Beam Dynamics Effects in the Muon *g*–2 Experiment

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on behalf of the Muon g–2 Collaboration

UNIVERSITÀ DI PISA INFN g-2 Fermilab

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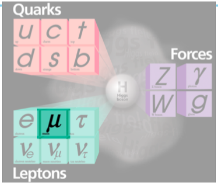
Bologna (Italy)

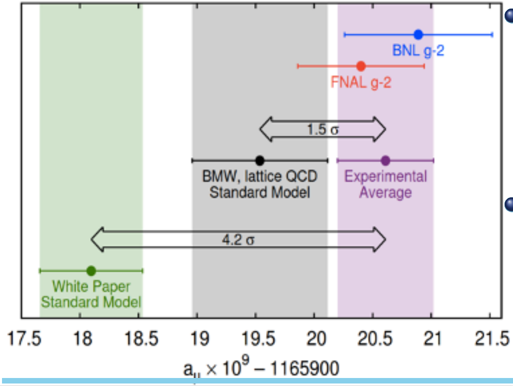
The muon anomaly

- **Muon:** elementary particle with spin-1/2 and magnetic moment proportional to spin through the **g-factor**:



$$\vec{\mu} = \mathbf{g}\frac{q}{2m_\mu}\vec{S} \quad \Rightarrow \quad \boxed{a_\mu = \frac{g-2}{2}} \quad \textbf{muon anomaly}$$

–> Comparison between theoretical value and exp. measurement allows for a **precise test of the Standard Model and to look for new physics**



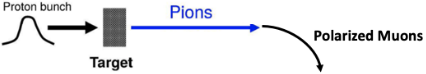
- New measurement from FNAL Muon $g-2$ Exp. Run-1 data in 2021 confirmed result from BNL:

$$\mathbf{a}_\mu(\mathbf{FNAL}) = 116592040(54)\cdot 10^{-11}\ (460\text{ppb})$$
$$\mathbf{a}_\mu(\mathbf{BNL}) = 116592089(63)\cdot 10^{-11}\ (540\text{ ppb})$$
$$\mathbf{a}_\mu(\mathbf{Exp}) = 116592061(41)\cdot 10^{-11}\ (350\text{ ppb})$$

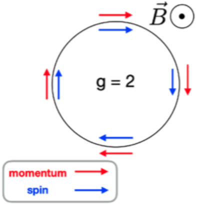
- Theory:
  - $\mathbf{a}_\mu$(**BMW, SM**) calculation with LatticeQCD
  - $\mathbf{a}_\mu$(**WP, SM**) calculation recommended by the Theory Initiative based on e^+e^- data

Experimental technique

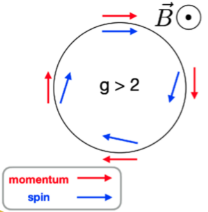
1. Inject polarized muons into a magnetic storage ring
2. Muons circulate around the ring at the cyclotron frequency:
$$\vec{\omega}_C = \frac{q}{\gamma m_\mu}\vec{B}$$
3. Muon spin precession frequency (Larmor) is given by:
$$\vec{\omega}_S = \frac{q}{\gamma m_\mu}\vec{B}(1+\gamma a_\mu)$$
4. Muon anomaly is proportional to **anomalous precession frequency** and $\vec{B}$:
$$\vec{\omega}_a \cong \vec{\omega}_S - \vec{\omega}_C \cong a_\mu \frac{q}{m_\mu}\vec{B}$$
5. Measure B and ω_a to extract the anomaly



If $g = 2 \Rightarrow \vec{\omega}_a = 0$



$g \neq 2 \Rightarrow \vec{\omega}_a \cong a_\mu \frac{e}{m}\vec{B}$



Final formula

Muon anomaly is determined with:

$$a_\mu = \underbrace{\boxed{\frac{\omega_a}{\tilde{\omega}'_p(T_r)}}}_{} \underbrace{\frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2}}_{}$$

ratio of frequencies (R_μ) measured by us

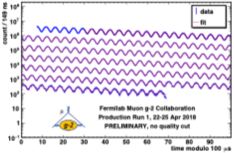
fundamental factors (combined uncertainty 25 ppb):

