Linear power corrections to e^+e^- shape variables in the three-jet region

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Part I

Introduction:
Renormalons & Power Corrections

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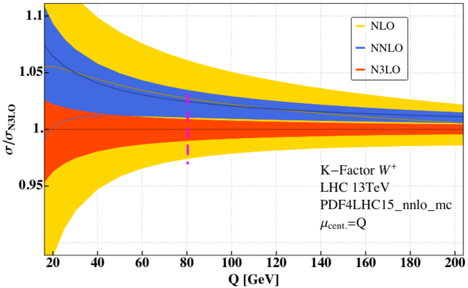

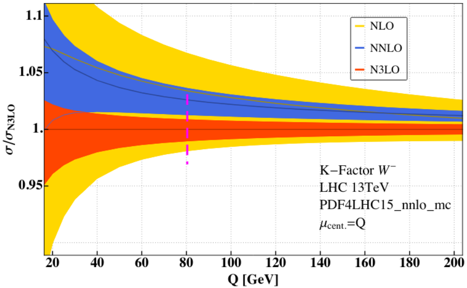

Drell-Yan N^{3}LO $\sim$ 1% correction

[Duhr, Dulat, Mistlberger; JHEP 11 (2020) 143]

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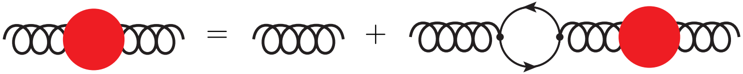
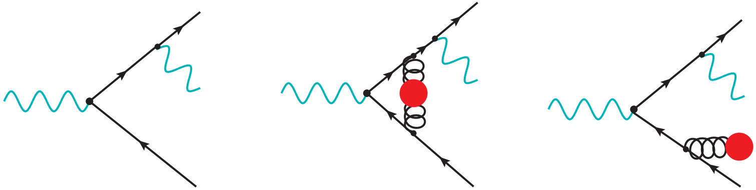
$\rightarrow$ **non-perturbative corrections may become relevant**

non-perturbative physics: Renormalons

- Renormalon model identifies simple class of diagrams that dominate in the large n_f limit
  [Beneke, Braun, Dokshitzer, Marchesini, Smye, Webber, etc.]



- example: 3-jet event $Z^*/\gamma^* \to q\bar{q}\gamma$



- each diagram can be computed perturbatively,

$$d\sigma = d\sigma^{(0)} + \left(\frac{\alpha_s}{\pi}\right) d\sigma^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 n_f \, d\sigma^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 n_f^2 \, d\sigma^{(3)} + \ldots \quad (1)$$

non-perturbative physics: Renormalons

- can resum leading-n_f contributions via integral

$$\int_0^Q dk\ k^{p-1}\alpha_s(k) = \alpha_s(Q)\,Q^p \sum_{n=0}^{\infty} \underbrace{\left(\frac{\beta_0}{2\pi}\alpha_s(Q)\right)^n \frac{1}{p^{n+1}} n!}_{\text{factorial growth}}, \tag{2}$$

$$\text{with} \quad \alpha_s(\mu) = \frac{1}{\frac{\beta_0}{2\pi}\log\frac{\mu}{\Lambda_{\mathrm{QCD}}}}, \quad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f\,. \tag{3}$$

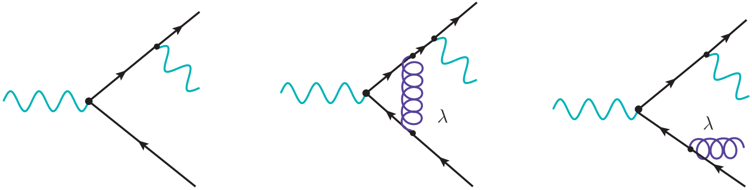
- series is not Borel summable, ambiguity given by

$$\int dk\ k^{p-1}\frac{2\pi}{\beta_0}\frac{\Lambda_{\mathrm{QCD}}}{k-\Lambda_{\mathrm{QCD}}} = \pm i\pi\frac{2\pi}{\beta_0}\Lambda_{\mathrm{QCD}}^p$$

$\rightarrow$ ambiguity removed by non-perturbative power corrections $\Lambda_{\mathrm{QCD}}^p/Q^p$

linear power corrections

- power corrections can be computed by considering perturbative corrections with massive gluon of mass λ



- direct relation between $\lambda^p \to \Lambda^p_{\mathrm{QCD}}$
- for phenomenological applications only linear terms λ/Q are relevant, higher orders in λ are surpressed by $\mathcal{O}\left(\Lambda^2_{\mathrm{QCD}}/Q^2\right)$

[S. Ferrario Ravasio, P. Nason, C. Oleari, JHEP 01 (2019) 203]

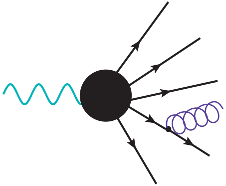
[S. Ferrario Ravasio, G. Limatola, P. Nason, JHEP 06 (2021) 018]

Part II

Linear power corrections to the C-parameter

Event shapes: The C-parameter

- event shapes describe the *geometry* of the collision (C-parameter, thrust, ...)



