Lund and Cambridge multiplicity for precision physics

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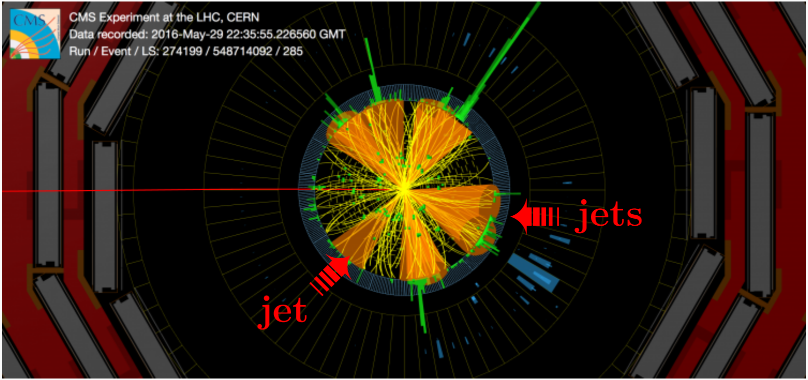
Context

- QCD jets appear in every LHC collision. Their structure spans energy scales from the $\mathcal{O}(100\,\mathrm{MeV})$ to the $\mathcal{O}(5\,\mathrm{TeV})$
- This range makes jets and jet substructure a powerful tool for both new particle searches, as well as precision measurements



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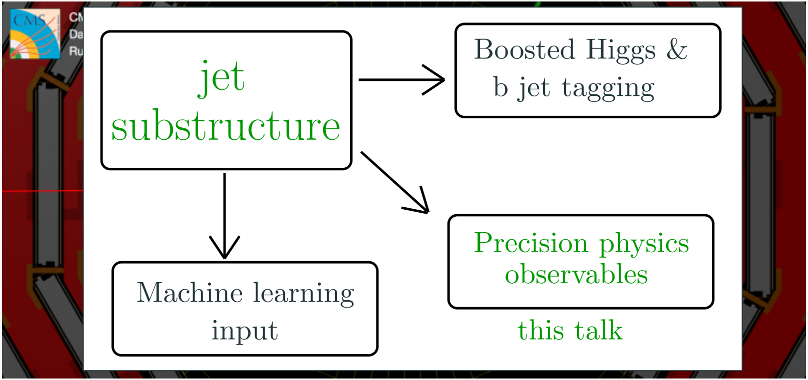


Outline

1. Lund multiplicity
   A new way of counting jets

2. Predicting the average multiplicity
   New advance in high-resolution regime

3. Phenomenology at LEP and the LHC
   Uncertainties below 5%

Lund multiplicity

Lund multiplicity algorithm

Is a new way of evaluating the multiplicity at colliders.

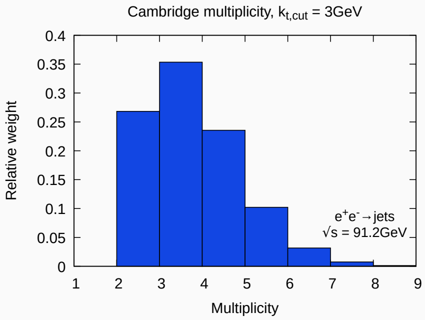
1. Begin with an anti-k_t jet (LHC), an event hemisphere (e^+e^-), any object.
2. Choose a *resolution parameter* $k_{t,\text{cut}}$
3. Re-cluster the object with Cambridge/Aachen
4. Traverse back through the clustering tree and count the number of declusterings with $k_t > k_{t,\text{cut}}$, adding +1 for the initial jet.*

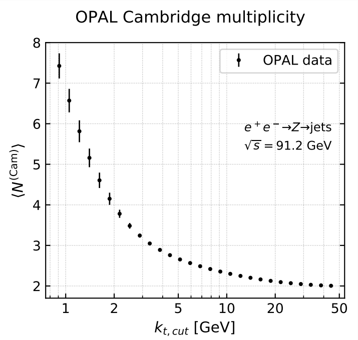
* $k_t = \min(E_1, E_2)\sin\theta_{12}$ for $\ell^+\ell^-$ collider.
For pp collisions use $k_t = \min(k_{\perp,j_1}, k_{\perp,j_2})\Delta R_{j_1,j_2}$.

