Measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ and its contribution to the muon *g-2*

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on behalf of the BABAR Collaboration

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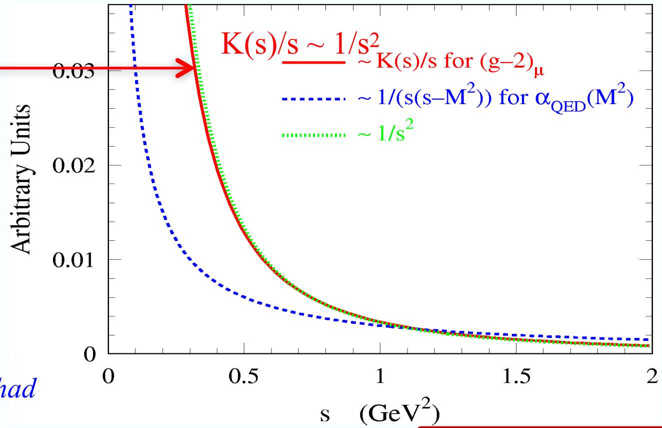
Hadronic contribution to a_μ

Dominated by the Hadronic Vacuum Polarization LO.
Can be calculated by using dispersion relations

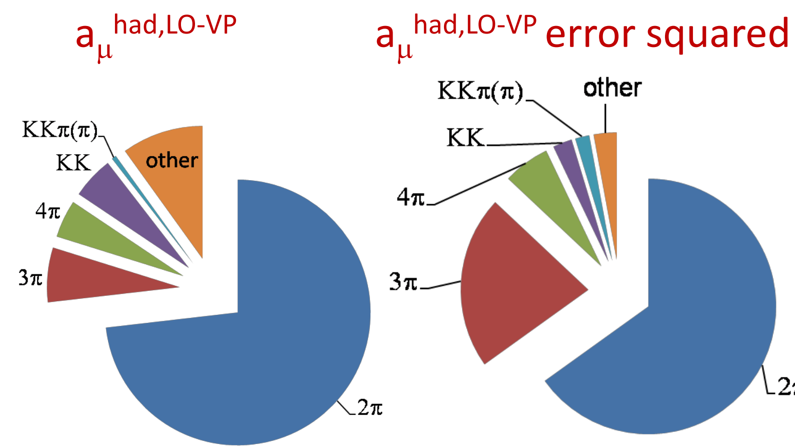
$$a_\mu^{had} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma^0 \left[e^+e^- \to \text{hadrons}\,(\gamma)\right]}{\sigma_{pt}}$$

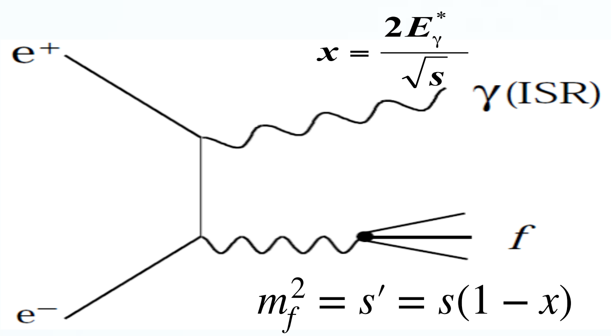
σ^{had} => experimental input to a_μ^{had} calculation



The process $e^+e^- \to \pi^+\pi^-\pi^0$ gives the second largest contribution to $a_\mu^{had,LO-VP}$ and its uncertainty



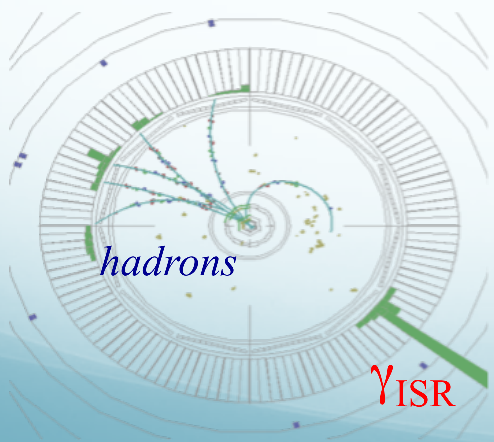
The Initial State Radiation method



$$\frac{d\sigma_{e^+e^-\to f\gamma}(s,m_f)}{dm_f\, d\cos\theta_\gamma^*} = \frac{2m_f}{s} W(s,x,\theta_\gamma^*)\cdot\sigma_{e^+e^-\to f}(m_f)$$

- The hadronic cross section $e^+e^- \to f$ can be extracted from the ISR cross section $e^+e^- \to \gamma f$.
- The radiator function $W(s,x)$ is calculated in QED with accuracy better than 1% level

BABAR display of a typical ISR event

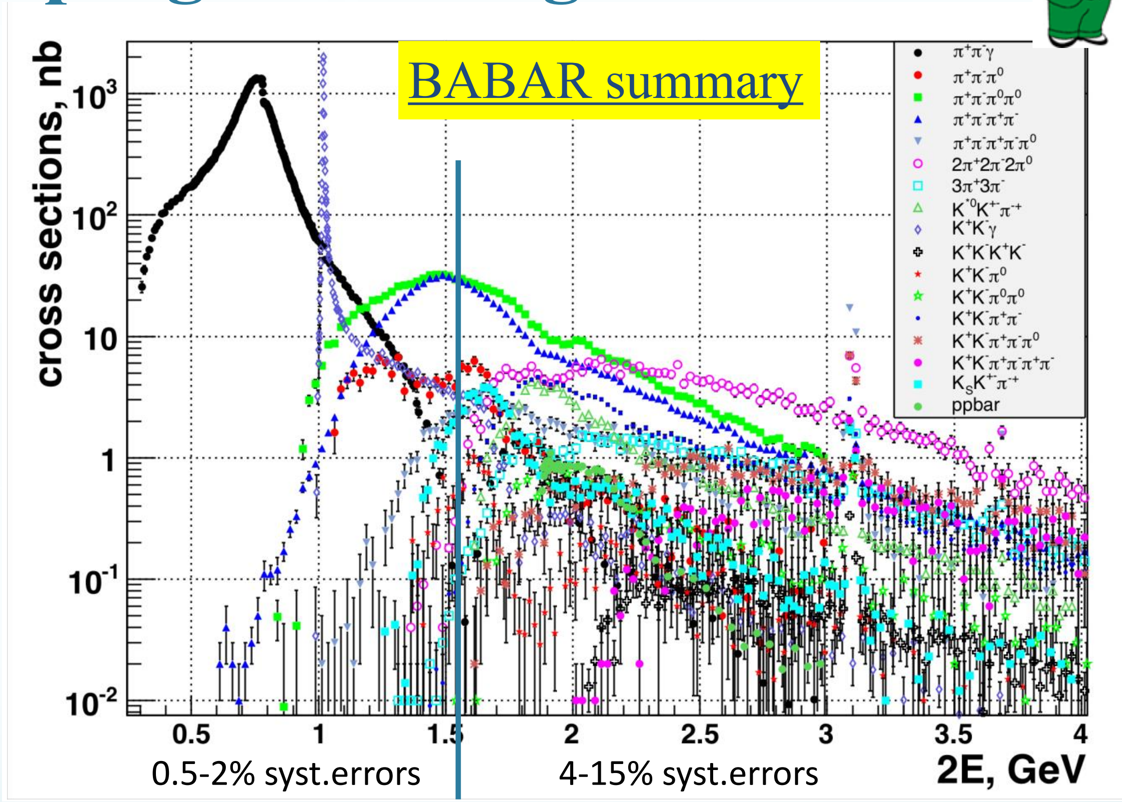


Common ISR analysis strategy

- Tagged analysis (E_γ^*>3 GeV)
- Back-to-back topology btw ISR γ and the rest of the event
- π/K/*p* discrimination based on dE/dx e Cherenkov angle
- Kinematic fit for 4-momentum conservation
- Fitted χ^2 used for signal selection and background subtraction
- Detector acceptances and selection efficiencies estimated with MC simulation

