓
$L_{w \geq 1;=1}^{D;D^*}$ Shape	$\alpha_s/m_Q, 1/m_Q^2$	✓	✓	✓	—	—	✓	✓	✓
NoL	$\alpha_s/m_Q, 1/m_Q^2$	—	—	—	—	—	✓	✓	✓
$L_{w \geq 1; \geq 1}^{D;D^*}$	$\alpha_s/m_Q, 1/m_Q^2$	—	✓	✓	✓	✓	✓	✓	✓
$L_{w \geq 1;=1}^{D;D^*}$ NLO	$\alpha_s, 1/m_Q$	✓	✓	✓	—	—	✓	✓	✓
$L_{w \geq 1;=1}^{D;D^*}$ +SR	$\alpha_s/m_Q, 1/m_Q^2$	—	✓	✓	—	—	✓	✓	✓
$L_{w \geq 1;=1}^{D;D^*}$ 17	$\alpha_s/m_Q, 1/m_Q^2$	—	✓	✓	—	—	✓	✓	—
$L_{w \geq 1;=1}^{D;D^*}$ 19	$\alpha_s/m_Q, 1/m_Q^2$	—	✓	✓	—	—	✓	—	✓
$L_{w \geq 1; \geq 1 [h_{A_1}]}^{D;D^*}$	$\alpha_s/m_Q, 1/m_Q^2$	—	✓	✓	✓	—	✓	✓	✓

Form factors and ratios in various fit scenarios

S1



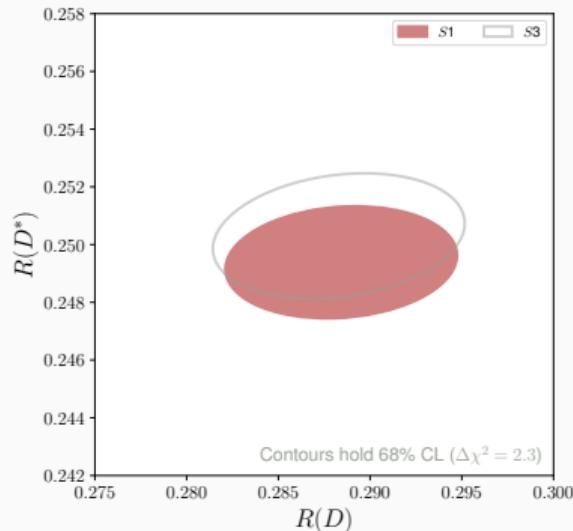
S3



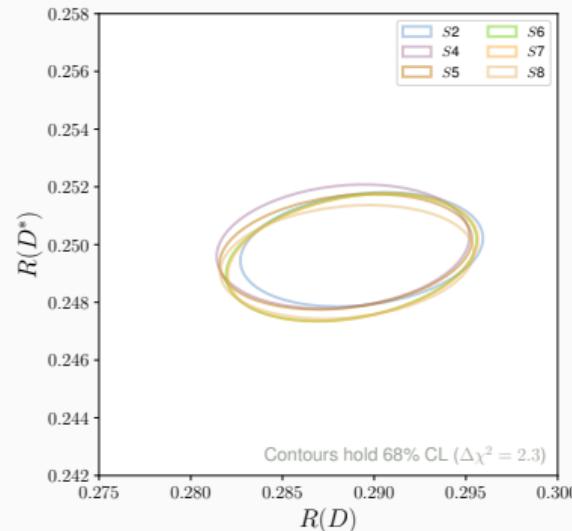
- The uncertainties in all form factor ratios are well controlled.

Predictions in various fit scenarios

S1 & S3



Other Scenarios



- Predictions for $R(D^{(*)})$ are stable for all terminating nodes / fit hypotheses.
- Truncation of model parameters does not introduce model dependence.

Input Data

Experimental Data

- $\bar{B} \rightarrow D\ell\nu$ tagged ('Belle 16')
- $\bar{B}^0 \rightarrow D^{*+}\ell\nu$ tagged ('Belle 17')
- $\bar{B}^0 \rightarrow D^{*+}\ell\nu$ untagged ('Belle 19')

QCDSR Input

- $\hat{\chi}_2(1) = -0.06 \pm 0.02$
- $\hat{\chi}'_2(1) = 0 \pm 0.02$
- $\hat{\chi}'_3(1) = 0.04 \pm 0.02$
- $\hat{\eta}(1) = 0.62 \pm 0.2$
- $\hat{\eta}'(1) = 0 \pm 0.2$

Baseline fit scenario: $L_{w \geq 1;=1}^{D;D^*}$

Lattice Input at zero-recoil

- $\bar{B} \rightarrow D\ell\nu$:

$$\mathcal{G}(1)_{\text{LQCD}} = 1.054(9)$$

- $\bar{B} \rightarrow D^*\ell\nu$

$$\mathcal{F}(1)_{\text{LQCD}} = 0.906(13) = h_{A_1}(1)$$

Lattice Input beyond zero-recoil

Form factor	$w = 1.0$	$w = 1.08$	$w = 1.16$
f_+	1.1994(95)	1.0941(104)	1.0047(123)
f_0	0.9026(72)	0.8609(77)	0.8254(94)
Form factor	$w = 1.03$	$w = 1.10$	$w = 1.17$
h_{A_1}	0.877(16)	0.807(15)	0.745(22)
h_{A_2}	-0.624(84)	-0.586(81)	-0.492(82)
h_{A_3}	-0.391(95)	1.259(78)	1.213(75)
h_V	1.103(74)	0.989(86)	1.270(46)