

Shapes and sizes of diquarks in lattice QCD

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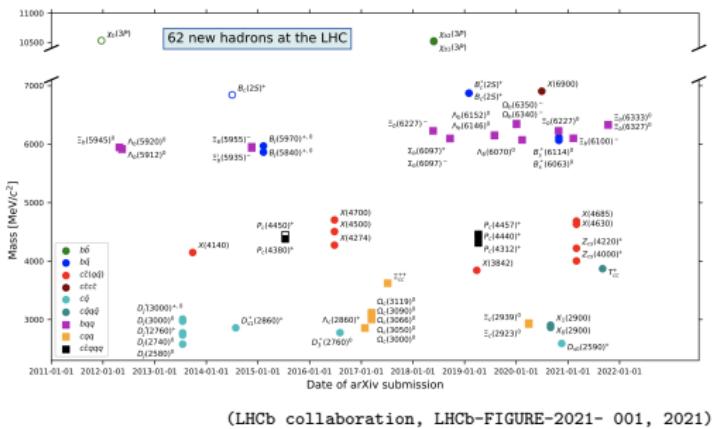
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based on JHEP 05 (2022) 062 [arXiv:2106.09080]

Many new and exotic hadrons observed, e.g.
at the LHC



... many not explained in theory

QCD often approximated in models
~~ many interpretations

model	building blocks
"plain"	$q_{(i,c)}, \bar{q}_{(i,c)}$
diquark	$[qq]_{(i,j,c)} & q/\bar{q}$
triquark	$[qq\bar{q}]_{(i,j,k,c)} & q/\bar{q}$
hydro-onium	$[Q\bar{Q}]_{(i,j)}, [q\bar{q}]_{(i,j)}, [qqq]_{(i,j,k)}$
molecular	$[Q\bar{q}]_{(i,j)}, [q\bar{Q}]_{(i,j)}, [qqQ]_{(i,j,k)}, \dots$

Goal: Add non-perturbative insights into exotic hadrons in full QCD to the discussion

Diquarks - an attractive concept

"The concept of diquarks is almost as old as the quark model, and actually predates QCD [1]"

↔ arXiv:2203.16583; [1] PR 155, 1601 (1967)

- Successful for low-lying baryons and exotic hadrons.
 - Well founded in QCD with many predictions.
 - But, experimental evidence has been elusive.
- Light diquarks:
 - special "good" ($\bar{3}_F, \bar{3}_c, J^P = 0^+$) configuration
 - "good" diquarks experience attraction effect
 - large mass splitting in good, bad and not-even-bad
 - non-vanishing size or compact?
- HQSS-limit: A diquark acts as an antiquark $[QQ] \leftrightarrow \bar{Q}$.
↔ currently one motivation for T_{QQ} -type hadrons

3 types of diquark:
good, bad and not-even bad

Diquark operator:

$$D_\Gamma = q^c C \Gamma q'$$

↔ c, C = charge conjugation

↔ Γ acts on Dirac space

J^P	C	F	Op: Γ
0^+	$\bar{3}$	$\bar{3}$	$\gamma_5, \gamma_0 \gamma_5$
1^+	$\bar{3}$	6	γ_i, σ_{i0}
0^-	$\bar{3}$	6	$\mathbb{1}, \gamma_0$
1^-	$\bar{3}$	$\bar{3}$	$\gamma_i \gamma_5, \sigma_{ij}$

Towards a clearer understanding and footing in QCD using lattice calcs

1. **spectrum:** [diquark] mass differences are fundamental characteristics of QCD
(Jaffe '05, arXiv:hep-ph/0409065)
2. **spatial correlations:** study attraction and special status of the "good" diquark
3. **structure:** estimate size and shape of the "good" diquark

A gauge invariant probe - static quark as spectator

- A problem for the lattice is that diquarks are colored, i.e. not-gauge invariant.
 - Could fix a gauge, but then properties are gauge-dependent (masses, sizes,...)

↔ lattice and Dyson-Schwinger, see e.g. [15-20] in 2106.09080

- **Alternative:** Static spectator quark Q ($m_Q \rightarrow \infty$) cancels in mass differences.
 - Diquark properties exposed in a gauge-invariant way.