The *BABAR* ISR program for light hadrons

- 30 publications for more than 40 final states studied
- Almost any channel from 2 to 7 light hadrons in the final state
- Many first measurements and significant precision improvement in most cases
- Discoveries (*e.g.* $\phi(2170)$ in $e^+e^- \rightarrow \phi(1020)f_0(980)$)
- Most precise measurement of $e^+e^- \rightarrow \pi^+\pi^-$

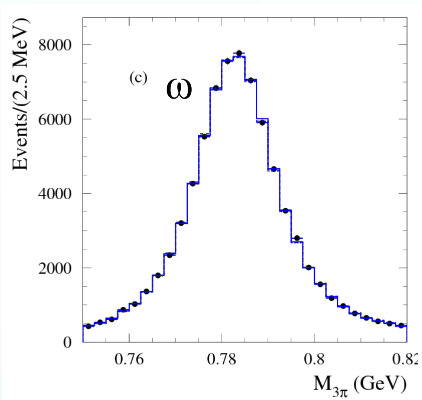


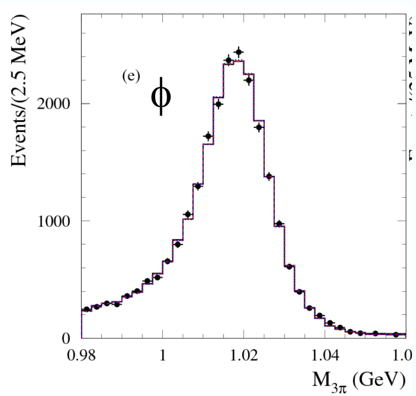
Today's presentation:
$e^+e^- \rightarrow \pi^+\pi^-\pi^0$ updated with the full data sample of 460 fb^{-1} **[Phys.Rev.D104, 112003 (2021)]**
Preliminary results on $e^+e^- \rightarrow K^+K^-\pi^0\pi^0\pi^0$, $e^+e^- \rightarrow K_S^0K^\pm\pi^\mp\pi^0\pi^0$, $e^+e^- \rightarrow K_S^0K^\pm\pi^\mp\pi^+\pi^-$

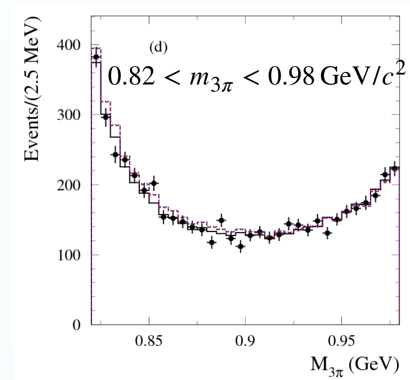
Fit to the 3π mass spectrum

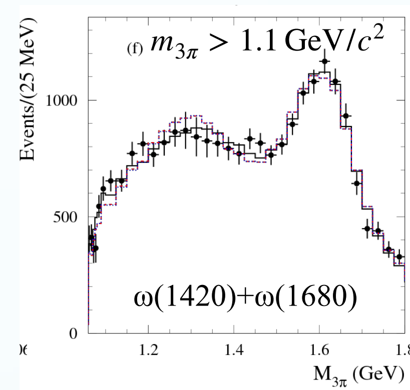
The fit to the measured mass spectrum is based on the VMD model with $\omega(782)+\omega(1420)+\omega(1680)+\phi(1020)+\rho(770)$

- The true spectrum is smeared to account for data-MC difference in the mass resolution, and then multiplied by the transfer matrix obtained from simulation for the unfolding









- Parameters fitted for ω and ϕ in good agreement with world average

$$\Gamma_{\omega\to e^+e^-}\mathscr{B}_{\omega\to\pi^+\pi^0\pi^0} = (0.5698 \pm 0.0031 \pm 0.0082)\ \text{keV}$$
$$\Gamma_{\phi\to e^+e^-}\mathscr{B}_{\phi\to\pi^+\pi^0\pi^0} = (0.1841 \pm 0.0021 \pm 0.0080)\ \text{keV}$$

PDG: $\Gamma_\omega \times \mathscr{B} = 0.557 \pm 0.011$ keV
$\Gamma_\phi \times \mathscr{B} = 0.1925 \pm 0.0043$ keV

- The $\rho \to 3\pi$ decay needed to describe the data
- The significance of $\rho \to 3\pi$ is greater than 6σ
- In agreement with SND PRD68, 052006 (2003)

	BF($\rho\to3\pi$)x 10^4	φ
BABAR	$0.88 \pm 0.23 \pm 0.30$	$-(99 \pm 9 \pm 15)^o$
SND	$1.01^{+0.54}_{-0.36} \pm 0.34$	$-(135^{+17}_{-13} \pm 9)^o$

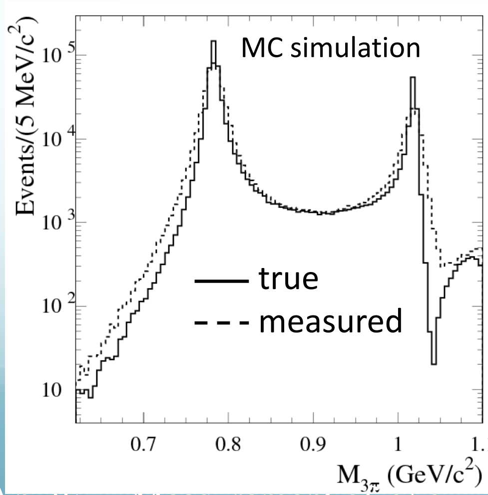
$e^+e^- \to \pi^+\pi^-\pi^0$ cross section for $m_{3\pi}$<1.1 GeV/c^2