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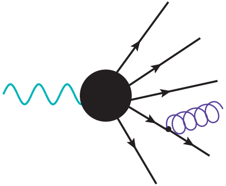

- definition of C-parameter:

$$C = 3 - 3\sum_{i>j}^{N} \frac{(p_i p_j)^2}{(p_i q)\,(p_j q)}. \tag{4}$$

p_i: momentum of particle i
q: sum of all momenta p_i
N: number of final-state particles

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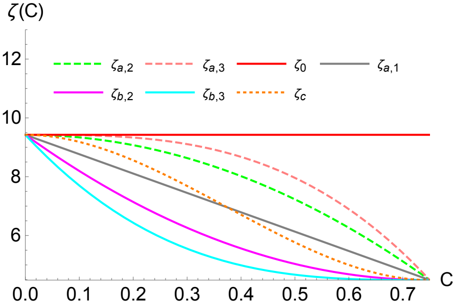

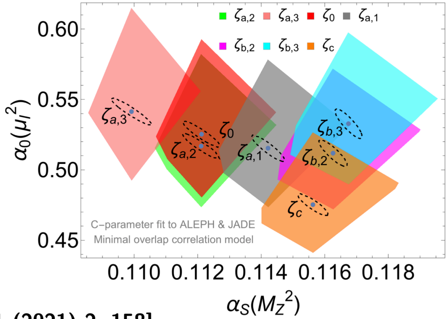

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- for α_s determination, we need **analytic** results in entire 3-jet region!

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- *virtual* corrections **do not** induce linear corrections
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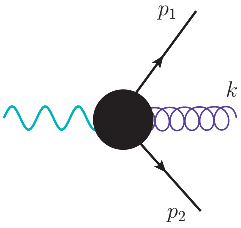
  - hard region **does not** induce linear corrections
  - soft radiation at next-to-soft approximation may lead to linear corrections

soft radiation at next-to-soft approximation



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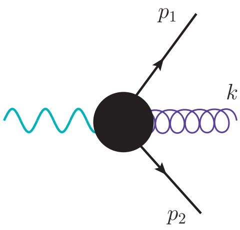

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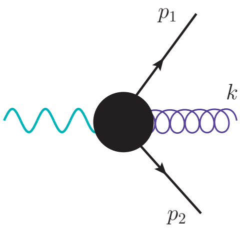

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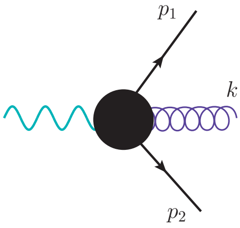

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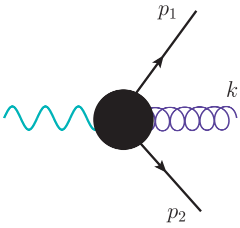

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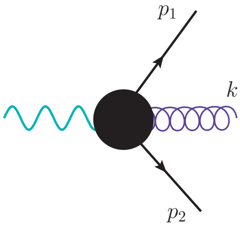

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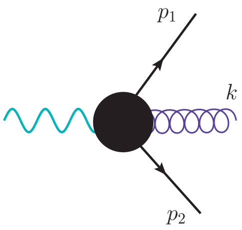

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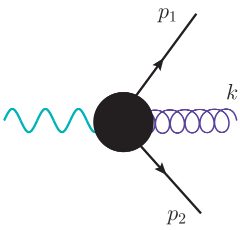

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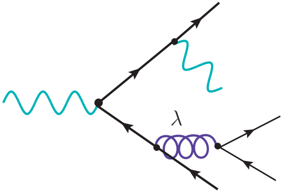
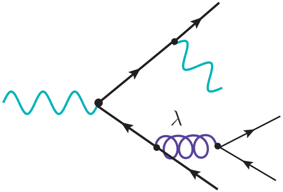
linear power corrections



event shape variables are sensitive to number of final-state particles

linear power corrections



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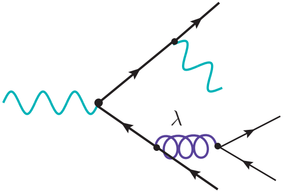
$\rightarrow$ include $g^*(k) \rightarrow q(l)\overline{q}(\bar{l})$ splitting

linear power corrections



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- can factorise out phase-space of soft partons q, $\bar{q}$

