Short and intermediate distance HVP contributions to muon g-2: SM (lattice) prediction versus e+e- annihilation data

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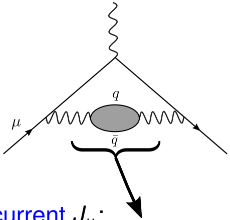
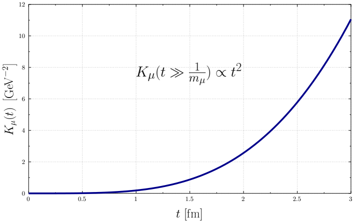
Talk based on ETM Collaboration, C. Alexandrou et al. arXiv:2206.15084 (June 30)





e.m. quark current J_μ:

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left(\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right) \Pi(Q^2)$$

$$\Downarrow$$

$$a_\ell^{\rm HVP} = 4\alpha_{em}^2 \int_0^\infty dQ^2\, \frac{1}{m_\ell^2}\, f\left(\frac{Q^2}{m_\ell^2}\right) \cdot \left(\Pi(Q^2) - \Pi(0)\right)$$

(Blum, 2002)

Time-Momentum representation (Bernecker & Meyer, 2011)

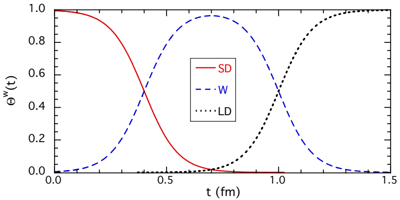
$$a_\ell^{\rm HVP} = 2\alpha_{em}^2 \int_0^\infty dt\ [2K_\ell(t)]\ V(t), \qquad V(t) \equiv -\frac{1}{3} \sum_{i=1,2,3} \int d\vec{x}\, \langle J_i(\vec{x}, t) J_i(0) \rangle$$

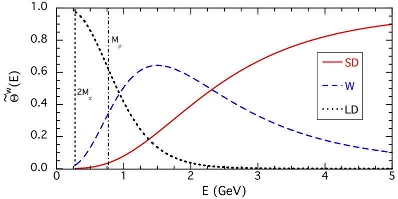
$a_\mu^{\rm HVP}$ window observables $\to$ probing $R^{\rm had}(E)$

RBC/UKQCD window decomposition: $a_\mu^{\rm HVP} \equiv a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$,

i.e. observables defined using Euclidean-time modulating functions $\Theta^{\rm SD,W,LD}(t)$ s.t.

$$a_\mu^w = 2\alpha_{em}^2 \int_0^\infty dt \left[t^2 K(m_\mu t)\right] \underline{\Theta^w(t)}\; V(t) \qquad w = \{SD, W, LD\} ,$$





probe the $R^{\rm had}$ ratio of $e^+e^- \to$ hadrons in different regions of c.o.m. energy E

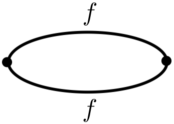
$$a_\mu^w = \frac{2\alpha_{em}^2 m_\mu^{-1}}{9\pi^2} \int_{E_{thr}}^\infty dE \left\{ \frac{m_\mu^3}{E^3}\, \widetilde{K}\left(\frac{E}{m_\mu}\right) \underline{\widetilde{\Theta}^w(E)} \right\} R^{had}(E) ,$$

$\Rightarrow$ a key test of SM (lattice QCD+QED) v.s experiment (independent of $g_\mu - 2$!)

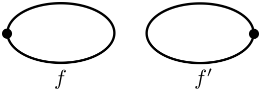
Sketch of the lattice computation of a_μ^{SD} and a_μ^{W}

We compute the $\ell \equiv u/d, s, c$, quark-line connected and disconnected contributions to a_μ^{SD} and a_μ^{W} – on twisted mass fermion gauge ensembles in the limit $m_u = m_d$

$$V_{conn}^{ff'}(t) \equiv -\frac{1}{3} \sum_{i=1,2,3} \int d^3x \langle J_i^{ff'}(\vec{x},t)\, J_i^{f'f}(0)\rangle = q_f^2 \times$$



$$V_{disc}^{fh}(t) \equiv -\frac{1}{3} \sum_{i=1,2,3} \int d^3x \langle J_i^{ff}(\vec{x},t)\, J_i^{hh}(0)\rangle = -q_f q_h \times$$



- physical point quark masses (interpolated), linear size $L \sim 5.4 \div 7.6$ fm
- 3 (or 4) lattice spacings $\times$ 2 UV regularizations , nice a^2 (or $a^2 + a^4$) scaling
- a_μ^{SD} for the first time – with exact removal of $O(a^2\, log(a^2))$ artifacts
- local vector currents with very precise (better than 0.1%) chiral covariant normalization (from WTI & hadronic methods)
- using PT (via "rhad" 2002 package) for $a_\mu^{SD}(b)$ and $a_\mu^{SD}(QED)$; lattice BMW-20 data for isospin breaking effect $a_\mu^{W}(IB)$: all tiny & accurate ...

Lattice setup and analysis - 1

Extensive Monte Carlo simulations of Lattice QCD with $2+1+1$ sea quarks:

- Iwasaki gluons + Wilson-clover twisted mass fermions $\Rightarrow$ only $\mathrm{O}(a^{2n})|_{n\geq 1}$ artifacts

- mixed action setup (dealing flexibly with terms in $V(t)$ of different size / accuracy)

$$V(t) = V^{\ell\ell}_{conn,r}(t) + V^{ss}_{conn,r}(t) + V^{cc}_{conn,r}(t) + V^{\rm all}_{disc,OS}(t) \;, \qquad \text{regularization } r = \{tm, OS\}$$

- tuning of M_π to 135 MeV, continuum limit and finite-L corrections: e.g. via fit ansatz

$$\left[a^w_\mu(..) + \Delta a^w_\mu(L) + F^r_1 a^2 \frac{\partial}{\partial M_\pi} \Delta a^w_\mu(L)\right] \left[1 + A(M_\pi - M^{phys}_\pi) + D^r_1 \frac{a^2}{[\log(a^2/w_0^2)]^{n_r}} + D^r_2 a^4\right]$$

$a^w_\mu(..)$, A (r–independent), F^r_1, D^r_1 and D^r_2 are free fitting parameters ($n_r = 0, ..., 3$)

$\Delta a^w_\mu(L)$: from a 2-pion (MLLGS) model of Finite Size Effects

tuned to reproduce known lattice data (needed only for $.. = \ell$)

term $\propto F^r_1$ describes a^2-dependent FSEs due to $\mathrm{O}(a^2)$ distortions of the pion spectrum.

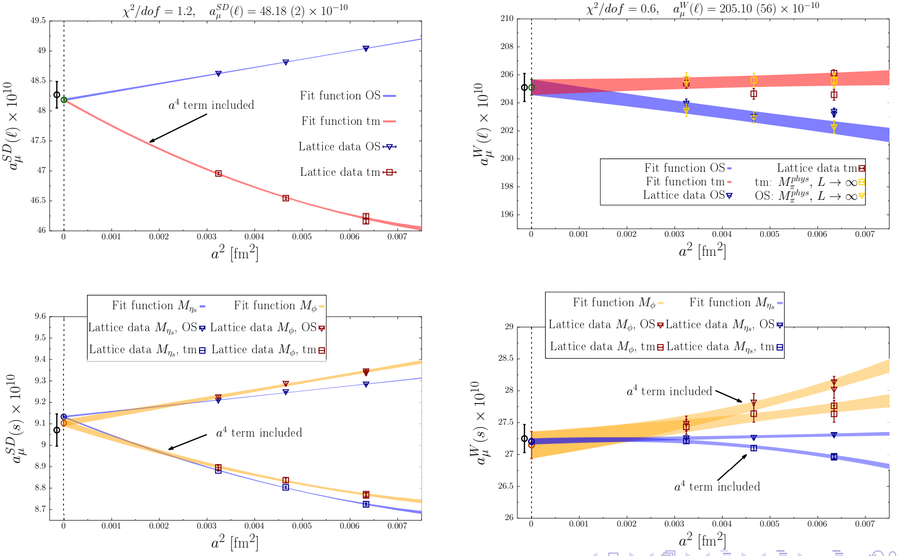
- for each $a^w_\mu(..)[.. = \ell, s, c, disc]$ O(50) different fits are separately decided & done,

results (X_k) from the (N) fits with good χ^2/dof are kept and combined according to

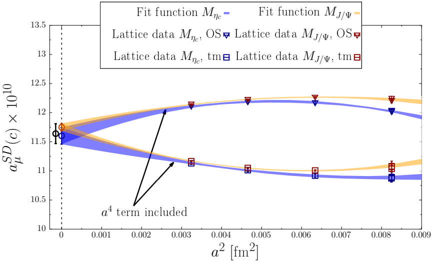
$$X = \sum_{k=1}^N \frac{1}{N} X_k \;, \qquad \sigma^2_X = \sum_{k=1}^N \frac{1}{N}\sigma^2_k + \sum_{k=1}^N \frac{1}{N}(X_k - X)^2$$

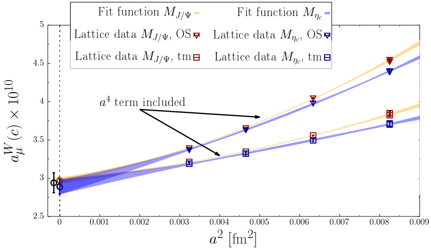
Lattice setup and analysis - 2

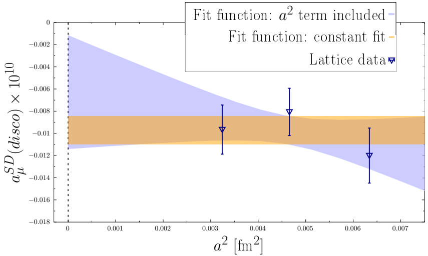
Isospin symmetric QCD inputs: $M_\pi^{iso} = 135.0(2)$ MeV, $f_\pi^{iso} = 130.4(2)$ MeV

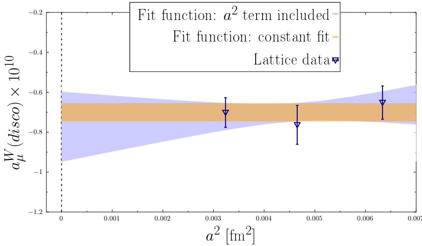


Isospin symmetric QCD inputs: $M_K^{iso} \Leftrightarrow M_\phi^{exp} \Leftrightarrow M_{\eta s}^{iso}$, $M_D^{exp} \Leftrightarrow M_{J/\psi}^{exp} \Leftrightarrow M_{\eta c}^{exp}$









a_μ^{HVP} window observables: lattice results + comparison

Ref.	$a_\mu^{SD}(\ell)\ 10^{10}$	$a_\mu^{SD}(s)\ 10^{10}$	$a_\mu^{SD}(c)\ 10^{10}$	$a_\mu^{SD}(disc.)\ 10^{10}$
ETMC-22	48.27 (0.22)	9.071 (75)	11.64 (0.16)	−0.006 (5)
–	–	–	–	–

Ref.	$a_\mu^{W}(\ell)\ 10^{10}$	$a_\mu^{W}(s)\ 10^{10}$	$a_\mu^{W}(c)\ 10^{10}$	$a_\mu^{W}(disc.)\ 10^{10}$
ETMC-22	205.1 (1.0)	27.27 (0.24)	2.95 (0.13)	−0.77 (0.17)
BMW-20	207.3 (1.4)	27.18 (0.03)	2.7 (0.1)	−0.85 (0.06)
CLS/Mainz-22	207.0 (1.5)	27.68 (0.28)	2.89 (0.14)	−0.81 (0.09)
χQCD-22	206.7 (1.5)	26.7 (0.3)	–	–
average	206.30 (0.67)	27.18 (0.03)	2.82 (0.08)	−0.83 (0.05)

- individual a_μ^W terms all self-consistent $\Rightarrow$ clear success of LQCD computations
- a_μ^{SD} terms above + "rhad" PT terms $a_\mu^{SD}(b) = 0.32\ 10^{-10} + a_\mu^{SD}(QED) = 0.03\ 10^{-10}$

$$\Rightarrow a_\mu^{SD}(\mathrm{ETMC}-22) = 69.33(29)\ 10^{-10}$$

- a_μ^W terms above + "BMW-20" QED + strong IB correction $a_\mu^W(IB) = 0.43(4)\ 10^{-10}$

$$\Rightarrow a_\mu^{W}(\mathrm{ETMC}-22) = 235.0(1.1)\ 10^{-10}$$

→ compatible at 1.0 $\sigma_{\mathrm{combined}}$ level with $a_\mu^W(\mathrm{BMW}-20) = 236.7(1.4)\ 10^{-10}$

→ compatible at 1.3 $\sigma_{\mathrm{combined}}$ level with $a_\mu^W(\mathrm{CLS}-22) = 237.30(1.46)\ 10^{-10}$

$a_\mu^{\rm HVP}$ window observables: SM (lattice) vs. experiment ($R^{\rm had}$)

SM predictions from lattice QCD + QED (col. 2,3,4) against $R^{\rm had}$ data driven results (col. 5, 6)

latt. "aver." $\leftrightarrow$ our average of the "independent" results from ETMC-22, CLS-22 and BMW-20

WP-proc.('22) $\leftrightarrow$ 2205.12963 (Colangelo et al.) with merging procedure of 2006.04822 (WP)

KNT('19-'22) $\leftrightarrow$ Keshavarzi, Nomura, Teubner: 1911.00367 + private communication (2022)

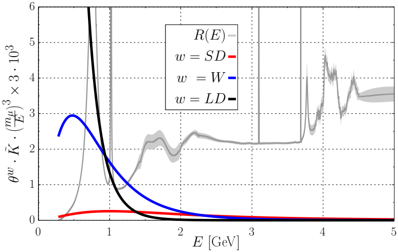
obs.(HVP-LO)	ETMC-22	BMW-20	latt. "aver."	WP-proc.('22)	KNT('19-'22)
a) $a_\mu^{\rm SD}\ 10^{10}$	69.33(29)	–	–	68.4(5)	*68.44(48)*
b) $a_\mu^{\rm W}\ 10^{10}$	235.0(1.1)	236.7(1.4)	236.08(74)	229.4(1.4)	*229.51(87)*
c) $a_\mu^{\rm HVP}\ 10^{10}$	–	707.5(5.5)	–	693.0(3.9)	*692.78(2.42)*

a) Agreement at 1.6 $\sigma_{combined}$ level

b) Tension at 4.2 (or *5.8*) $\sigma_{combined}$ level ! [BACKUP]

c) Tension at 2.1 (or *2.4*) $\sigma_{combined}$ level

$$a_\mu^w \propto \int_{E_{thr}}^{\infty} dE \left\{ \frac{m_\mu^3}{E^3}\, \widetilde{K}\left(\frac{E}{m_\mu}\right) \widetilde{\Theta}^w(E) \right\} R^{had}(E)$$



Final remarks & questions

"... accurate lattice results in the short and intermediate windows hint at possible deviations of the e^+e^- cross section data with respect to SM predictions distributed somewhere in the low (and possibly intermediate) energy regions, but not in the high energy region." (ETMC-22)

- a_μ^W represents a strong & theoretically clean probe of $e^+e^- \to 2$ (3) pions physics

- new ETMC result on a_μ^{SD} shows agreement within errors of SM theory with $R^{\rm had}(E)$ large E [in line with CLS/Mainz 2203.08676 work on $\Delta\alpha$], as needed for consistency of the photon HVP with EW precision tests [see e.g. Sirlin et al. 2006.12666, Crivellin et al. 2003.04886]

- now experiments are challenged to reduce errors on e^+e^- data (e.g. resolving tensions between KLOE + BESIII and BABAR) ... pushing the a_μ^W discrepancy to the discovery level?

- research on the original $g_\mu - 2$ puzzle (discrepancy between experimental measurement and data-driven+SM determination of a_μ) has led to find a possible failure of the SM in the description of $e^+e^- \to$ hadrons data at low and intermediate E $\Rightarrow$ the "photon HVP problem"

- further lattice studies are needed (in progress) to improve accuracy on a_μ^{LD} and see whether a pure SM prediction for $a_\mu^{\rm HVP}$ brings a_μ to agree with experiment (as suggested by BMW-20) or not. Anyway, the "photon HVP problem" (if confirmed) sheds a new light on the $g_\mu - 2$ puzzle!

Further remarks & outlook

- search for New Physics scenarios explaining "photon HVP problem" and $g_\mu - 2$ puzzle while fullfilling all known constraints has just started. For instance models of "light" NP, with a new vector boson of mass $\lesssim 1$ GeV: Darmé et al. 2112.09139,Di Luzio et al. 2112.08312. Else?

- lattice and data-driven results for window and full HVP (LO) a_μ-observables look compatible, within errors, with an overall few-percent shift of the $e^+e^- \to \pi^+\pi^-$ data at c.o.m $E < 1$ GeV

obs.	a_μ^w(LQCD)	$a_\mu^w(e^+e^-)$ [**]	Δa_μ^w	$a_\mu^w(2\pi)$ [**]	$\Delta a_\mu^w / a_\mu^w(2\pi)$
$a_\mu^{\rm SD}$	69.3 (0.3) [*]	68.4 (0.5)	0.9 (0.6)	13.7 (0.1)	0.066 (43)
$a_\mu^{\rm W}$	235.0 (1.1) [*]	229.4 (1.4)	5.6 (1.8)	138.3 (1.2)	0.040 (13)
$a_\mu^{\rm HVP}$	707.5 (5.5) [***]	693.0 (3.9)	14.5 (6.7)	494.3 (3.6)	0.029 (14)

[*] = ETMC-22; [**] = Colangelo et al. 2205.12963; [***] BMW-20

- "photon HVP problem" seen by pushing theory to $\lesssim$ 0.5% accuracy in the vector channel. Any other tensions in hadronic physics if/when SM-lattice theory is pushed to such a high precision?

- Technical outlook about ETMC "homework"

i) Direct computation of QED and strong IB correction on ETMC ensembles

ii) New ensembles at very large L to evaluate a_μ^{LD} (controlling finite L effects)

iii) New ensembles at physical m_π with one finer and one coarser lattice spacing

Thanks to organizers, convenors and audience ...

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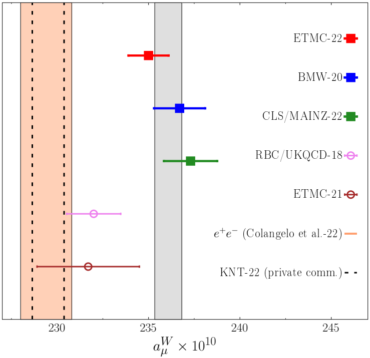
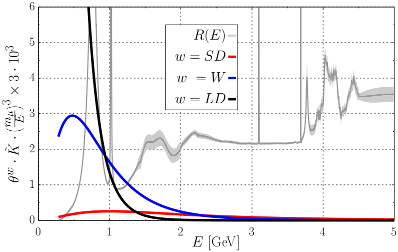
THANKS FOR YOUR ATTENTION !

Backup: average of lattice a_μ^W-results & tension with exp. data



$$a_\mu^w \propto \int_{E_{thr}}^{\infty} dE \left\{ \frac{m_\mu^3}{E^3} \, \widetilde{K}\left(\frac{E}{m_\mu}\right) \widetilde{\Theta}^w(E) \right\} R^{had}(E)$$



- our average *(grey band)*: based on results with the dominating contributions evaluated using

  i) at least 3 lattice spacings (for the limit $a \to 0$), ii) some ensembles with physical pion mass.

This excludes RBC/UKQCD-18 (due to i)) and ETMC-21 (due to ii), superseded by ETMC-22).

- strong tension with a_μ^W (HVP-LO) results driven by experimental e^+e^- data :

  at $\sim 4.2\sigma_{combined}$ if WP-proc.('22) (2205.12963, Colangelo et al.), see *light-red band*, is used

  at $\sim 5.8\sigma_{combined}$ if KNT('19-'22) (1911.00367 + private comm.), see *dashed lines*, is used