Lund multiplicity algorithm

Proof of concept: Both hemispheres in e^+e^- collisions at LEP

- Because of their angular-ordered nature, conventional Cambridge jet multiplicity is closely related to Lund multiplicity, so we computed both. In this talk I will focus on the former.
- Multiplicity varies from event to event. This talk is about computing the average multiplicity $\langle N \rangle$

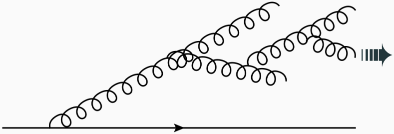




Predicting the average Lund and Cambridge multiplicities

Structure of average multiplicity at low $k_{t,\text{cut}}$

- As $k_{t,\text{cut}} \ll \sqrt{s}$, the dominant Feynman diagrams become



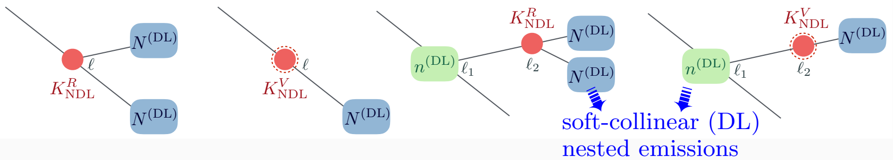
- In this limit, two large logarithms $L = \ln\left(k_{t,\text{cut}}/\sqrt{s}\right)$ appear for each emission – $\alpha_s L^2$ for 1, or $[\,\alpha_s L^2\,]^n$ for n emissions
- As $\alpha_s L^2 \sim 1$, one sums the effect of an infinite number of emissions, so-called "double-logarithmic" (DL) counting

$$\langle N(\alpha_s, L)\rangle = \langle N(\alpha_s, 0)\rangle \Big[\underbrace{\text{DL}}_{\left(\alpha_s L^2\right)^n} + \underbrace{\text{NDL}}_{\alpha_s^n L^{2n-1}} + \underbrace{\text{NNDL}}_{\alpha_s^n L^{2n-2}} + \ldots \Big]$$

Structure of multiplicity at low $k_{t,\mathrm{cut}}$

$$\langle N(\alpha_s, L)\rangle = \langle N(\alpha_s, 0)\rangle \left[\underbrace{\mathrm{DL}}_{(\alpha_s L^2)^n} + \underbrace{\mathrm{NDL}}_{\alpha_s^n L^{2n-1}} + \underbrace{\mathrm{NNDL}}_{\alpha_s^n L^{2n-2}} + \dots \right]$$

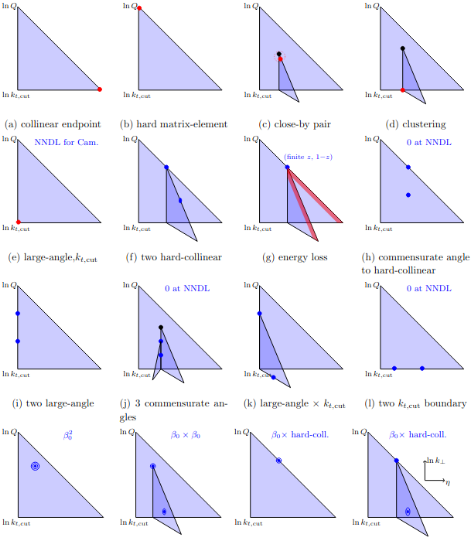
- We developed a novel resummation formalism which works as
  1. Enumerate the possible corrections going from $\mathrm{N}^k\mathrm{DL} \to \mathrm{N}^{k+1}\mathrm{DL}$
  2. For each contribution, compute a fixed-order correction kernel K
  3. Use K and $\mathrm{N}^{\leq k}\mathrm{DL}$ to account for the effect at all orders.
- For example, going from DL → NDL, exactly one emission in a chain of n is *not* soft-and-collinear (done in 1991)



⇒With this we computed $\langle N\rangle$ up to NNDL accuracy

NNDL accurate average multiplicity

Based on emission phase-space we first identified 16 contributions...



(a) collinear endpoint (b) hard matrix-element (c) close-by pair (d) clustering

(e) large-angle, $k_{t,\mathrm{cut}}$ (f) two hard-collinear (g) energy loss (h) commensurate angle to hard-collinear

(i) two large-angle (j) 3 commensurate angles (k) large-angle × $k_{t,\mathrm{cut}}$ (l) two $k_{t,\mathrm{cut}}$ boundary

(m) squared running coupling (n) running coupling × hard-collinear

NNDL accurate average multiplicity

Based on emission phase-space we first identified 16 contributions...