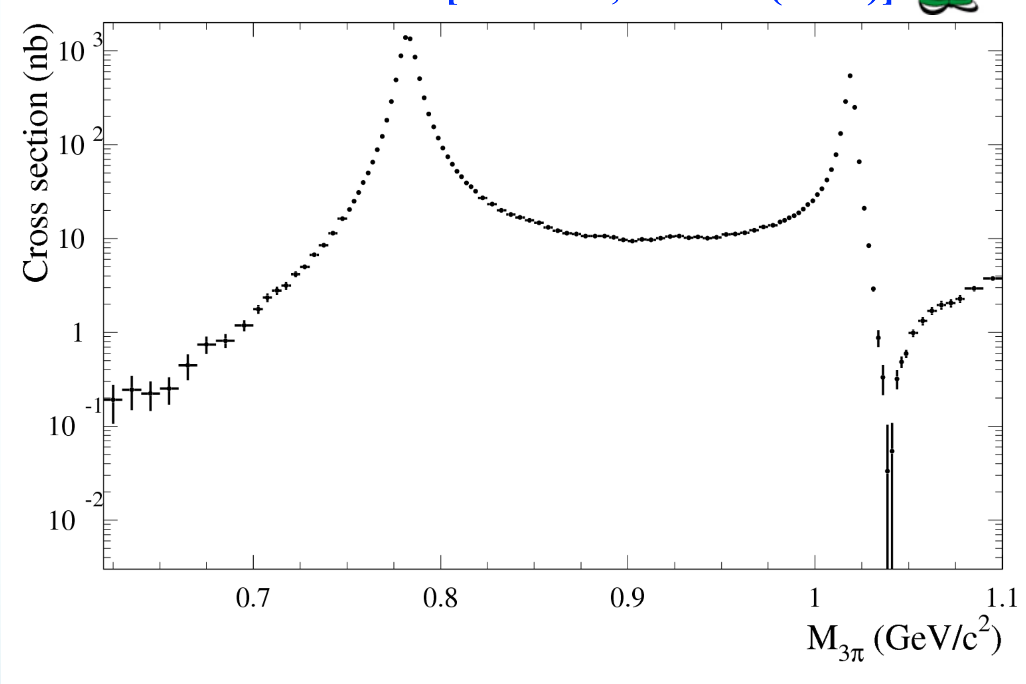
$$\frac{dN}{dm} = \sigma_{3\pi}(m)\frac{d\mathcal{L}}{dm} R\,\varepsilon \qquad \frac{d\mathcal{L}}{dm} = \frac{\alpha}{\pi x}\left((2-2x+x^2)\log\frac{1+C}{1-C} - x^2 C\right)\frac{2m}{s}\mathcal{L}.$$

[PRD104, 112003 (2021)]

- The mass spectrum has sharp structures and unfolding is required to obtain the true spectrum.

- Unfolding performed with the IDS (iterative, dynamically stabilized) method B. Malaescu, arXiv:0907.3791



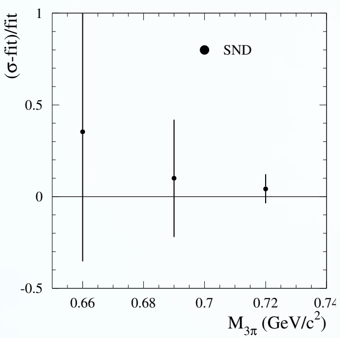


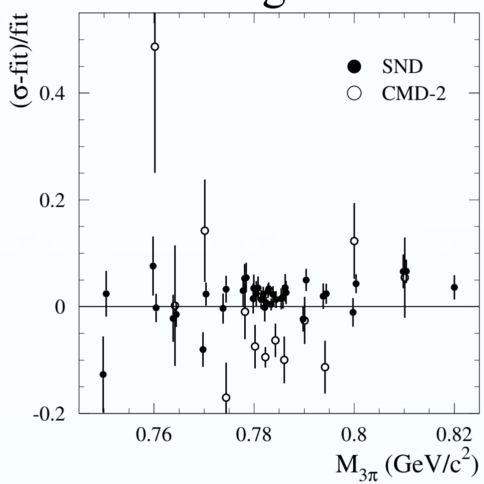
- Bin-width in the peaks regions: 2.5 MeV
- Systematic uncertainties at resonance peaks amount to about 1.3 % (most precise results)

Comparison with existing measurements

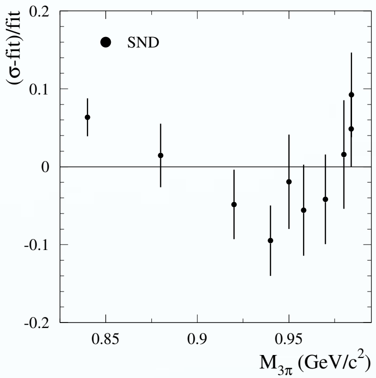
[PRD104, 112003 (2021)]

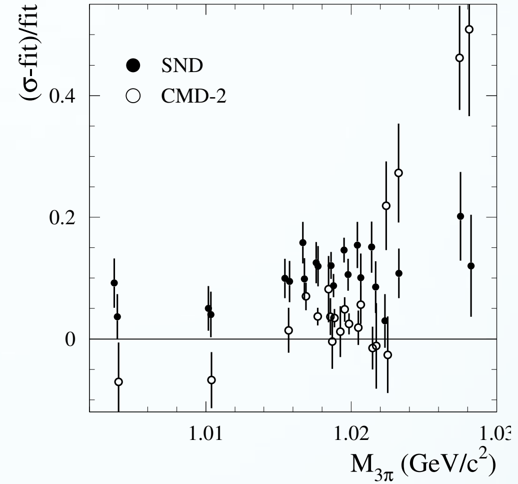
ω region





φ region





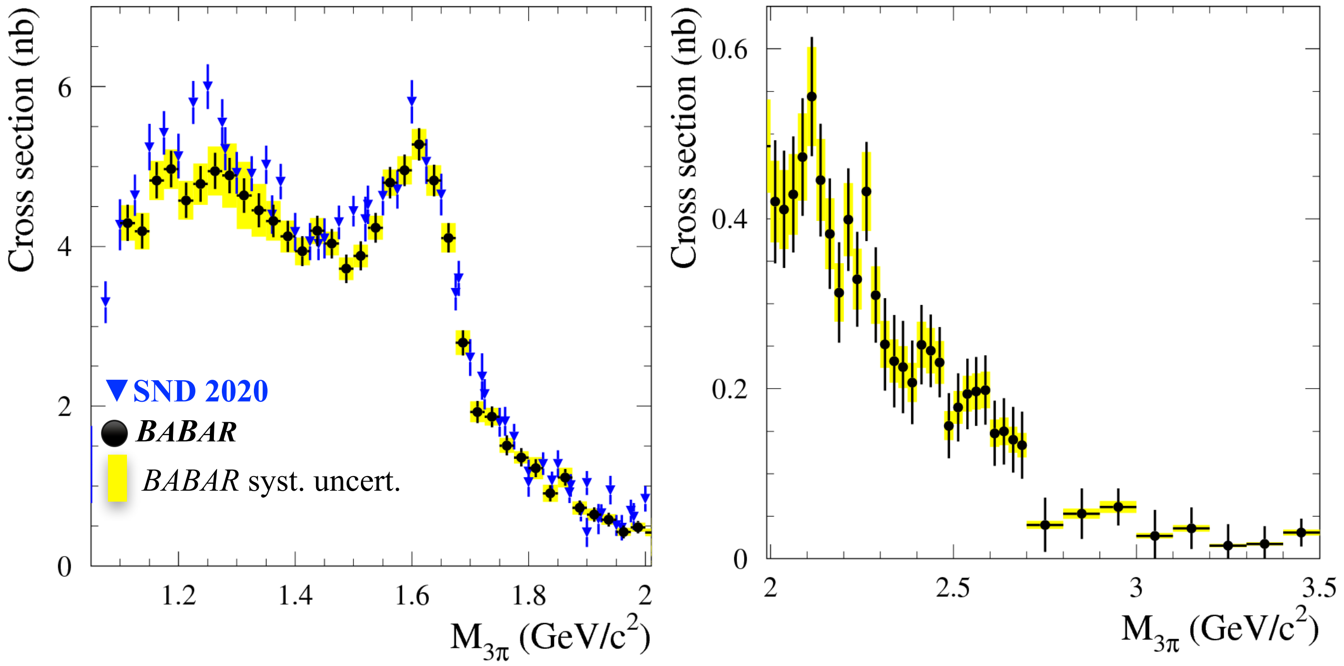
- Only statistical errors included in the plots
- **In the ω region** there is good consistency between *BABAR* and SND, while CMD-2 data lie about 7% below *BABAR*, with a difference of order 2.5σ
- **At the** φ CMD-2 data (2.5% syst. uncert.) are about 3% above *BABAR*, while SND data (5% syst. uncert.) are about 11% higher