↔ hep-lat/0510082, hep-lat/0509113, hep-lat/0609004, arxiv:1012.2353

$$C_\Gamma(t) \sim \exp \left[-t \left(m_{D_\Gamma} + m_Q + \mathcal{O}(m_Q^{-1}) \right) \right]$$

↔ $t \rightarrow \text{large}$, $m_Q \rightarrow \text{large}$

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

$$C_\Gamma(t) = \sum_{\vec{x}} \langle [D_\Gamma Q](\vec{x}, t) [D_\Gamma Q]^\dagger(\vec{0}, 0) \rangle$$

↔ static quark= Q and $D_\Gamma = q^c C \Gamma q$

↔ flavor combinations ud , ℓs , ss'

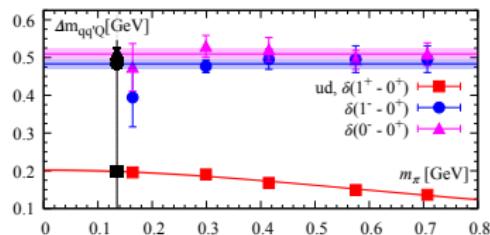
↔ static-light mesons $[\bar{Q} \Gamma q]$

↔ Lattice setup details in the extra info slides

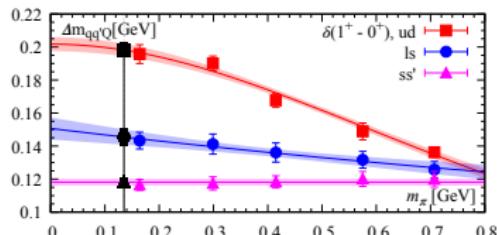
I. Diquark spectroscopy

Lattice spectroscopy - diquark-diquark differences

$ud\ 0^+$ versus $1^+, 0^-$ and 1^-



$(1^+ - 0^+)_{qq'}$ splitting



We consider differences of $qq'Q$ baryons:

$$C_\Gamma^{qq'Q}(t) - C_{\gamma_5}^{qq'Q}(t)$$

$\leadsto Q$ drops out

\leadsto measures diquark-diquark mass difference

Bad-good diquark splitting:

- Special status of good diquark observed
- Good 0^+ ud diquark lies lowest in the spectrum
- Bad 1^+ ud diquark 100-200 MeV above
- 0^- and 1^- ud diquarks ~ 0.5 GeV above
- Pattern repeated in ls and ss'

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral limit: $\sim \text{const}$
- Heavy-quark limit: decreases $\sim 1/(m_{q_1} m_{q_2})$, with $m_\pi \sim (m_{q_1} + m_{q_2})$

$$\delta(1^+ - 0^+)_{q_1 q_2} = A / \left[1 + (m_\pi / B)^{n \in 0, 1, 2} \right]$$

\leadsto Throughout continuum limit pending but effects expected to be small. Focus on coverage in $m_\pi \in [164, 909]\text{MeV}$.

Lattice spectroscopy - diquark-quark differences

We also consider differences of a $qq'Q$ baryon and a light-static meson:

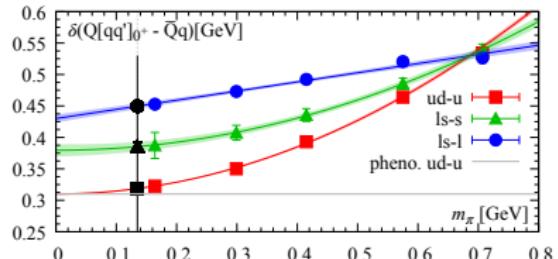
$$C_{\Gamma=\gamma_5}^{qq'Q}(t) - C_{\gamma_5}^{q'\bar{Q}}(t)$$

$\rightsquigarrow Q$ drops out
 \rightsquigarrow diquark-quark mass difference

$\Delta m_{qq'Q}(m_\pi)$ dependence:

- Chiral vs. heavy-quark limiting behaviours, as before

$Qqq' - \bar{Q}q'$ splittings



$$\delta(Q[q_1 q_2]_{0+} - \bar{Q}q_2) = C [1 + (m_\pi/D)^{n \in 0,1,2}]$$

Diquark-quark splitting:

- Established relative masses between a good diquark and an [anti]quark
- May prove useful in identifying favourable tetra-, pentaquark channels
- Omits possible distortions through additional light quarks, Pauli-blocking, spin-spin interactions ...