↓

ω_a: muon anomalous precession frequency

Extract from decay positron time spectra

$N(t) = N_0 e^{-t/\tau_\mu}[1 + A\cos(\omega_a t + \phi)]$



$\mu'_p(T_r)/\mu_e(H)$ from [Metrologia **13**, 179 (1977)]

$\mu_e(H)/\mu_e$ from [Rev. Mod. Phys. **88** 035009 (2016)]

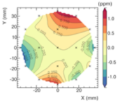
m_μ/m_e from [Phys. Rev. Lett. **82**, 711 (1999)]

$g_e/2$ from [Phys. Rev. A **83** 052122 (2011)]

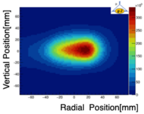
↓ Next Talk by R. Reinmann ↓

$\tilde{\omega}'_p(T_r)$: magnetic field B in terms of (shielded) proton precession frequency (proton NMR $\hbar\omega_P = 2\mu_p B$) **and** weighted by the muon distribution

(shielded = measured in spherical water sample at $T_r = 34.7\,^\circ$C)



Map the magnetic field



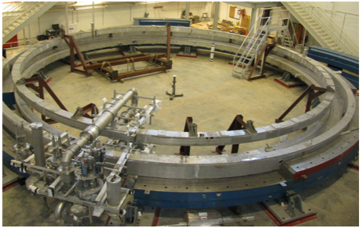
Muon distribution in the storage ring

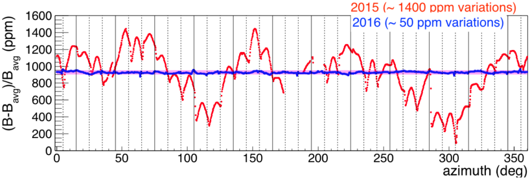
Production of the muon beam

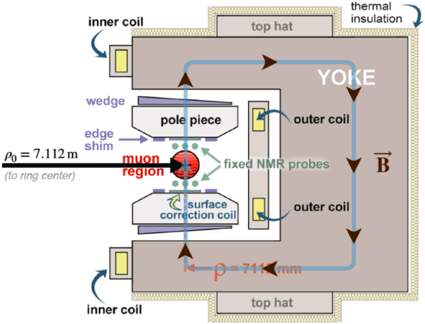
- **Recycler Ring:** 8 GeV protons from Booster are divided in 4 bunches
- **Target Station:** p-bunches are collided with target and π^+ with 3.1 GeV/c ($\pm10\%$) are collected
- **Beam Transport and Delivery Ring:** magnetic optics select μ^+ from $\pi^+ \to \mu^+\nu_\mu$ then μ^+ are separated from p and π^+ in circular ring
- **Muon Campus:** polarized μ^+ are ready to be injected into the **storage ring**

Storage ring magnet

- Three superconducting coils provide 1.45 T vertical magnetic field
- Vacuum chambers surrounded by a cryosystem and C-shaped **yokes** to allow the decay positrons to reach the detectors.
- Achieved 50 ppm on field uniformity thanks to low-carbon steel **poles**, **edge shims**, **steel wedges**, **surface correction coil**



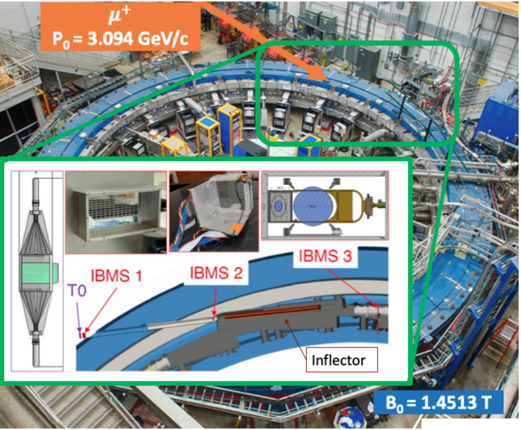




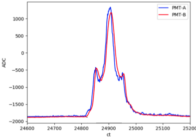
final field ~ 3 times more uniform than at BNL

Injection of the muons into the ring

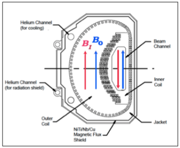
- Beam enters the ring through a 2.2 m-long 10 cm hole in the iron yoke