SND:
- PRD 63, 072002 (2001)
- PRD 68, 052006 (2003)

CMD-2:
- PLB 578, 285 (2004)
- PLB 642, 203 (2006)

$e^+e^- \to \pi^+\pi^-\pi^0$ cross section above 1.1 GeV/c^2

[PRD104, 112003 (2021)]



- No narrow structures
- Bin size 25 MeV (100 MeV for $m_{3\pi} > 2.7$ GeV)

=> no need for unfolding

- Systematic uncertainties (4-15%) dominated by background subtraction

- SND 2020: Eur. Phys. J. C80, 993 (2020)
- Sizable difference between SND and BABAR data near 1.25 and 1.5 GeV.
- General agreement elsewhere.
- SND systematic uncertainties are 4.4%

Calculation of the contribution to a_μ

$$a_\mu = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{K(s)}{s}R(s)\,ds,\quad R(s) = \frac{\sigma_0(e^+e^- \to \text{hadrons})(s)}{4\pi\alpha^2/s}.$$

$$\sigma_0(e^+e^- \to \pi^+\pi^-\pi^0)(s) = \sigma_{3\pi}(s)|1-\Pi(s)|^2$$

$M_{3\pi}$ GeV$/c^2$		$a_\mu^{3\pi} \times 10^{10}$
0.62–1.10		$42.91 \pm 0.14 \pm 0.55 \pm 0.09$
1.10–2.00	*BABAR*	$2.95 \pm 0.03 \pm 0.16$
< 2.00	**PRD 104, 112003 (2021)**	$45.86 \pm 0.14 \pm 0.58$
< 1.8[1]	DHNZ	$46.21 \pm 0.40 \pm 1.40$
< 1.97[50]	KNT	46.74 ± 0.94
< 2[51]	Jegerlehner	44.32 ± 1.48
< 1.8[52]	HHK	$46.2 \pm 0.6 \pm 0.6$

Result consistent with calculations using previous data

Uncertainty on $a_\mu^{3\pi}$ improved by a factor of about 2

BABAR preliminary

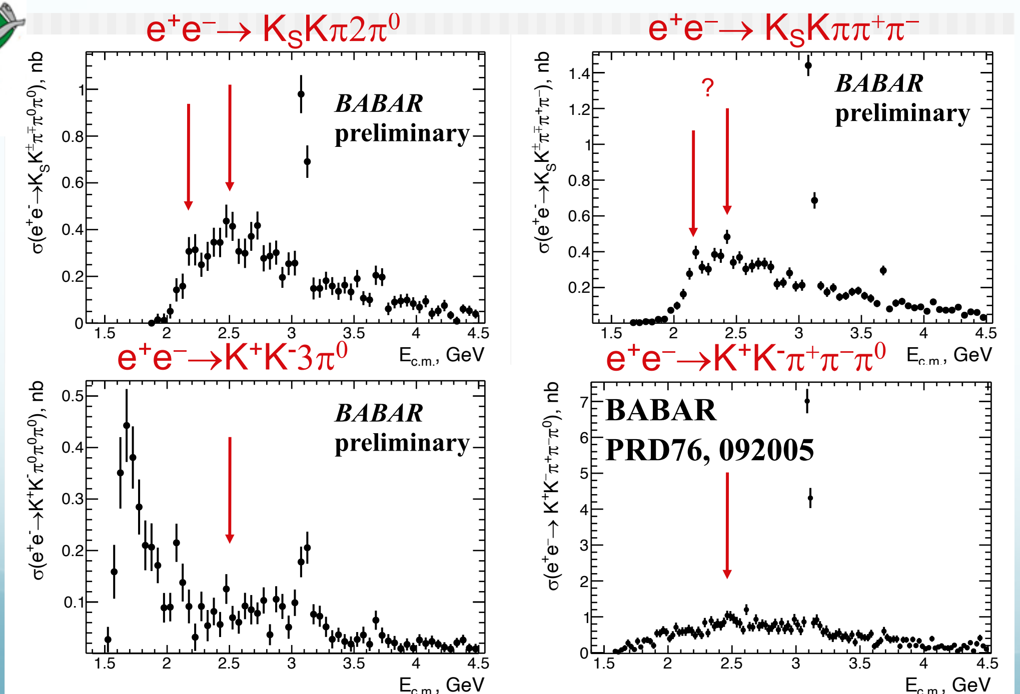
$$e^+e^- \to K^+K^-\pi^0\pi^0\pi^0$$

$$e^+e^- \to K_S^0K^\pm\pi^\mp\pi^0\pi^0$$

$$e^+e^- \to K_S^0K^\pm\pi^\mp\pi^+\pi^-$$

$e^+e^- \to 2K3\pi$

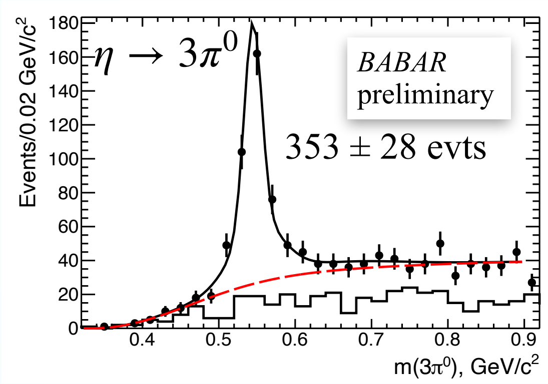
- Previously 2K3π final state studied: $e^+e^- \to K^+K^-\pi^+\pi^-\pi^0$ [PRD76, 092005 (2007)]
- Main motivations:
  - systematic deviation seen between sum-of-exclusive cross section near 2 GeV and pQCD predictions
  - direct measurement of the final states reduces the need of isospin relations for a_μ calculation
  - study of intermediate states, look for new states or new decay modes of recently discovered states
- Analysis method similar to that for $e^+e^- \to \pi^+\pi^-\pi^0$, but tuned for multi-hadron final states