Diquark spectroscopy - comparing results

- We want to compare our results with phenomenology
 - more details in extra info slides
 - Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
 - For pheno estimates use charm and bottom hadron masses where leading $\mathcal{O}(1/m_Q)$ ($Q = c, b$) can be cancelled
- The main spectroscopy results are summarised as:

All in [MeV]	$\delta E_{\text{lat}}(m_\pi^{\text{phys}})$	δE_{pheno}	$\delta E_{\text{bottom}}^{\text{pheno}}$	$\delta E_{\text{charm}}^{\text{pheno}}$
$\delta(1^+ - 0^+)_{ud}$	198(4)	206(4)	206	210
$\delta(1^+ - 0^+)_{\ell s}$	145(5)	145(3)	145	148
$\delta(1^+ - 0^+)_{ss'}$	118(2)			
$\delta(Q[ud]_{0^+} - \bar{Q}u)$	319(1)	306(7)	306	313
$\delta(Q[\ell s]_{0^+} - \bar{Q}s)$	385(9)	397(1)	397	398
$\delta(Q[\ell s]_{0^+} - \bar{Q}\ell)$	450(6)			

◦ use the bottom estimate for static
◦ use charm-bottom difference as estimate for deviation from static
⇒ $\lesssim \mathcal{O}(7)\text{MeV}$ deviation

- Overall, very good agreement observed.

II. Diquark structure

Diquarks - spatial correlations

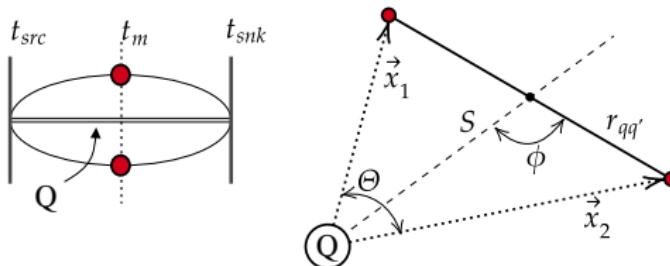
We access (good) diquark structure information through density-density correlations:

$$C_{\Gamma}^{dd}(\vec{x}_1, \vec{x}_2, t) = \left\langle \mathcal{O}_{\Gamma}(\vec{0}, 2t) \rho(\vec{x}_1, t) \rho(\vec{x}_2, t) \mathcal{O}_{\Gamma}^{\dagger}(\vec{0}, 0) \right\rangle$$

$\rightsquigarrow \mathcal{O}_{\Gamma} = q^c C \Gamma q$ and $\rho(\vec{x}, t) = \bar{q}(\vec{x}, t) \gamma_0 q(\vec{x}, t)$

$\rightsquigarrow t_m = (t_{snk} + t_{src})/2$ to minimize excited states

Main tool: Correlations between two light quarks' relative positions to the static quark



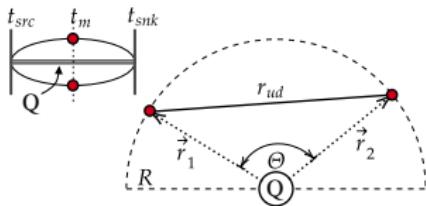
$\rightsquigarrow \vec{r}_{ud} = \vec{x}_2 - \vec{x}_1$ and $\vec{S} = (\vec{x}_1 - \vec{x}_2)/2$

$$\rho_2(r_{ud}, S, \phi; \Gamma) = C_{\Gamma}^{dd}(\vec{x}_1, \vec{x}_2, t_m)$$

Note, when S and r_{ud} fixed, distance between static quark Q and light quarks q, q' is

- o Minimized for $\phi = \pi$, possible disruption due to Q is largest
- o Maximized for $\phi = \pi/2$, possible disruption due to Q is smallest

Good diquark attraction



Setting $\phi = \pi/2$:

- $|\vec{x}_1| = |\vec{x}_2| = R$, use R, Θ :

$$\rho_2^\perp(R, \Theta) = \rho_2(r_{ud}, S, \pi/2)$$
- Attraction visible through increase in ρ_2^\perp for small Θ at any fixed R

Two limiting cases for the two quarks:

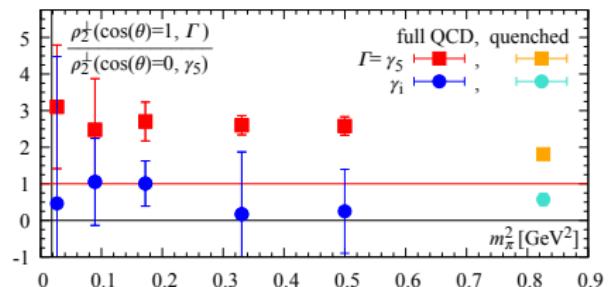
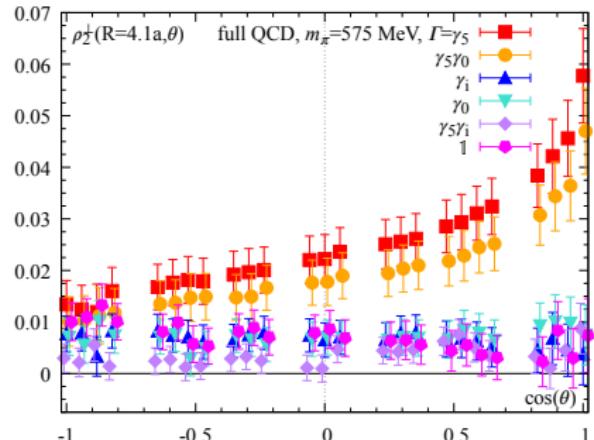
- $\cos(\Theta) = 1$ on top of each other
- $\cos(\Theta) = -1$ opposite each other

"Lift" as qualitative criterion:

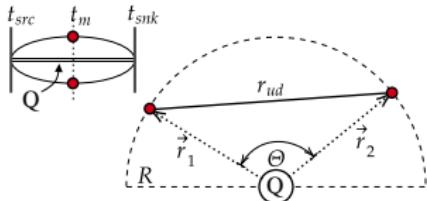
$$\frac{\rho_2^\perp(R, \Theta = 0, \Gamma)}{\rho_2^\perp(R, \Theta = \pi/2, \gamma_5)}$$

Increase observed in good diquark only

Spatial correlation over Θ



Good diquark size



- Distance between quarks:

$$r_{ud} = R \sqrt{2(1 - \cos(\Theta))}$$

\rightsquigarrow different visualisation

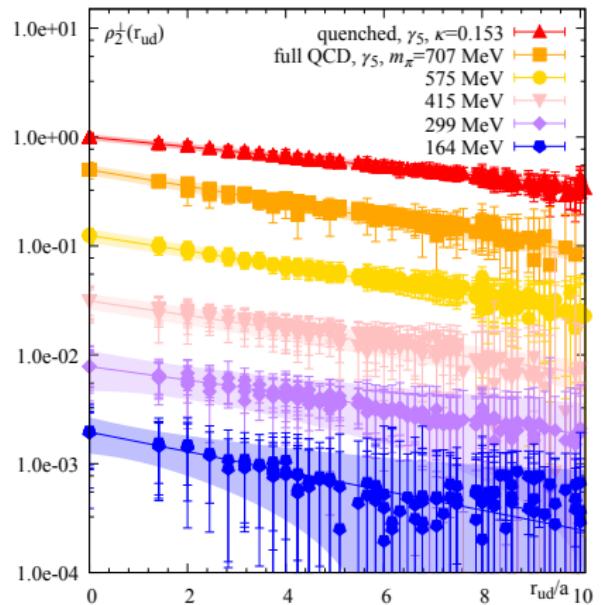
- $\rho_2^\perp(R, r_{ud}) \sim \exp(-r_{ud}/r_0)$
 \rightsquigarrow "characteristic size" r_0
- Need to control:

- interference from Q
 \rightsquigarrow we limit analysis to $r_{ud} < R$
- periodicity effects
 \rightsquigarrow in practice we find $L = 5r_0$

- Further checks:
 $A(R, r_{ud} = 0) \sim \exp(-R/R_0)$

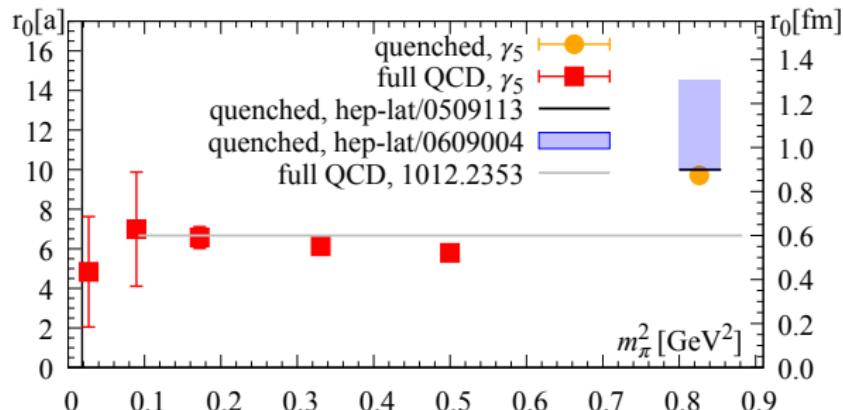
Data well described by (single)
exponential Ansatz