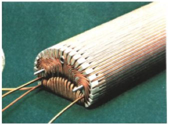


- **T0 Counter** (thin scintillator read out by PMTs) to measure **beam time profile**

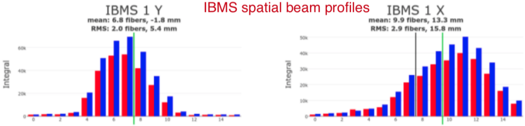


- **Inflector magnet** provides nearly field free region for muons to enter the storage region



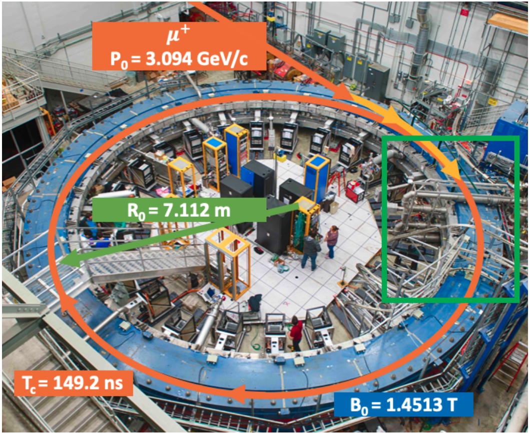


- **Inflector Beam Monitoring System** (scintillator fiber grids) to measure **beam spatial profile**



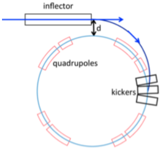
Muon storage

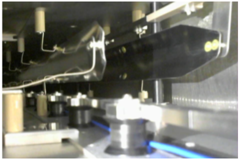
- Injected beam is 77 mm off from storage region center

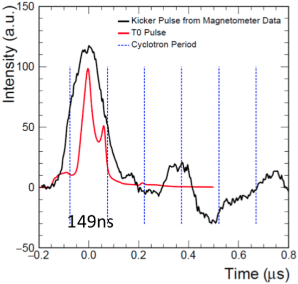


Kicker Magnets

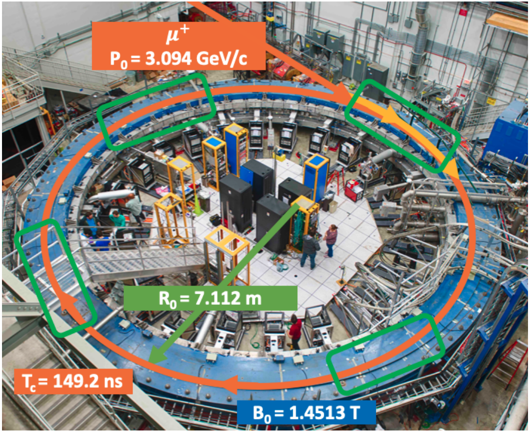
- 3 pulsed magnets deflect beam ~10 mrad onto the closed storage orbit in less than 150 ns





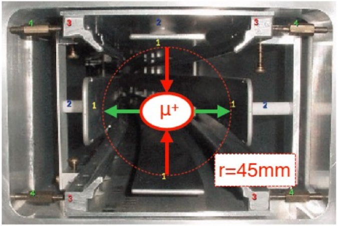


Vertical focusing



Electrostatic Quadrupoles

- 4 sets of quads provide vertical beam focusing

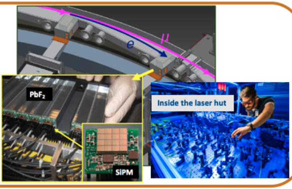


- $E-$field component cancels out (at first order) when muons at *magic momentum* :

$$\vec{\omega}_a \cong -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \underbrace{\frac{1}{\gamma^2 - 1}}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

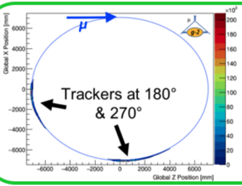
~0 if γ=29.3 *i.e.*, p_μ=3.094 GeV/c

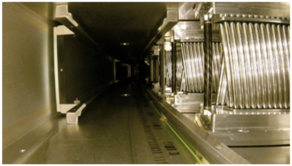
The ring is equipped with detectors and field probes



24 Calos around the ring

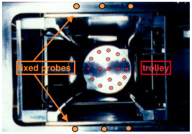
- Each made of 6×9 PbF_2 crystals read out by large-area SiPMs
- 1296 channels individually calibrated by 405nm-laser system