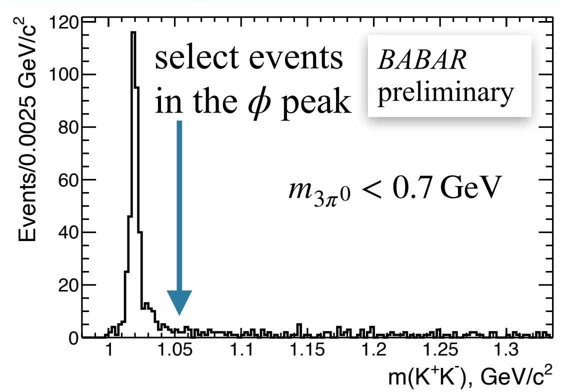


Measured cross sections:

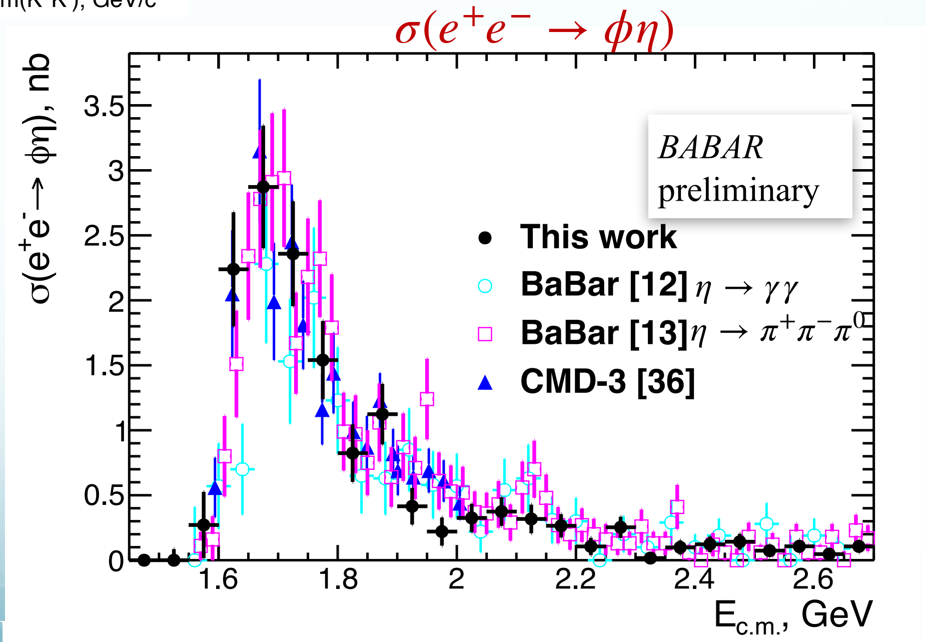
- systematic uncertainties ~10%
- smaller than $K^+K^-\pi^+\pi^-\pi^0$ but sizeable
- observation of correlated production of $K^*\bar{K}^*$, $K^*\rho$ in the top channels, and $\phi\eta$ in the $K^+K^-3\pi^0$ final state
- possible "bumps" around 2.4 and 2.17 GeV present in all plots?

$e^+e^- \to K^+K^-\pi^0\pi^0\pi^0$ intermediate states



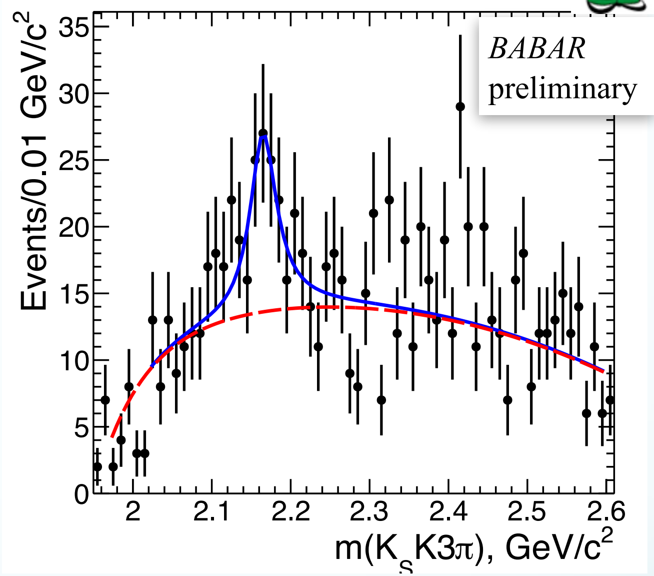


- The cross section is dominated by the channel $e^+e^- \to \phi\eta$
- To extract the cross section:
    - select events with $m_{KK} < 1.05$ GeV
    - divide the $2K3\pi^0$ mass spectrum in 50 MeV wide bins
    - fit the $3\pi^0$ mass distribution to get the events in the η peak
    - account for $\phi \to K^+K^-$ and $\eta \to 3\pi^0$ branching fractions



Other structures? $\phi(2170)$

- A little bump visible in $e^+e^- \to K_S^0 K^\pm \pi^\mp \pi^+ \pi^-$ at ~2170 MeV
  - also in the other channels but with less statistical significance
- It might represent a new decay channel of the $\phi(2170)$, discovered by BABAR in $e^+e^- \to \phi f_0(980)$
- Refine the mass spectrum, with bin size = 10 MeV
- Require $m(\pi^+\pi^-) < 0.7$ GeV, to remove the contribution from the $\rho(770)$
- Fit the peak with a BW + a 2nd order polynomial
- Parameters consistent with PDG. **Significance >3σ**



BABAR preliminary

$$N = 86 \pm 34 \text{ events},$$
$$m = 2.164 \pm 0.006 \ \text{GeV}/c^2,$$
$$\Gamma = 0.041 \pm 0.020 \ \text{GeV},$$

$\phi(2170)$ [PDG 2022]

$$m = 2.162 \pm 0.007 \ \text{GeV}/c^2$$
$$\Gamma = 0.100^{+0.031}_{-0.023} \ \text{GeV}$$

Summary

- BABAR pioneered the use of the ISR method to precisely measure low-energy exclusive hadronic cross sections

- Recent *BABAR* measurement of the process $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ **[Phys.Rev.D104, 112003]**
  - Most precise measurement ever of the cross section from 0.62 up to 3.5 GeV
  - Systematic uncertainties at the ω and ϕ resonance peaks are ~1.3%
  - The precision on $a_\mu^{3\pi}$ is improved by a factor about 2 (for $m_{3\pi} < 2$ GeV)
- New measurements on hadronic cross sections expected from *BABAR* and other experiments (BES III, CMD-3, SND), possibly also Belle II in the future.
  - Preliminary results, not published, by BESIII on $\pi^+\pi^-\pi^0$ consistent with BABAR [arXiv:1912.11208]

- Preliminary results on final states with **2K and 3π** have been presented
  - the very rich dynamic of the process has been explored
  - most channels are measured for the first time
  - evidence for a new decay mode of the $\phi(2170)$, and hints for a confirmation of the X(2400)
  - The article is in the final review stage and should be soon submitted for publication

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Thanks for your attention!