And resummed them with our developed formalism

$$
\begin{aligned}
2\pi h_3^{(q)} = {} & D_{\text{end}}^{q\to qg} + \left(D_{\text{end}}^{g\to gg} + D_{\text{end}}^{g\to q\bar{q}}\right)\frac{C_F}{C_A}(\cosh\nu - 1) + D_{\text{hme}}^{qqg}\cosh\nu \\
& + \frac{C_F}{C_A}\left[(1-c_\delta)D_{\text{pair}}^{q\bar{q}}(\cosh\nu - 1) + \left(K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar{q}}\right)\frac{\nu}{2}\sinh\nu\right] \\
& + C_F\left[\left(\cosh\nu - 1 - \frac{1-c_\delta}{4}\nu^2\right)D_{\text{clust}}^{(\text{prim})} + (\cosh\nu - 1)D_{\text{clust}}^{(\text{sec})}\right] \\
& + \frac{C_F}{C_A}\left[D_{\text{e-loss}}^{g}\frac{\nu}{2}\sinh\nu + \left(D_{\text{e-loss}}^{q} - D_{\text{e-loss}}^{g}\right)(\cosh\nu - 1)\right] \\
& + \frac{C_F}{2}\Big\{(B_{gg} + c_\delta B_{gq})^2\nu^2\cosh\nu + 8\left[2c_\delta B_{gg} - 2c_\delta B_q - (1-3c_\delta^2)B_{gq}\right]B_{gq}\cosh\nu \\
& \quad + \left[4B_q(B_{gg} + (2c_\delta + 1)B_{gq}) - (B_{gg} + c_\delta B_{gq})(B_{gg} + 9c_\delta B_{gq})\right]\nu\sinh\nu \\
& \quad + 4(1-c_\delta^2)B_{gq}^2\nu^2 + 8\left[2c_\delta B_q - 2c_\delta B_{gg} + (1-3c_\delta^2)B_{gq}\right]B_{gq}\Big\} \\
& + \frac{C_F}{C_A}\frac{\pi\beta_0}{2}\Big\{(B_{gg} + c_\delta B_{gq})\nu^3\sinh\nu + \left[2B_q - 2B_{gg} + (6-8c_\delta)B_{gq}\right]\nu\sinh\nu \\
& \quad + 2(B_q + B_{gg} + B_{gq})\nu^2\cosh\nu - 4(1-c_\delta)B_{gq}(2\cosh\nu - 2 + \nu^2)\Big\} \\
& + \frac{C_F}{C_A}\frac{\pi^2\beta_0^2}{8C_A}\left[3\nu(2\nu^2 - 1)\sinh\nu + (\nu^4 + 3\nu^2)\cosh\nu\right]
\end{aligned}
\tag{4.71}
$$

all coefficients are analytic

quark-initiated jets

for quarks, and

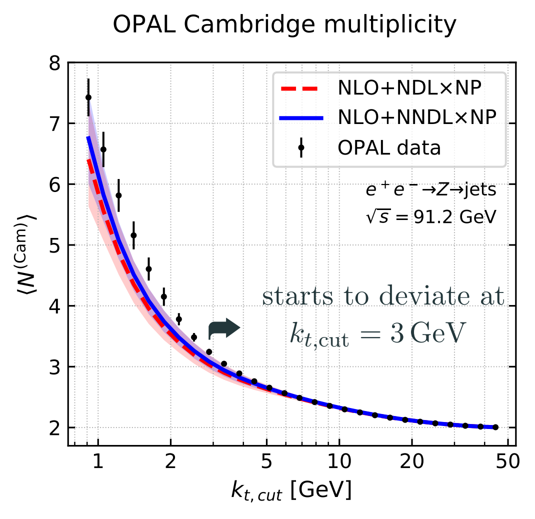
$$
\begin{aligned}
2\pi h_3^{(g)} = {} & \left(D_{\text{end}}^{g\to gg} + D_{\text{end}}^{g\to q\bar{q}}\right)\cosh\nu + \left[D_{\text{hme}}^{ggg}\cosh\nu + D_{\text{hme}}^{gq\bar{q}}\left(c_\delta\cosh\nu + 1 - c_\delta\right)\right] \\
& + \left[(1-c_\delta)D_{\text{pair}}^{q\bar{q}}(\cosh\nu - 1) + \left(K + D_{\text{pair}}^{gg} + c_\delta D_{\text{pair}}^{q\bar{q}}\right)\frac{\nu}{2}\sinh\nu\right] \\
& + C_A\left(D_{\text{clust}}^{(\text{prim})} + D_{\text{clust}}^{(\text{sec})}\right)(\cosh\nu - 1) + D_{\text{e-loss}}^{g}\frac{\nu}{2}\sinh\nu \\
& + \frac{C_A}{2}\Big\{(B_{gg} + c_\delta B_{gq})^2\nu^2\cosh\nu - 8(1-c_\delta^2)B_{gq}^2(\cosh\nu - 1) \\
& \quad + \left[(B_{gg} + c_\delta B_{gq})(3B_{gg} - 5c_\delta B_{gq}) + 4(1+c_\delta)B_{gq}B_q\right]\nu\sinh\nu\Big\} \\
& + \frac{\pi\beta_0}{2}\Big\{(B_{gg} + c_\delta B_{gq})\nu^3\sinh\nu + 6(1-c_\delta)B_{gq}\nu\sinh\nu + 2\left[2B_{gg} + (1+c_\delta)B_{gq}\right]\nu^2\cosh\nu \\
& \quad - 8B_{gq}(1-c_\delta)(\cosh\nu - 1)\Big\} + \frac{\pi^2\beta_0^2}{8C_A}\left[3\nu(2\nu^2 - 1)\sinh\nu + (\nu^4 + 3\nu^2)\cosh\nu\right]
\end{aligned}
\tag{4.72}
$$

gluon-initiated jets

Phenomenology at LEP and the LHC

Predictions vs LEP data

Modelling non-perturbative effects with parton showers and matching to exact $\mathcal{O}\left(\alpha_s^2\right)$ we compared our predictions to LEP measurements.