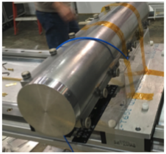




2 in-vacuum straw trackers

- Each with 8 modules consisting of 128 gas filled straws





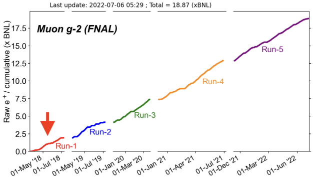
2 types of field probes

- 378 fixed NMR probes above and below storage region
  - → measure B-field 24/7
- Trolley with 17-probe NMR
  - → 2D profile of B over the entire azimuth when beam is OFF

First production Run

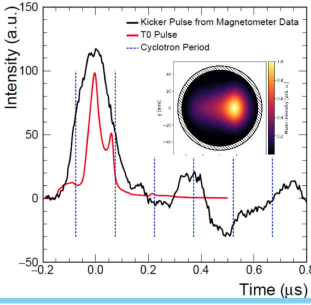
Statistics:

- March 26 – July 7 2018 : **Run-1**
- **1.2** $\times$ **BNL** after data quality selection

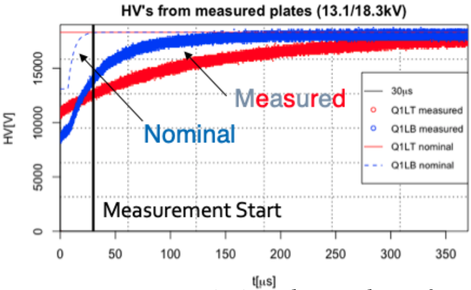


Main challenges for Run-1:

- Non-ideal kick
  - $\longrightarrow$ low amplitude and ringing
  - $\longrightarrow$ beam not centered in storage region



- some HV Quad resistors were damaged
  - $\longrightarrow$ slow recovery time



- Temperature variations larger than 1°C

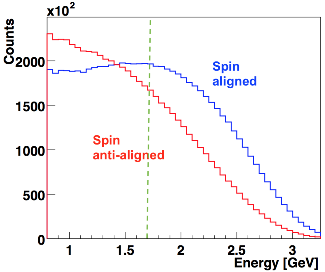
Measuring ω_a

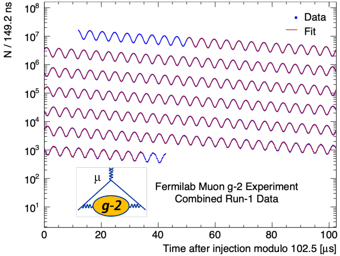
- Polarized muon decay:

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$$

- High energy e^+ are preferentially emitted in direction of μ^+ spin (parity violation of the weak decay)
- Energy spectrum modulates at the ω_a frequency
- Counting the number of e^+ with $E_{e^+} > E_{\text{threshold}}$ as a function of time (wiggle plot) leads to $\underline{\omega_a}$:

$$N(t) = \underbrace{N_0}_{\text{normalization}} e^{\overbrace{-t/\tau}^{\text{muon lab-frame lifetime}}}[1 + \underbrace{A}_{\text{g-2 asymmetry}} \cos(\omega_a t + \overbrace{\varphi}^{\text{g-2 phase}})]$$

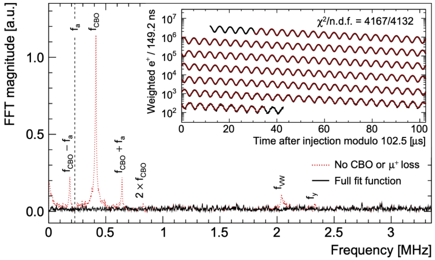




E_{e^+} and t are measured by the calorimeters with a blinding factor applied to the digitization rate