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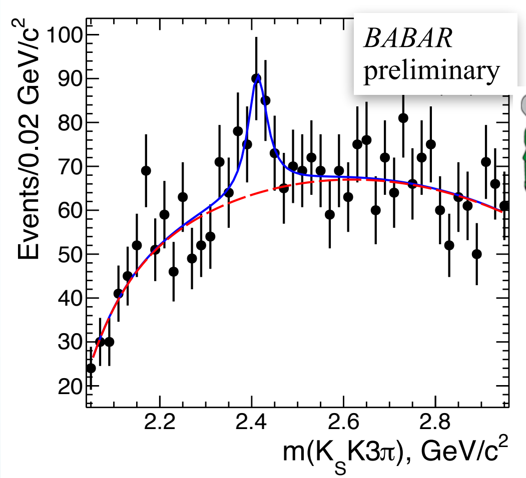
BACKUP SLIDES

Other structures? X(2400)

- An accumulation at ~2400 MeV seen particularly visible in $e^+e^- \to K_S^0 K^\pm \pi^\mp \pi^+ \pi^-$
- Visible with less statistical significance also in the other channels



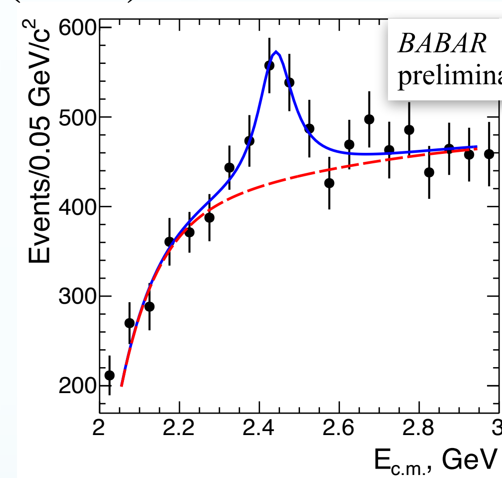
Fit with a BW and a polynomial

BABAR preliminary

$N = 108 \pm 50$ events,
$m = 2.41 \pm 0.01$ GeV/c^2,
$\Gamma = 0.051 \pm 0.027$ GeV.

Statistical significance: 2.9σ

- Adding the events from previously measured $e^+e^- \to K^+K^-\pi^+\pi^-\pi^0$ and $e^+e^- \to 2(\pi^+\pi^-)3\pi^0$ and make a similar fit:



BABAR preliminary

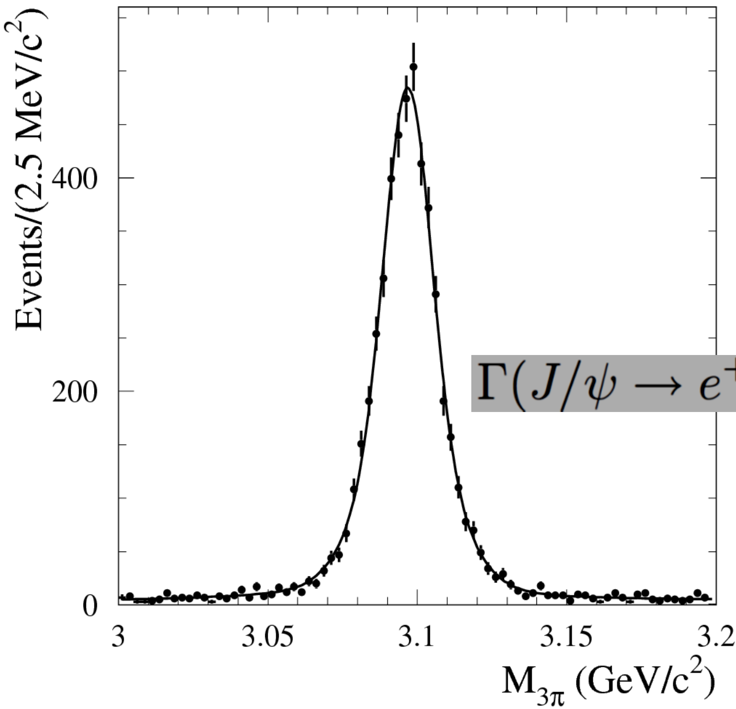
$N = 487 \pm 251$ events,
$m = 2.44 \pm 0.02$ GeV/c^2,
$\Gamma = 0.107 \pm 0.049$ GeV.

Statistical significance: 3.5σ

- Shen and Yuan [Chin.Phys.C34, 1045 (2010)] performed a fit to a structure called X(2400) using all available data from *BABAR* and Belle
- $m_X = 2.436 \pm 0.026$ GeV/c^2, $\Gamma_X = 0.121 \pm 0.035$ GeV
- Significance <3σ, could be also interpreted as threshold effect

The J/ψ region

[PRD104, 112003 (2021)]



$$\frac{d\sigma(s,\theta)}{d\cos\theta} = \frac{12\pi^2\Gamma(J/\psi\to e^+e^-)\mathcal{B}(J/\psi\to 3\pi)}{m_{J/\psi}s}W(s,x_{J/\psi},\theta)$$

$$N_{J/\psi} = 4921 \pm 74$$

$$\sigma_s^2 = 1.8 \pm 2.6\ \mathrm{MeV}^2/c^4,$$

$$M_{J/\psi} = 3.0962 \pm 0.0002\ \mathrm{GeV}/c^2 \quad -(0.7\pm0.2)\ \mathrm{MeV}/c^2$$

$$\Gamma(J/\psi\to e^+e^-)\mathcal{B}(J/\psi\to 3\pi) = (0.1248 \pm 0.0019 \pm 0.0026)\ \mathrm{keV}$$

$$\mathcal{B}(J/\psi\to 3\pi) = (2.265 \pm 0.034 \pm 0.062)\%$$

PDG $(2.10 \pm 0.08)\%$

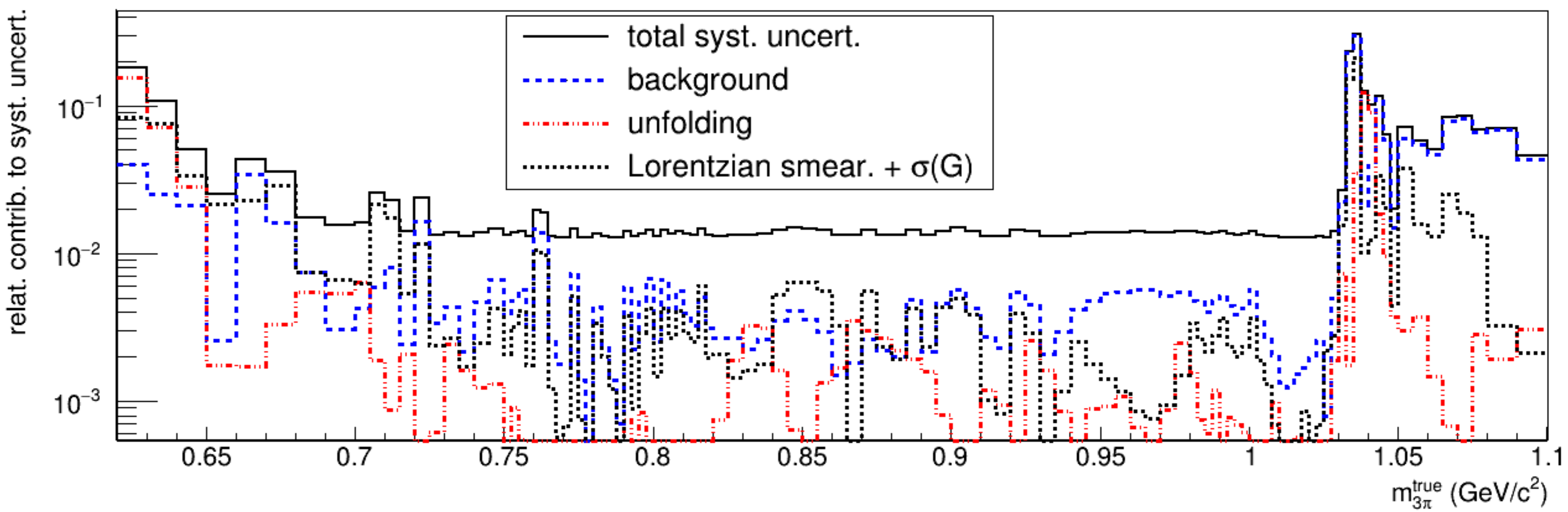
BESIII $(2.137 \pm 0.064)\%$

Systematic uncertainties on resonance parameters

TABLE VI: Contributions to the systematic errors of fit parameters from different effects ($P_1 = \Gamma(\omega \to e^+e^-)\mathcal{B}(\omega \to \pi^+\pi^-\pi^0)$, $P_2 = \Gamma(\phi \to e^+e^-)\mathcal{B}(\phi \to \pi^+\pi^-\pi^0)$, $P_3 = \mathcal{B}(\rho \to \pi^+\pi^-\pi^0)$, $P_4 = \phi_\rho$).

Effect	P_1 (%)	P_2 (%)	P_3 (%)	P_4 (deg)
Luminosity	0.4	0.4	0.4	–
Radiative correction	0.5	0.5	0.5	–
Detection efficiency	1.1	1.1	1.1	–
MC statistics	0.1	0.2	0.2	–
Lorentzian smearing	0.3	0.4	4.7	12
Γ_ω	0.4	0.2	13.0	8
Γ_ϕ	0.0	0.0	0.3	0
ϕ_ϕ	0.2	3.1	6.1	1
Background subtraction	0.1	0.2	7.3	2
$\omega(1680) \to \rho(1450)\pi$	0.4	2.7	30.0	0
total	1.4	4.3	34.5	15

Breakdown of systematic uncertainties on $\sigma(e^+e^- \to \pi^+\pi^-\pi^0)$



- Between 0.7 and 1.03 GeV the systematic uncertainty is dominated by the uncertainties in the luminosity, radiative correction and detection efficiency (1.3%) and is independent of mass.
- Below 0.65 GeV the largest contribution comes from the unfolding procedure, while above 1.03 GeV from the FSR background.

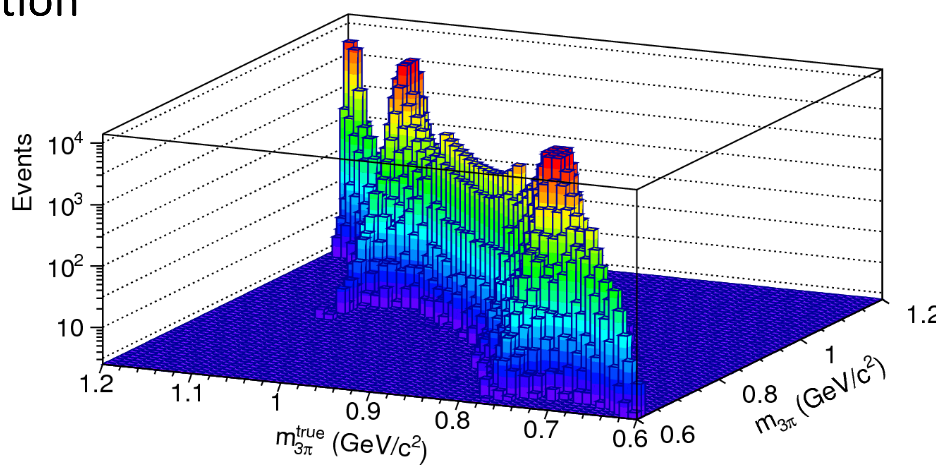
Unfolding the 3π cross section below 1.1 GeV

To obtain "true" mass spectrum, unfolding is applied to the measured $M_{3\pi}$ spectrum. Similar to the previous K^+K^- and $\pi^+\pi^-$ BABAR analyses, we use the IDS (iterative, dynamically stabilized) method developed by Bogdan Malaesku.

We reweight the signal MC simulation using the results of the fit to the measured mass spectrum and obtain the folding matrix P_{ij}. The matrix is then corrected to take into account the data-MC difference in mass resolution

$$P^*_{ij} = (1-\epsilon)\sum P_{ik}G_{kj} + \epsilon L_{ij}$$

The unfolding procedure uses the transfer matrix $\boldsymbol{A_{ij} = P^*_{ij}T_j}$, where $\boldsymbol{T_j}$ is the true spectrum obtained in the fit.