Spatial correlation over r_{ud}



- $r_{ud} = 0$ normalised, offset for each m_π
- all R shown simultaneously
- combined fits over $\forall R$ with shared r_0

Size dependence $r_0(m_\pi)$



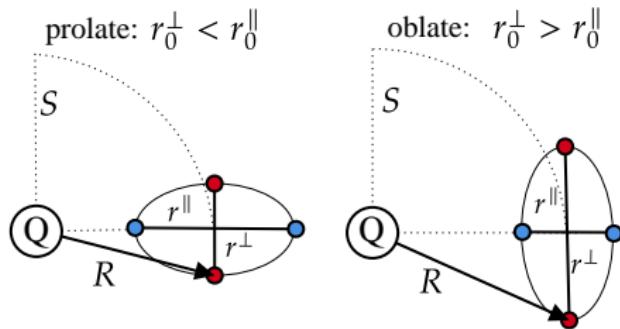
Good diquark size:

- Agreement w/ prev. quenched and dynamical
- Refinement through our results
- $r_0 \simeq \mathcal{O}(0.6)$ fm weak m_π dependence
 $\rightsquigarrow \sim r_{\text{meson, baryon}}$, arXiv:1604.02891

$r_0(m_\pi)$ dependence:

- $m_{q,q'} \uparrow$ should produce more compact object
- But, diquark attraction \downarrow works opposite
- Former effect dominates at large m_π ?
- But, in quenched diquarks definitely larger...

Shape of good diquarks - studying wavefunction "oblateness"



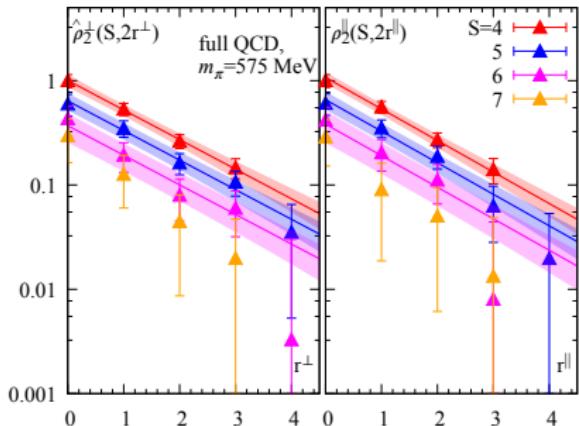
Tangential and radial spatial correlation decay

As opposed to before $R \neq \text{fixed}$:

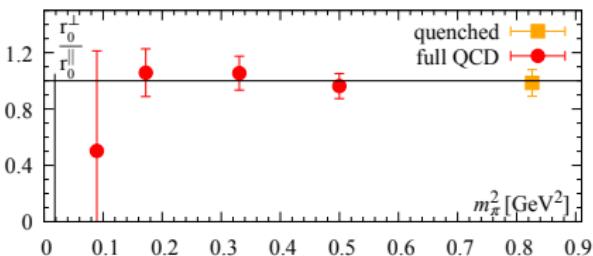
- $\phi = \pi$: radial correlation,
size $\sim r_0^{\parallel}$
- $\phi = \pi/2$: tangential correlation,
size $\sim r_0^{\perp}$
- $r_0^{\perp} / r_0^{\parallel}$ gives information on shape:
 $= 1$, spherical
 $\neq 1$, prolate/oblate

- Probe $J = 0$ nature of good diquark (spherical, S -wave expectation)
- Diquark polarisation through static quark?

Oblateness results at $m_\pi = 575\text{MeV}$



Shape dependence $r_0^\perp/r_0^\parallel(m_\pi)$



- Goal:

- r_0^\perp, r_0^\parallel at fixed S

Technical issue:

- (\parallel) as before:
 $R = S$
- (\perp) different: $R = \sqrt{(r^\perp)^2 + S^2}$

Solution:

- Introduce "nuisance" parameter R_0
- Adjusted in figure
- Parallel lines $\leadsto r_0^\perp = r_0^\parallel$

- $r_0^\perp/r_0^\parallel(m_\pi)$ dependence:

- Ratio $\simeq 1$ for all m_π
- Consistent w/ scalar, $J = 0$, shape
- No diquark polarisation through Q observed