Fitting procedure

- FFT analyses of fit residuals shows that simple 5-parameter model is inadequate
- Fit result improves using a 22-parameter fit function that includes:
  - beam systematics effects like vertical and radial beam oscillations and lost muons



$$\boxed{N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A \cdot A_{BO}(t)\cos(\omega_a t + \phi \cdot \phi_{BO}(t))) \cdot N_{\mathrm{CBO}}(t) \cdot N_{\mathrm{VW}}(t) \cdot N_y(t) \cdot N_{2\mathrm{CBO}}(t) \cdot J(t)}$$

$$A_{\mathrm{BO}}(t) = 1 + A_A\cos(\omega_{\mathrm{CBO}}(t) + \phi_A)e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$\phi_{\mathrm{BO}}(t) = 1 + A_\phi\cos(\omega_{\mathrm{CBO}}(t) + \phi_\phi)e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$\boldsymbol{\omega_{CBO}}$, $\boldsymbol{\omega_{2CBO}}$ radial oscillations

$$N_{\mathrm{CBO}}(t) = 1 + A_{\mathrm{CBO}}\cos(\omega_{\mathrm{CBO}}(t) + \phi_{\mathrm{CBO}})e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$N_{2\mathrm{CBO}}(t) = 1 + A_{2\mathrm{CBO}}\cos(2\omega_{\mathrm{CBO}}(t) + \phi_{2\mathrm{CBO}})e^{-\frac{t}{2\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{VW}}(t) = 1 + A_{\mathrm{VW}}\cos(\omega_{\mathrm{VW}}(t)t + \phi_{\mathrm{VW}})e^{-\frac{t}{\tau_{\mathrm{VW}}}}$$

$\boldsymbol{\omega_y}$, $\boldsymbol{\omega_{VW}}$ vertical oscillations

$$N_y(t) = 1 + A_y\cos(\omega_y(t)t + \phi_y)e^{-\frac{t}{\tau_y}}$$

Red = free parameters
Blue= fixed parameters

$$J(t) = 1 - k_{LM}\int_{t_0}^{t}\Lambda(t)dt$$

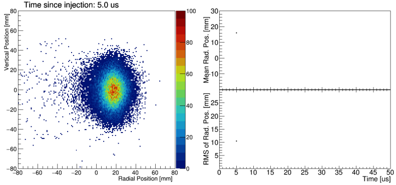
Lost muons

$$\omega_{\mathrm{CBO}}(t) = \omega_0 t + Ae^{-\frac{t}{\tau_A}} + Be^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F\omega_{\mathrm{CBO}(t)}\sqrt{2\omega_c/F\omega_{\mathrm{CBO}}(t) - 1}$$

$$\omega_{\mathrm{VW}}(t) = \omega_c - 2\omega_y(t)$$

The muon beam oscillates and breathes:



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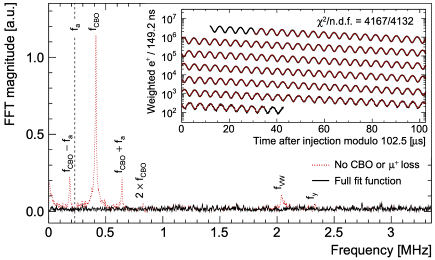

$$\boxed{N_0\, e^{-\frac{t}{\gamma\tau}}\,(1 + A \cdot A_{BO}(t)\cos(\omega_a t + \phi \cdot \phi_{BO}(t))) \cdot N_{\mathrm{CBO}}(t) \cdot N_{\mathrm{VW}}(t) \cdot N_y(t) \cdot N_{2\mathrm{CBO}}(t) \cdot J(t)}$$

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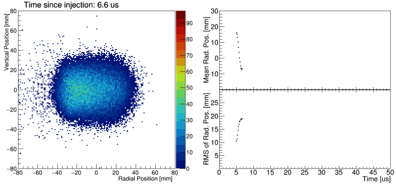

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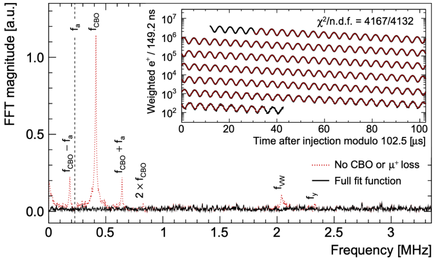

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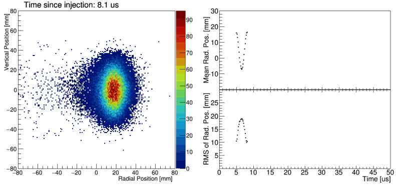