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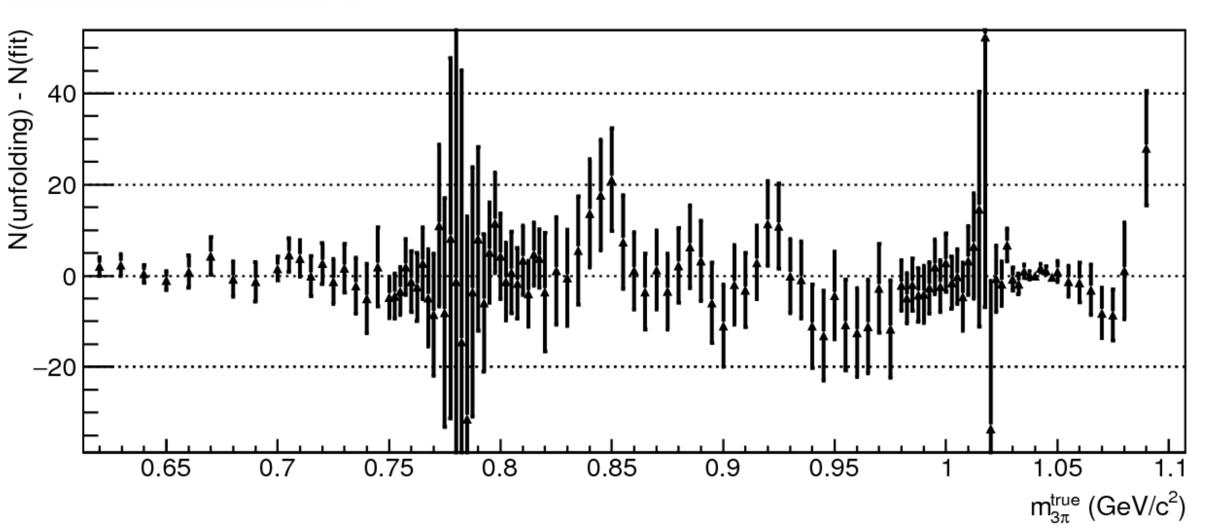
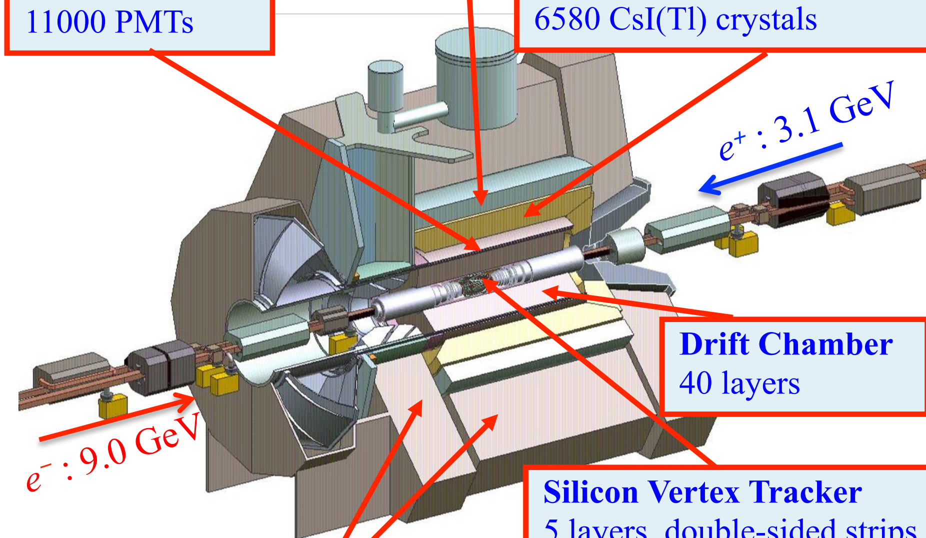


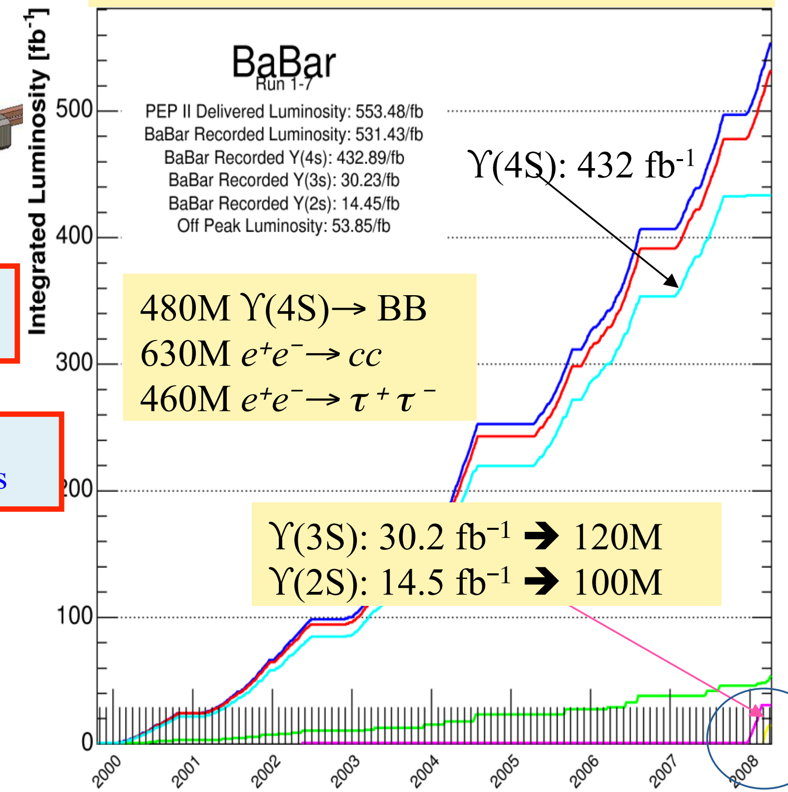
The unfolded spectrum is after the first iteration step. Further iterations do not improve the result.

Good agreement between the fit result and unfolding is seen, which confirms correctness of the model used in the fit.

BABAR detector and collected data sample



BABAR recorded luminosity and data set:



Detector details and performances in:
- NIM **A479**,1 (2002),
- NIM **A729**, 615 (2013)

Ten years of operation from 1999 to 2008
- Integrated luminosity: **~530 fb^{-1}**

ISR method in a nutshell

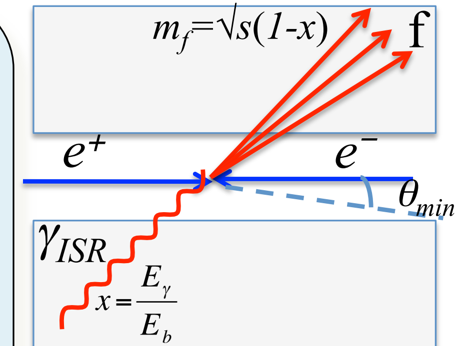
Born approximation $\dfrac{d\sigma_{e^+e^-\to f\gamma}(s,m_f,\theta^*_\gamma)}{dm\, d\cos\theta^*_\gamma} = \dfrac{2m}{s} W(s,x,\theta^*_\gamma)\cdot\sigma_{e^+e^-\to f}(m_f)$

$x = \dfrac{E_\gamma}{E_b}$ $\quad m^2 = s' = s(1\text{-}x)$

θ_γ*: ISR photon polar angle in the e^+e^- c.m.

Radiator function (at lowest order):

$$w_0(s,x,\theta^*) = \frac{\alpha}{\pi x}\left[\frac{\left(2-2x+x^2\right)\sin^2\theta^* - \frac{x^2}{2}\sin^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)^2} - \frac{4m_e^2}{s}\frac{(1-2x)\sin^2\theta^* - x^2\cos^4\theta^*}{\left(\sin^2\theta^* + \frac{4m_e^2}{s}\cos^2\theta^*\right)}\right]$$



cross section $\sigma_0(e^+e^- \to f)$

$$\sigma_0(m_i) = \frac{\Delta N(m_i)}{\Delta m}\frac{1}{\varepsilon(s,m_i)\left(1+\delta_{rad}\right)\mathrm{d}\mathcal{L}(m_i)/\mathrm{d}m}$$

reconstruction efficiency

radiative corrections

ISR luminosity

ISR differential luminosity $\dfrac{d\mathcal{L}}{dm} = \dfrac{2m}{s}\dfrac{\alpha}{\pi\cdot x}\cdot\left(\left(2-2x+x^2\right)\log\dfrac{1+C}{1-C} - x^2C\right)L_{ee}$

Luminosity integrated by the collider

$\cos\theta^*_{\min}$ Detector angular acceptance

- obtained from integration of the radiator function over θ_γ*
- $20^o < \theta_\gamma^* < 160^o$ ==> **acceptance for ISR photon ~15% in *BABAR***
- known at <1% level