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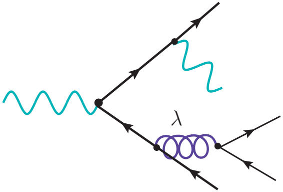

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- can factorise out phase-space of soft partons q, $\overline{q}$

  $\rightarrow$ leads to master equation:

$$\boxed{\begin{gathered} I_C(\{\tilde{p},\lambda\}) = \int [dk] \frac{J^\mu J^\nu}{\lambda^2} \theta\left(\omega_{\max} - \frac{(k\cdot q)}{\sqrt{q^2}}\right) \int [dl][d\overline{l}]\,(2\pi)^4\,\delta^{(4)}\big(k - l - \overline{l}\big) \\ \times\,\mathrm{Tr}\left[\hat{l}\gamma^\mu \hat{\overline{l}}\gamma^\nu\right]\left[C\big(\{\tilde{p}\},l,\overline{l}\big) - C(\{\tilde{p}\})\right] \end{gathered}} \tag{5}$$

linear power corrections: master equation

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- current J^μ is defined as

$$J^\mu = \frac{p_1^\mu}{p_1\cdot k} - \frac{p_2^\mu}{p_2\cdot k} \tag{7}$$

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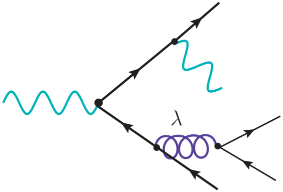
- shift in C-parameter is given by

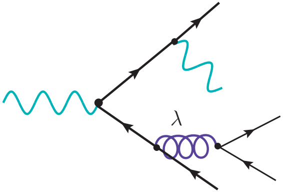
$$\Delta C = C(\{\tilde{p}\},l,\bar{l}) - C(\{\tilde{p}\}) = \sum_{i=1}^{3} \frac{(\tilde{p}_i\cdot l)^2}{(\tilde{p}_i\cdot q)\,(l\cdot q)} + (l\to\bar{l}) \tag{8}$$

linear power corrections



- integrate out quark and gluon momenta in rest frame of decaying particle

linear power corrections



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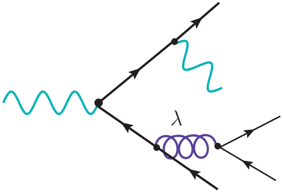
  - $\rightarrow$ one-dimensional integral over energy/velocity remaining:

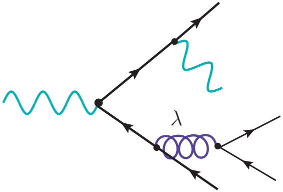
linear power corrections



- integrate out quark and gluon momenta in rest frame of decaying particle
  - $\rightarrow$ one-dimensional integral over energy/velocity remaining:

$$I_C(p_1, p_2, p_3, \lambda) = -\frac{3\lambda}{4\pi^3 q} \sum_{i=1}^{5} \int_0^{\beta_{\text{max}}} d\beta \, G_i(\beta, x, y) \tag{9}$$

linear power corrections



$$
\begin{aligned}
G_5 = {} & \frac{\sqrt{1-\beta^2}\ln\left(\frac{1+\beta}{1-\beta}\right)\ln\left(\frac{\sqrt{1-\beta^2 c_{12}^2}+\beta s_{12}}{\sqrt{1-\beta^2 c_{12}^2}-\beta s_{12}}\right)}{64\beta^8 s_{12} x(x(y-1)+1)(xy-1)\sqrt{1-\beta^2 c_{12}^2}} \\
& \times \Big(\beta^6 x\left[x^2(y-1)y + x\left(-4y^2+4y-5\right)+5\right] + \beta^4\left[x^2\left(54y^2-54y-17\right)\right. \\
& \left. - 21x^3(y-1)y + 55x - 38\right] + 5\beta^2\left[x^2\left(-24y^2+24y+5\right)\right. \\
& \left. + 11x^3(y-1)y - 17x + 12\right] - 35(x-2)\left(x^2(y-1)y + x - 1\right)\Big).
\end{aligned}
$$

- integrate out quark and gluon momenta in rest frame of decaying particle
  - $\rightarrow$ one-dimensional integral over energy/velocity remaining:

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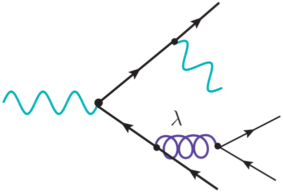

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& \times\Big(\beta^6 x\left[x^2(y-1)y+x\left(-4y^2+4y-5\right)+5\right]+\beta^4\left[x^2\left(54y^2-54y-17\right)\right. \\
& \left.-21x^3(y-1)y+55x-38\right]+5\beta^2\left[x^2\left(-24y^2+24y+5\right)\right. \\
& \left.+11x^3(y-1)y-17x+12\right]-35(x-2)\left(x^2(y-1)y+x-1\right)\Big).
\end{aligned}
$$