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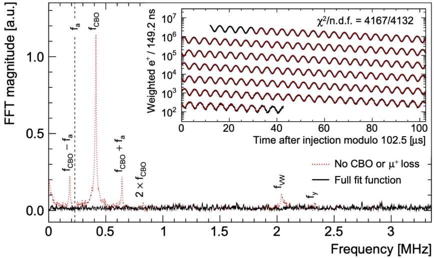
- Fit result improves using a 22-parameter fit function that includes:
  - beam systematics effects like vertical and radial beam oscillations and lost muons



$$\boxed{N_0 e^{-\frac{t}{\gamma\tau}} \left(1 + A \cdot A_{BO}(t) \cos(\omega_a t + \phi \cdot \phi_{BO}(t))\right) \cdot N_{\mathrm{CBO}}(t) \cdot N_{\mathrm{VW}}(t) \cdot N_y(t) \cdot N_{\mathrm{2CBO}}(t) \cdot J(t)}$$

$$A_{\mathrm{BO}}(t) = 1 + A_A \cos(\omega_{\mathrm{CBO}}(t) + \phi_A) e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$\phi_{\mathrm{BO}}(t) = 1 + A_\phi \cos(\omega_{\mathrm{CBO}}(t) + \phi_\phi) e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

ω_{CBO}, ω_{2CBO} radial oscillations

$$N_{\mathrm{CBO}}(t) = 1 + A_{\mathrm{CBO}} \cos(\omega_{\mathrm{CBO}}(t) + \phi_{\mathrm{CBO}}) e^{-\frac{t}{\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{2CBO}}(t) = 1 + A_{\mathrm{2CBO}} \cos(2\omega_{\mathrm{CBO}}(t) + \phi_{\mathrm{2CBO}}) e^{-\frac{t}{2\tau_{\mathrm{CBO}}}}$$

$$N_{\mathrm{VW}}(t) = 1 + A_{\mathrm{VW}} \cos(\omega_{\mathrm{VW}}(t) t + \phi_{\mathrm{VW}}) e^{-\frac{t}{\tau_{\mathrm{VW}}}}$$

$$N_y(t) = 1 + A_y \cos(\omega_y(t) t + \phi_y) e^{-\frac{t}{\tau_y}}$$

ω_y, ω_{VW} vertical oscillations

Red = free parameters
Blue= fixed parameters

$$J(t) = 1 - k_{LM} \int_{t_0}^{t} \Lambda(t) dt$$

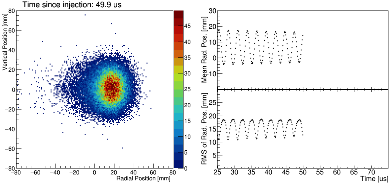
Lost muons

$$\omega_{\mathrm{CBO}}(t) = \omega_0 t + A e^{-\frac{t}{\tau_A}} + B e^{-\frac{t}{\tau_B}}$$

$$\omega_y(t) = F \omega_{\mathrm{CBO}(t)} \sqrt{2\omega_c / F \omega_{\mathrm{CBO}}(t) - 1}$$

$$\omega_{\mathrm{VW}}(t) = \omega_c - 2\omega_y(t)$$

The muon beam oscillates and breathes:



Electric Field
Pitch
Muon Loss
Phase Acceptance

+ beam dynamics corrections : $\omega_a = \omega_a^{meas} \cdot (1 + \boxed{C_e} + \boxed{C_p} + \boxed{C_{ml}} + \boxed{C_{pa}})$

Electric field and pitch corrections

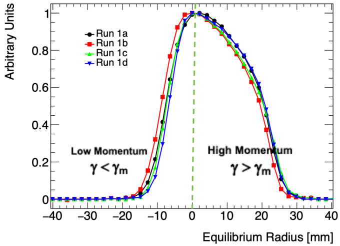
Electric Field

- due to momentum spread around p_{magic}

$$\vec{\omega}_a \cong -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

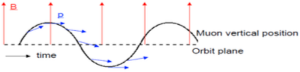
- measured using momentum distribution provided by the calorimeters in terms of equilibrium radius