Summary

Gauge invariant approach to diquarks in $n_f = 2 + 1$ lattice QCD

- Lattice setup with short chiral extrapolations, continuum limit still required

Diquark spectroscopy

- Special status of "good" diquark confirmed, attraction of 198(4)MeV over "bad"
- Chiral and flavor dependence modelled through simple Ansatz
- Very good agreement with phenomenological estimates

Diquark structure

- $q - q$ attraction in good diquark induces compact spatial correlation
- Good diquark size $r_0 \simeq \mathcal{O}(0.6)\text{fm} \sim r_{\text{meson, baryon}}$, weakly m_π dependent
- Good diquark shape appears nearly spherical

Outlook

- Results provide clear, quantitative support for the good diquark picture
- Hope to refine diquark model parameters
- Insights for studies of exotic tetraquarks (esp. doubly heavy), heavy-baryons, etc.
- Refinement towards diquarks in light baryons? Tetraquark diquark content? ...

Thank you for your attention.



Extra info

A gauge invariant probe - lattice calculation details

- **Lattice correlator:** Diquark embedded in a static-light-light baryon

$$C_\Gamma(t) = \sum_{\vec{x}} \left\langle [D_\Gamma Q](\vec{x}, t) [D_\Gamma Q]^\dagger(\vec{0}, 0) \right\rangle$$

~> static quark=Q and $D_\Gamma = q^c C \Gamma q$
~> flavor combinations ud , ℓs , ss'
~> static-light mesons $[\bar{Q} \Gamma q]$

setting up on the lattice - we recycle

- $n_f = 2 + 1$ full QCD, $32^3 \times 64$, $a = 0.090\text{fm}$, $a^{-1} = 2.194\text{GeV}$ (PACS-CS gauges)
- $m_\pi = 164, 299, 415, 575, 707\text{ MeV}$, $m_s \simeq m_s^{\text{phys}}$, propagators re-used from before
- Quenched gauge $a \simeq 0.1\text{fm}$, $m_\pi^{\text{valence}} = 909\text{ MeV}$, to match [hep-lat/0509113](#)

Diquark spectroscopy - phenomenological estimates

We want to compare our results with phenomenology

- o Key resource: (Jaffe '05, arXiv:hep-ph/0409065), updated with PDG 2021 input
- o For pheno estimates use charm and bottom hadron masses where leading $\mathcal{O}(1/m_Q)$ ($Q = c, b$) can be cancelled

Four estimates considered:

- o $\delta(1^+ - 0^+)_{ud} : \boxed{\frac{1}{3} (2M(\Sigma_Q^*) + M(\Sigma_Q)) - M(\Lambda_Q)}$

- o $\delta(1^+ - 0^+)_{us} : \boxed{\frac{2}{3} (M(\Xi_Q^*) + M(\Sigma_Q) + M(\Omega_Q)) - M(\Xi_Q) - M(\Xi'_Q)}$

- o $\delta(Q[ud]_{0^+} - \bar{Q}u) : \boxed{M(\Lambda_Q) - \frac{1}{4} (M(P_{Qu}) + 3M(V_{Qu}))}$

$\rightsquigarrow P_{Qu}, V_{Qu}$ are the ground-state, heavy-light mesons

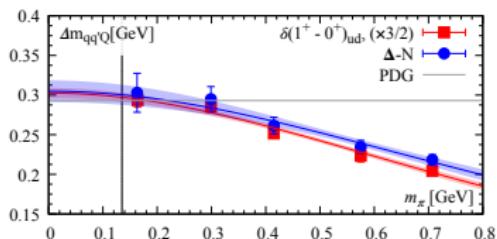
- o $\delta(Q[us]_{0^+} - \bar{Q}s) :$

- $$\boxed{M(\Xi_Q) + M(\Xi'_Q) - \frac{1}{2}(M(\Sigma_Q) + M(\Omega_Q)) - \frac{1}{4}(M(P_{Qs}) + 3M(V_{Qs}))}$$

$\rightsquigarrow P_{Qs}, V_{Qs}$ are the ground-state, heavy-strange mesons

Δ -Nucleon mass difference

$[\Delta - N](m_\pi)$



Measured the mass difference of $\Delta - N$

- Prediction: $\delta(\Delta - N) = 3/2 \times \delta(1^+ - 0^+)_{ud}$
- Same Ansatz as before
- Prediction holds well, even at fairly large m_π