- integrate out quark and gluon momenta in rest frame of decaying particle
  - $\rightarrow$ one-dimensional integral over energy/velocity remaining:

$$
I_C(p_1,p_2,p_3,\lambda) = -\frac{3\lambda}{4\pi^3 q}\sum_{i=1}^{5}\int_0^{\beta_{\max}} d\beta\, G_i(\beta,x,y) \tag{9}
$$

$\rightarrow$ require class of *elliptic multiple polylogarithms* (eMPLs)

linear power corrections

- performing analytic integration leads to long expressions

linear power corrections

$$
\begin{aligned}
I_1 = &- \frac{\omega_{\max}}{\lambda}\frac{2(x-1)\left(3x^2(y-1)y+x\left(-4y^2+4y+1\right)-1\right)}{3(x(y-1)+1)^2(xy-1)^2} \\
&- \frac{1}{\beta_{\min}^5}\frac{7}{16(x(y-1)+1)^2(xy-1)^2}\left[x^4(y-1)^2y^2-2x^3y\left(y^3-2y^2+5y-4\right)\right. \\
&\left.+3x^2\left(6y^2-6y-1\right)+x\left(-10y^2+10y+6\right)-3\right] \\
&+ \frac{1}{\beta_{\min}^3}\frac{5}{288(x(y-1)+1)^2(xy-1)^2}\left[31x^4(y-1)^2y^2+2x^3y\left(-37y^3+74y^2\right.\right. \\
&\left.\left.-227y+190\right)+141x^2\left(6y^2-6y-1\right)+x\left(-466y^2+466y+282\right)-141\right] \\
&- \frac{1}{\beta_{\min}}\frac{1}{384(x(y-1)+1)^2(xy-1)^2}\left[27x^4(y-1)^2y^2+2x^3y\left(-71y^3+142y^2\right.\right. \\
&\left.-1239y+1168\right)+x^2\left(5286y^2-5286y-817\right) \\
&\left.+x\left(-2950y^2+2950y+1634\right)-817\right],
\end{aligned} \tag{B.6}
$$

$$
\begin{aligned}
I_2 = &\frac{1}{\beta_{\min}^5}\frac{7(x-1)\left(5x^3(y-1)y+x^2\left(-7y^2+7y+2\right)-3x+1\right)}{8x(x(y-1)+1)^2(xy-1)^2} \\
&- \frac{1}{\beta_{\min}^3}\frac{5\left((x-1)\left(217x^3(y-1)y+x^2\left(-299y^2+299y+82\right)-111x+29\right)\right)}{144\left(x(x(y-1)+1)^2(xy-1)^2\right)} \\
&+ \frac{1}{\beta_{\min}}\frac{(x-1)\left(4121x^3(y^2-y)-x^2\left(5875(y^2-y)-1754\right)-2895x+1141\right)}{576x(x(y-1)+1)^2(xy-1)^2} \\
&- \frac{\pi^2(x-1)\left(85x^3(y-1)y+x^2\left(-127y^2+127y+42\right)-83x+41\right)}{256\left(x(x(y-1)+1)^2(xy-1)^2\right)},
\end{aligned} \tag{B.7}
$$

$$
\begin{aligned}
I_4 = &\frac{1}{\beta_{\min}^5}\frac{7}{16x(x(y-1)+1)^2(xy-1)^2}\left[x^5(y-1)^2y^2-2x^4y\left(y^3-2y^2+10y-9\right)\right. \\
&\left.+7x^3\left(6y^2-6y-1\right)-8x^2\left(3y^2-3y-2\right)-11x+2\right] \\
&- \frac{1}{\beta_{\min}^3}\frac{5}{288\left(x(x(y-1)+1)^2(xy-1)^2\right)}\left[31x^5(y-1)^2y^2-74x^4y\left(y^2(y-2)\right.\right. \\
&\left.\left.+12y-11\right)+x^3\left(1878(y^2-y)-305\right)-x^2\left(1064(y^2-y)-668\right)-421x+58\right] \\
&+ \frac{1}{\beta_{\min}}\frac{1}{1152x(x(y-1)+1)^2(xy-1)^2}\left[81x^5(y-1)^2y^2-2x^4y\left(213y^2(y-2)\right.\right. \\
&\left.+7838y-7625\right)+x^3\left(35850(y^2-y)-5959\right)-200x^2\left(103(y^2-y)-71\right) \\
&\left.-10523x+2282\right] \\
&+ \frac{w_{\max}}{\lambda}\frac{4(1-x)\left((2x^3-3x^2)(y^2-y)+(1-x)^2\right)\left(\log\frac{\lambda}{2w_{\max}}+1\right)}{\cdots}
\end{aligned} \tag{B.8}
$$

$$
\begin{aligned}
I_5 = &- \frac{1}{\beta_{\min}^5}\frac{7(x-2)}{16x}+\frac{1}{\beta_{\min}^3}\frac{5\left(31x^3(y-1)y+x^2\left(-74y^2+74y+23\right)-81x+58\right)}{288x(x(y-1)+1)(xy-1)} \\
&+ \frac{1}{\beta_{\min}}\frac{1}{1152x(x(y-1)+1)^2(xy-1)^2}\left[-81x^5(y-1)^2y^2+2x^4y\left(213y^3-426y^2\right.\right. \\
&\left.-404y+617\right)+x^3\left(4134y^2-4134y-1057\right)+x^2\left(-2900y^2+2900y+4396\right) \\
&\left.-5621x+2282\right]-\frac{\pi^2(x-1)^2}{1024s_{12}x(x(y-1)+1)^3(xy-1)^3}\times \\
&\times\left(160x^4(y-1)^2y^2+192x^3(y-1)y+x^2\left(-68y^2+68y+37\right)+45x-82\right) \\
&+ \frac{1}{1024x(xy-1)^2(xy-x+1)^2}\left[3x^5y^4-6x^5y^3+3x^5y^2-2x^4y^4+4x^4y^3-196x^4y^2\right. \\
&\left.+194x^4y+430x^3y^2-430x^3y-37x^3-236x^2y^2+236x^2y-8x^2+127x-82\right]\times \\
&\times\left[-4\,\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ 0 & \frac{1+s_{12}}{1-s_{12}}\end{smallmatrix};1,\vec{q}\right)+4\,\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ 0 & \frac{1+s_{12}}{-1+s_{12}}\end{smallmatrix};1,\vec{q}\right)-4\,\mathrm{G}\left(0,\frac{s_{12}+1}{1-s_{12}};1\right)\right. \\
&\left.+4\,\mathrm{G}\left(0,\frac{s_{12}+1}{s_{12}-1};1\right)+\pi^2\right]+\mathrm{E}_4\left(\begin{smallmatrix}0\\ 0\end{smallmatrix};1,\vec{q}\right)\left[\frac{5\pi^2(x-1)^2\left(x^2(y-1)y+1\right)}{8s_{12}x(x(y-1)+1)^2(xy-1)^2}\frac{\eta_1}{\omega_1}\right. \\
&\left.+\frac{5\pi^2(x-1)^2\left(2x^4(y-1)^2y^2+7x^3(y-1)y+x^2\left(-8y^2+8y+3\right)-5x+2\right)}{96s_{12}x(x(y-1)+1)^3(xy-1)^3}\right] \\
&+ \left(\frac{5(x-1)^2\left(x^2(y-1)y+1\right)}{8s_{12}x(x(y-1)+1)^2(xy-1)^2}+\frac{5(x-1)^2\left(x^2(y-1)y+1\right)}{8x(x(y-1)+1)^2(xy-1)^2}\right)\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & \frac{s_{12}+1}{1-s_{12}}\end{smallmatrix};1,\vec{q}\right) \\
&+ \left(-\frac{5(x-1)^2\left(x^2(y-1)y+1\right)}{8s_{12}x(x(y-1)+1)^2(xy-1)^2}-\frac{5(x-1)^2\left(x^2(y-1)y+1\right)}{8x(x(y-1)+1)^2(xy-1)^2}\right)\times \\
&\times\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & \frac{s_{12}+1}{s_{12}-1}\end{smallmatrix};1,\vec{q}\right)+\left(\frac{5(x-1)^2\left(x^2(y-1)y+1\right)}{8s_{12}x(x(y-1)+1)^2(xy-1)^2}\right. \\
&\left.+\frac{-3x^3(y-1)y+x^2\left(2y^2-2y+37\right)-115x+78}{256x(x(y-1)+1)(xy-1)}\right)\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & -1\end{smallmatrix};1,\vec{q}\right) \\
&+\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & 1\end{smallmatrix};1,\vec{q}\right)\left(\frac{3x^3(y-1)y+x^2\left(-2y^2+2y-37\right)+115x-78}{256x(x(y-1)+1)(xy-1)}\right. \\
&\left.-\frac{5(x-1)^2\left(x^2(y-1)y+1\right)}{8s_{12}x(x(y-1)+1)^2(xy-1)^2}\right)+\frac{5(x-1)^2\left(x^2y^2-x^2y+1\right)}{16s_{12}x(xy-1)^2(xy-x+1)^2}\times \\
&\times\left[-\mathrm{E}_4\left(\begin{smallmatrix}2 & 1\\ \frac{s_{12}+1}{1-s_{12}} & \frac{s_{12}+1}{1-s_{12}}\end{smallmatrix};1,\vec{q}\right)+\mathrm{E}_4\left(\begin{smallmatrix}2 & 1\\ \frac{s_{12}+1}{1-s_{12}} & \frac{s_{12}+1}{s_{12}-1}\end{smallmatrix};1,\vec{q}\right)+\mathrm{E}_4\left(\begin{smallmatrix}2 & 1\\ \frac{s_{12}+1}{s_{12}-1} & \frac{s_{12}+1}{1-s_{12}}\end{smallmatrix};1,\vec{q}\right)\right. \\
&-\mathrm{E}_4\left(\begin{smallmatrix}2 & 1\\ \frac{s_{12}+1}{s_{12}-1} & \frac{s_{12}+1}{s_{12}-1}\end{smallmatrix};1,\vec{q}\right)+\mathrm{E}_4\left(\begin{smallmatrix}2 & 1\\ -1 & -1\end{smallmatrix};1,\vec{q}\right)-\mathrm{E}_4\left(\begin{smallmatrix}2 & 1\\ -1 & 1\end{smallmatrix};1,\vec{q}\right)-\mathrm{E}_4\left(\begin{smallmatrix}2 & 1\\ 1 & -1\end{smallmatrix};1,\vec{q}\right)
\end{aligned} \tag{B.14}
$$

$$
\begin{aligned}
I_3 = &\frac{1}{\beta_{\min}^5}\frac{7(x-2)}{16x}-\frac{1}{\beta_{\min}^3}\frac{5\left(31x^3y^2-31x^3y-74x^2y^2+74x^2y+23x^2-81x+58\right)}{288(x(xy-1)(xy-x+1))} \\
&+ \frac{1}{\beta_{\min}}\frac{1}{1152x(x(y-1)+1)^2(xy-1)^2}\left[81x^5(y-1)^2y^2+2x^4y\left(-213y^3+426y^2\right.\right. \\
&\left.+404y-617\right)+x^3\left(-4134y^2+4134y+1057\right)+4x^2\left(725y^2-725y-1099\right) \\
&\left.+5621x-2282\right]+\frac{w_{\max}}{\lambda}\frac{4}{3x}\left[\log s_{12}-\log\frac{\lambda}{2w_{\max}}-1\right] \\
&+ \frac{(x-1)^2}{512s_{12}x(xy-1)^3(xy-x+1)^3}\left(160x^4y^4-320x^4y^3+160x^4y^2+192x^3y^2\right. \\
&\left.-192x^3y-68x^2y^2+68x^2y+37x^2+45x-82\right)\left\{\frac{\pi^2}{4}+\mathrm{G}\left(0,\frac{s_{12}+1}{1-s_{12}},1\right)\right. \\
&-\mathrm{G}\left(0,\frac{s_{12}+1}{s_{12}-1},1\right)+\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & \frac{s_{12}+1}{1-s_{12}}\end{smallmatrix};1,\vec{q}\right)-\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & \frac{s_{12}+1}{s_{12}-1}\end{smallmatrix};1,\vec{q}\right) \\
&+\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ 0 & \frac{s_{12}+1}{1-s_{12}}\end{smallmatrix};1,\vec{q}\right)-\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ 0 & \frac{s_{12}+1}{s_{12}-1}\end{smallmatrix};1,\vec{q}\right)+\mathrm{E}_4\left(\begin{smallmatrix}-1 & 1\\ \infty & -1\end{smallmatrix};1,\vec{q}\right)
\end{aligned} \tag{B.1}
$$

linear power corrections

- performing analytic integration leads to long expressions
- after some non-trivial simplifications and several intermediate steps
- ...

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$\rightarrow$ we obtain a remarkable simple and compact result:

$$\boxed{\mathcal{T}_\lambda[I_C] = \frac{15}{128\pi} \frac{s_{12}^3}{1-z_3} \left(\frac{\lambda}{q}\right) \left[\frac{(1+z_3)}{2} K\left(c_{12}^2\right) - (1 - z_1 z_2)\, E\left(c_{12}^2\right)\right]} \tag{10}$$

linear power corrections

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$\rightarrow$ we obtain a remarkable simple and compact result:

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$\rightarrow$ simplicity of result calls for an explanation and suggests deeper structure

Part III

Factorisation of linear power corrections

Factorisation

$$I_C(\{\tilde{p},\lambda\}) = \int [dk] \frac{J^\mu J^\nu}{\lambda^2}\,\theta\left(\omega_{\max} - \frac{(k\cdot q)}{\sqrt{q^2}}\right)\int [dl][d\bar{l}]\,(2\pi)^4\,\delta^{(4)}\big(k-l-\bar{l}\big) \\ \times \mathrm{Tr}\left[\hat{l}\gamma^\mu\hat{\bar{l}}\gamma^\nu\right]\left[C\big(\{\tilde{p}\},l,\bar{l}\big) - C(\{\tilde{p}\})\right] \tag{11}$$

- we need to approach the computation in a different manner

Factorisation

$$I_C(\{\tilde{p},\lambda\}) = \int [dk] \frac{J^\mu J^\nu}{\lambda^2} \boxed{\theta\left(\omega_{\max} - \frac{(k\cdot q)}{\sqrt{q^2}}\right)} \int [dl][d\bar{l}]\,(2\pi)^4\,\delta^{(4)}(k - l - \bar{l})$$
$$\times\, \mathrm{Tr}\left[\hat{l}\gamma^\mu \hat{\bar{l}}\gamma^\nu\right]\left[C(\{\tilde{p}\}, l, \bar{l}) - C(\{\tilde{p}\})\right] \tag{11}$$

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- result should be independent of choice of regulator

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$$I_C(\{\tilde{p},\lambda\}) = \int [dk] \frac{J^\mu J^\nu}{\lambda^2} \cancel{\theta\left(\omega_{\max} - \frac{(k\cdot q)}{\sqrt{q^2}}\right)} \int [dl][d\bar{l}]\,(2\pi)^4\,\delta^{(4)}\big(k-l-\bar{l}\big) \times \mathrm{Tr}\Big[\hat{l}\gamma^\mu \hat{\bar{l}}\gamma^\nu\Big]\,\Big[C\big(\{\tilde{p}\},l,\bar{l}\big) - C(\{\tilde{p}\})\Big] \tag{11}$$

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- change order of integration and integrate out momentum of quark transverse to radiating dipole plane p_1, p_2

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$$\times\, \mathrm{Tr}\left[\hat{l}\gamma^\mu \hat{\bar{l}}\gamma^\nu\right]\left[C\left(\{\tilde{p}\},l,\bar{l}\right) - C(\{\tilde{p}\})\right] \tag{11}$$

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$\longrightarrow$

$$\boxed{I_C(\{\tilde{p},\lambda\}) = W_C \times \lambda\, F\left(p_1,p_2,\tilde{l}\right)} \tag{12}$$

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- rigorous derivation and generalisation of similar factor known from 2-jet limit (so-called Milan factor) **[Y. Dokshitzer, A. Lucenti, G. Marchesini, G. Salam, JHEP 05 (1998) 003]** and many more

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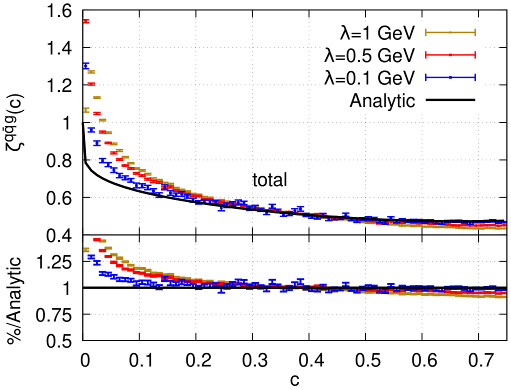
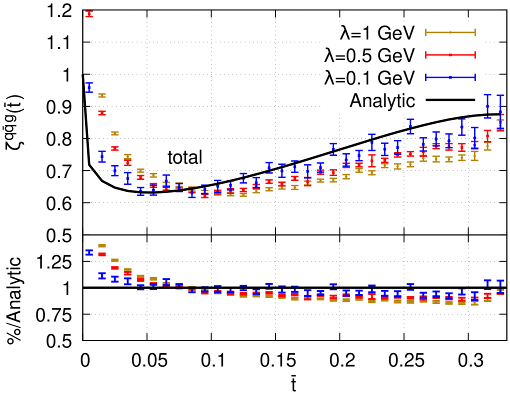
$$\boxed{W_T = \begin{cases} -\frac{1}{2\pi^3 q}\left[2E\left(n_{\mathrm{m},t}^2\right) - K\left(n_{\mathrm{m},t}^2\right)\right] & \text{if } \min\left(z_1, z_2, z_3\right) \neq z_3 \\ -\frac{n_{\mathrm{m},t}}{\pi^3 q}\left[E\left(\frac{1}{n_{\mathrm{m},t}^2}\right) - \frac{2n_{\mathrm{m},t}^2 - 1}{2n_{\mathrm{m},t}^2} K\left(\frac{1}{n_{\mathrm{m},t}^2}\right)\right] & \text{if } \min\left(z_1, z_2, z_3\right) = z_3 \end{cases}} \tag{17}$$

Part IV

Results

linear power corrections to C-parameter & thrust





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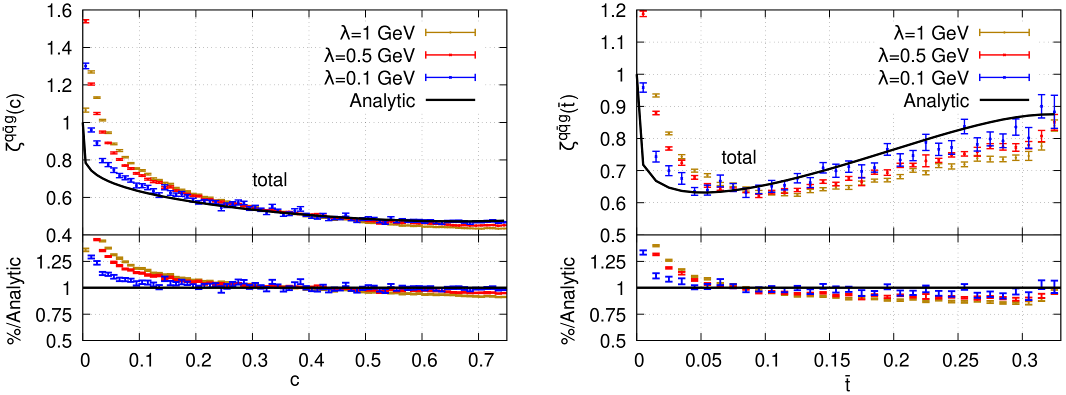

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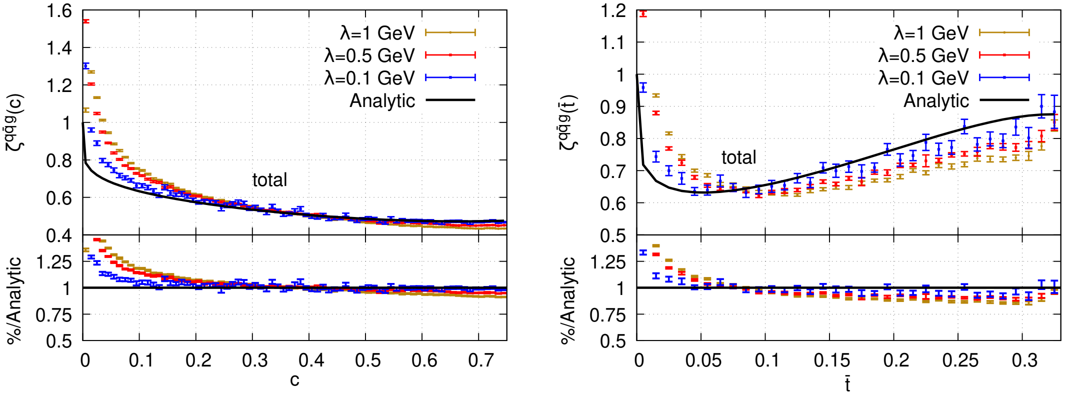

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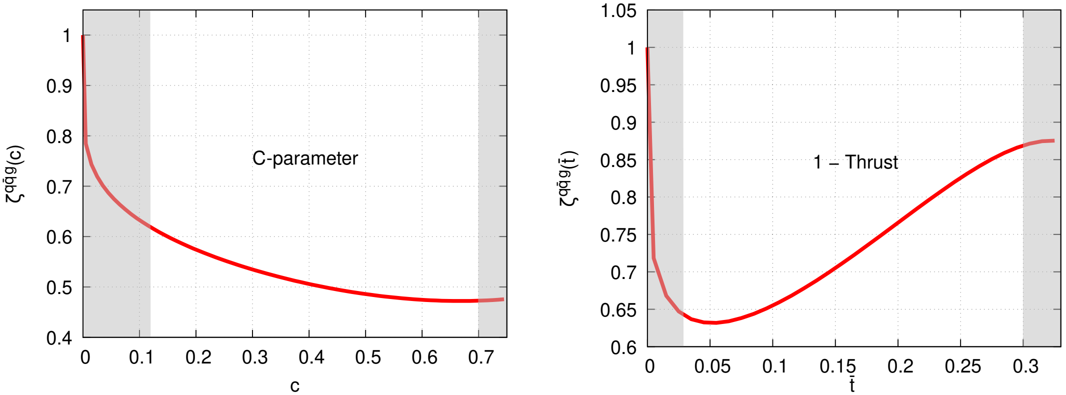

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Conclusions & Outlook

- improved understanding of analytic structure of linear power corrections
- derived a factorisation formula which allows us to easily compute linear power corrections for different observables
- shown that same universal factor appears for different observables
- computed analytically linear power corrections for C-parameter and thrust T in entire three-jet region $\rightarrow$ can now be used for pheno and α_s determination
- extended factorisation to arbitrary N-kinematics in usual Abelian approximation (Renormalons)
- future: investigate non-Abelian case
- future: pheno applications (α_s and m_t determinations)

Thank you for attention!