$$\boxed{C_e \sim 450 \text{ ppb} \quad \delta_{C_e} \sim 50 \text{ ppb}}$$



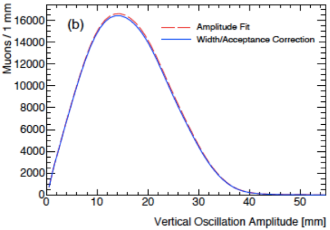
Pitch

- due to vertical beam oscillation



- measured using the beam vertical amplitude from the trackers, calorimeter data, and simulations

$$\boxed{C_p \sim 200 \text{ ppb} \quad \delta_{C_p} \sim 20 \text{ ppb}}$$

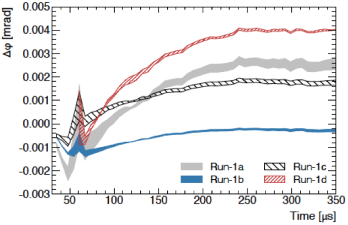


Muon loss and phase acceptance corrections

Muon losses cause a phase shift

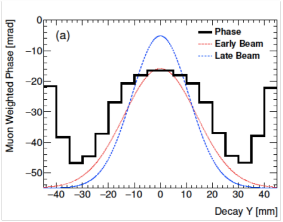
- because muon-spin-phase and muon loss rate are momentum-dependent
- measured using data-driven technique

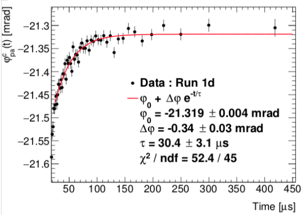
$C_{ml} < 20$ ppb $\delta_{C_{ml}} \sim 5$ ppb



Phase acceptance

- phase changes due to early to late variations of the beam
- worsened by damaged quads resistors (fixed after Run-1)
- measured using tracker data and simulations

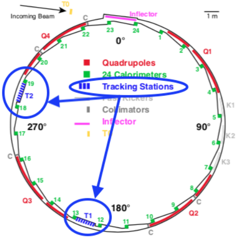


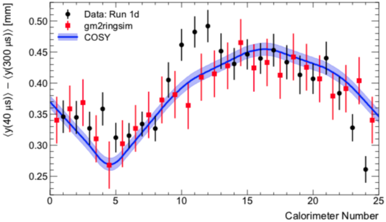


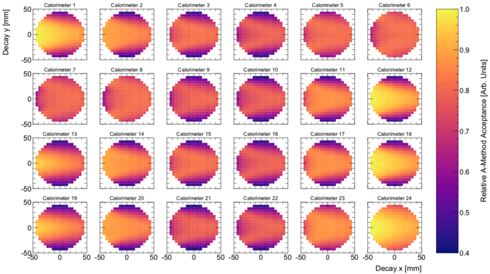
$C_{pa} \sim 200$ ppb $\delta_{C_{pa}} \sim 80$ ppb

Simulations for phase-acceptance

- Time-dependence of beam spatial distributions are measured by trackers in two locations
- Two independent **simulations** are used to extrapolate beam profile from tracker locations around the ring
  - based on **COSY-INFINITY** and **GEANT-4**
  - cross-checked against data
- The beam profiles in the ring are then folded with calorimeter acceptance maps produced with the **GEANT-4** based simulation







Summary and Conclusions

- FNAL $g-2$ Experiment measured a_μ with unprecedented precision analyzing Run-1 data and confirmed the previous experimental measurement performed at BNL
- Understanding and and correcting for beam dynamics effects was fundamental
- More to come: analysis of Run-2&3 ongoing, collected 5.5× BNL stat. in Run-4 and finishing collecting Run-5 data

Summary of Run1 results

Quantity	Correction Terms (ppb)	Uncertainty (ppb)
ω_a (statistical) Dominated by stat. unc.	–	434
ω_a (systematic)	–	56
C_e	489	53
C_p greatly reduced after Run-1	180	13
C_{ml}	-11	5
C_{pa} ω_a syst. dominated by PA	-158	75
$f_{\mathrm{calib}} \langle \omega'_p(x,y,\phi) \times M(x,y,\phi) \rangle$	–	56
B_k	-27	37
B_q	-17	92
$\mu'_p(34.7^\circ)/\mu_e$	–	10
m_μ/m_e	–	22
$g_e/2$	–	0
Total systematic	–	157
Total fundamental factors	–	25
Totals	544	462

More details in the papers!