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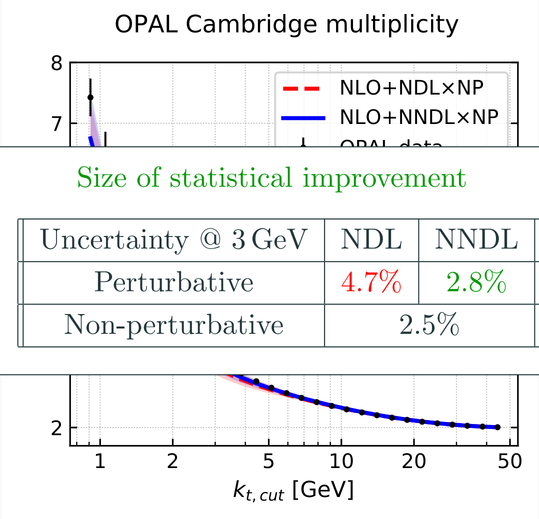
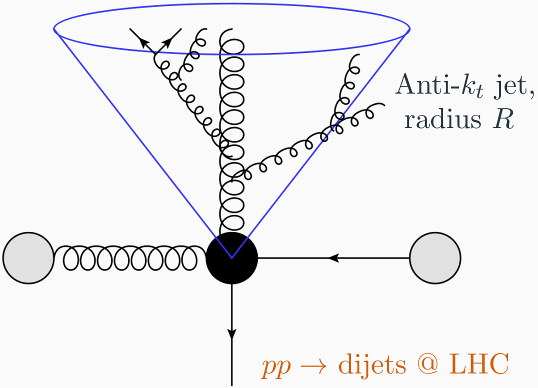


Size of statistical improvement

Uncertainty @ 3 GeV	NDL	NNDL
Perturbative	4.7%	2.8%
Non-perturbative	2.5%	

Lund multiplicity at the LHC [Work in progress]

- Looking inside jets at the LHC: dijets and Z+jet
  (i) Re-cluster a high-p_t jet with Cambridge/Aachen
  (ii) Traverse backwards and count the number of declusterings with $k_t > k_{t,\text{cut}}$, +1 for initial jet
- Currently working on predictions at NNDL accuracy



Conclusions

1. Lund multiplicity is a new observable for precision jet substructure measurements, applicable to LHC, FCC-ee, ...

   Valuable for α_s extraction, validation of new generation of parton showers

2. We have developed a new formalism to predict the average multiplicity in collider events

   First NNDL accurate multiplicity result

3. We have produced predictions that allow for sub-5% accuracy on measurements at lepton colliders

   Current on-going effort to extend the calculation to LHC jets

Backup Slides

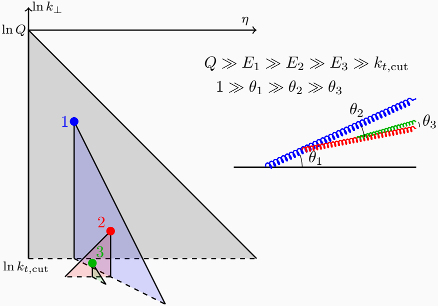
The Lund Jet plane

Lund multiplicity gets its name from the Lund jet plane, a modern jet substructure technique:

i) Begin with a jet of interest (e.g. anti-k_t jet at the LHC)

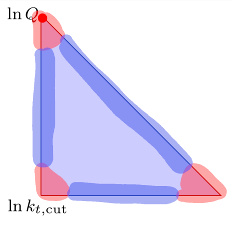
ii) Recluster it with the Cambridge/Aachen jet algorithm

iii) Traverse backwards through the clustering sequence and at each de-clustering record $\{k_t, \Delta R^2, m^2, ...\}$. This information can be represented with a Lund diagram



The resummation method: NDL example

- The correction kernel $K^{R,V}$ is dependent on the kinematic region.
  For NDL: blue region



- Example: hard-collinear correction

$$K^R = \frac{\alpha_s}{\pi}\int_0^1 dz \left[P_{gg}(z) - \frac{2C_A}{z}\right] = -K^V$$

- Schematically resum by adding towers of soft-collinear emissions after ($N^{(\mathrm{DL})}$) or before ($n^{(\mathrm{DL})}$) the correction: