# Studying light flavour resonances with polarised photon beams

Nadine Hammoud

Institute of Nuclear Physics, Polish Academy of Sciences, Kraków, Poland

Supervisors: dr hab. Robert Kamiński, dr inż Łukasz Bibrzycki

International Conference on High Energy Physics
July 9, 2022

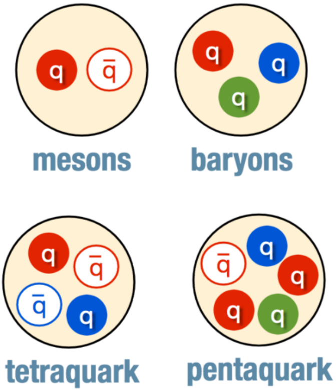
# Outline

- Introduction
- Motivation
- Double pion photoproduction:
  - Methodology
    - Deck Mechanism
    - pion-proton Scattering
    - Analysis of A and B amplitudes
  - differential cross Section
  - Moments
- Summary
- Current Work

# Introduction-Ordinary and Exotic Hadrons

According to quark models, quarks can be organized as triplets to form **Baryons** and $q\bar{q}$ pairs to form **Mesons**.

➜ QCD also predict that other kind of resonances could be formed i.e. multi-quarks configurations such as tetraquarks and pentaquarks.

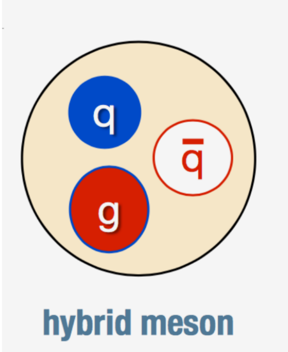


# Introduction- Hybrid Mesons

- Certain sets of quantum numbers cannot be formed from a quark and antiquark pair, such as:

$$J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}...$$

- These quantum numbers can be reached if excitations of the gluonic fields contribute.
- These mesons are known as hybrid mesons.



# Motivation and research problem

Searching for exotic mesons allows us to:

- confirm the theory of quantum chromodynamics,
- gain a better understanding of the fundamental quark-antiquark interactions,
- understand the role of gluons and the origin of confinement,nucleon mass.etc..

Hybrids can be photproduced through $\gamma + p \rightarrow \eta\pi + p$ at JLab, and due to the fact that there are a lot of experimental data for the double pion photoproduction.

➡ which makes studying such process easier and gives us a better understanding of the photoproduction processes.

**Deck Mechanism :**

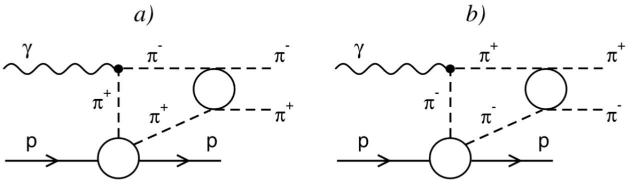



Figure: Diagrams for the pion photoproduction (Deck Mechanism)

$\epsilon_\lambda$ : photon polarization vector,
$q$: photon momentum vector,
$p_1, p_2$ : momentum vector of initial and recoiling proton respectively,
$k_1$,$k_2$ : momentum vector of $\pi^+$ and $\pi^-$ respectively.

## Deck Mechanism :

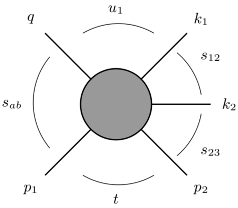



Figure: Relevant Mandelstam variables

$$s = (p_1 + q)^2\,, \tag{1}$$
$$t = (p_1 - p_2)^2\,, \tag{2}$$
$$s_{12} = (k_1 + k_2)^2\,, \tag{3}$$
$$s_{23} = (k_2 + p_2)^2\,, \tag{4}$$
$$s_{ab} = (q + p_2)^2\,. \tag{5}$$

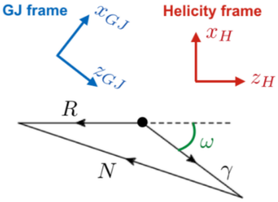
**Deck Mechanism :**
Our photoproduction amplitude can be written as:

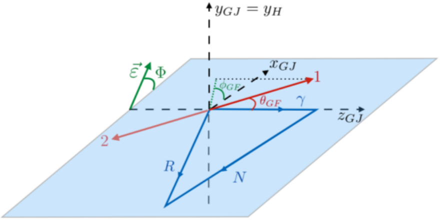
$$M = e\left[\frac{\epsilon_\lambda.\boldsymbol{k_1}}{\boldsymbol{q}.\boldsymbol{k_1}}T^- - \frac{\epsilon_\lambda.\boldsymbol{k_2}}{\boldsymbol{q}.\boldsymbol{k_2}}T^+ + \frac{\epsilon_\lambda.(\boldsymbol{p_1}+\boldsymbol{p_2})}{\boldsymbol{q}.(\boldsymbol{p_1}+\boldsymbol{p_2})}(T^+ - T^-)\right], \tag{6}$$

where $T^+$ corresponds for $\pi^+ p \rightarrow \pi^- p$ while $T^-$ represents $\pi^- p \rightarrow \pi^+ p$.
**Note** that $M = 0$ when replacing $\epsilon_\lambda$ by $q$. This means that our amplitude is **Guage inveariant**.

# Methodology

## Frames:




$$\boldsymbol{p}_{\mathbf{1}}^{H} = |\vec{p_1}|(\sin\theta_1, 0, \cos\theta_1; E_1)\ ;\ \boldsymbol{p}_{\mathbf{1}}^{GJ} = |\vec{p_1}|(-\sin\theta_1, 0, \cos\theta_1; E_1) \quad (7)$$

$$\boldsymbol{p}_{\mathbf{2}}^{H} = |\vec{p_2}|(0, 0, -1; E_2)\ ;\ \boldsymbol{p}_{\mathbf{2}}^{GJ} = |\vec{p_2}|(-\sin\theta_2, 0, \cos\theta_2; E_2) \quad (8)$$

$$\boldsymbol{q}^{H} = |\vec{q}|(-\sin\theta_q, 0, \cos\theta_q; E_q)\ ;\ \boldsymbol{q}^{GJ} = |\vec{q}|(0, 0, 1; E_q) \quad (9)$$

$$\boldsymbol{k}_{\mathbf{1}}^{H} = \boldsymbol{k}_{\mathbf{1}}^{GJ} = |\vec{k_1}|(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta; \omega) \quad (10)$$

$$\boldsymbol{k}_{\mathbf{2}}^{H} = \boldsymbol{k}_{\mathbf{2}}^{GJ} = -\boldsymbol{k}_{\mathbf{1}}^{H} = -\boldsymbol{k}_{\mathbf{1}}^{GJ} \quad (11)$$

Pion-proton scattering:

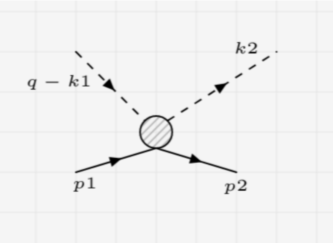



Figure: Feynman diagram for $\pi^- p \to \pi^- p$

Assuming that the intermediate pion is offshell, then the pion-proton scattering amplitude will read:

$$T_\lambda^- = \bar{u}_\lambda(p_2)\left[A^- + \frac{1}{2}\gamma_\mu(q - k_1 + k_2)^\mu B^-\right]u_\lambda(p_1), \tag{12}$$

# Methodology

Pion-proton scattering:

Similarly for the positive exchanged pion:

$$T_\lambda^+ = \bar{u}_\lambda(p_2)\left[A^+ + \frac{1}{2}\gamma_\mu(q-k_2+k_1)^\mu B^+\right]u_\lambda(p_1), \tag{13}$$

Where $A$ and $B$ are scalar functions, $u_\lambda(p_1)$ and $u_\lambda(p_2)$ are the spinors of the incoming and outgoing protons respectively.

In the $\pi N$ center of mass frame the t-channel $A$ and $B$ defined as follows:

$$\frac{1}{4\pi}A^\pm = \frac{\sqrt{s}+m_p}{Z_1^+ Z_2^+}f_1^\pm - \frac{\sqrt{s}-m_p}{Z_1^- Z_2^-}f_2^\pm, \tag{14}$$

$$\frac{1}{4\pi}B^\pm = \frac{1}{Z_1^+ Z_2^+}f_1^\pm - \frac{1}{Z_1^- Z_2^-}f_2^\pm. \tag{15}$$

Where $f_1$ and $f_2$ are called the reduced helicity amplitudes, $Z_i^{\pm} = \sqrt{E_i \pm m_p}$. The partial wave decomposition:

$$f_1 = \frac{1}{\sqrt{|\boldsymbol{p}_1||\boldsymbol{p}_2|}} \sum_{l=0}^{\infty} f_{l+}(s) P'_{l+1}(\cos\theta) - \frac{1}{\sqrt{|\boldsymbol{p}_1||\boldsymbol{p}_2|}} \sum_{l=2}^{\infty} f_{l-}(s) P'_{l-1}(\cos\theta), \tag{16}$$

$$f_2 = \frac{1}{\sqrt{|\boldsymbol{p}_1||\boldsymbol{p}_2|}} \sum_{l=1}^{\infty} [f_{l-}(s) - f_{l+}(s)] P'_l(\cos\theta). \tag{17}$$

# Analysis of A and B amplitudes

**Open Question:**

Dealing with an onshell pions, it's known that:

- $P_l(\cos\theta)$ diverge like $\cos\theta^l \sim 1/(q^2)^l$ for large $\cos\theta$.
- It is expected that our threshold will be finite, i.e. the $\cos\theta^l$ found in the Legendre polynomial is cancelled by a similar factor of $(q^2)^l$ in the amplitudes $f^{\pm}_{l_\pm}$.

**➜ Do we need an additional factor to ensure this cancelation continues in the offshell case?**

**➜ What is the correct momentum dependence of such factor?**

We tried the following two options:

$$\mathrm{R}_1 = \Big(\frac{|P_{\text{off}}|}{|P_{\text{on}}|}\Big)^l \tag{18}$$

$$\mathrm{R}_2 = \Big(\frac{|P_{\text{off}}|}{|P_{\text{on}}|}\Big)^{l+\frac{1}{2}}. \tag{19}$$

# Threshold Correction

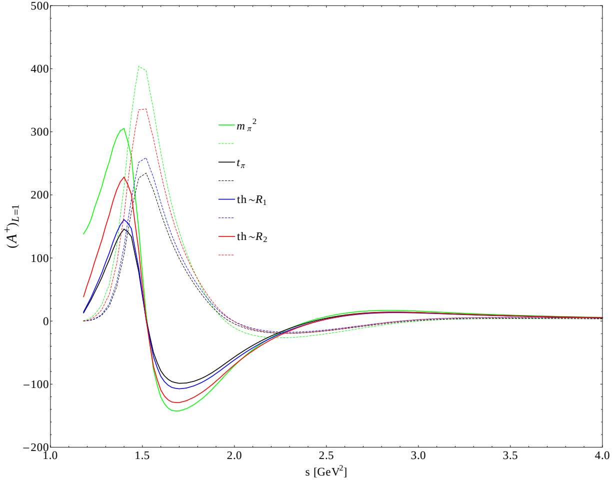



Figure: $A^+$ amplitude for $(L = 1)$ with onshell pion, offshell pion of virtuality $t_\pi = -0.1GeV^2$ in all cases i.e. with and without the different factor options; solid lines: Real, dashed lines: Imaginary.

# Differential Cross Section

The differential cross section is given as:

$$\frac{d^5\sigma}{dtd\sqrt{s_{12}}d\Omega d\Phi} = I(\Omega,\Phi) = I^0(\Omega) + \boldsymbol{I}(\Omega)\cdot P_\gamma(\Phi) \tag{20}$$

where $\Omega = (\theta,\phi)$ and the intensity vector is defined as:

$$I^0(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \mathcal{M}_{\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \tag{21}$$

$$I^1(\Omega) = \frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \mathcal{M}_{-\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \tag{22}$$

$$I^2(\Omega) = i\frac{\kappa}{2}\sum_{\lambda,\lambda_1,\lambda_2} \lambda\mathcal{M}_{-\lambda;\lambda_1\lambda_2}(\Omega)\mathcal{M}^*_{\lambda;\lambda_1\lambda_2}(\Omega) \tag{23}$$

where $0 < P_\gamma < 1$ is the degree of linear polarization of the photon beam.

The phase space factor $\kappa$ is:

$$\kappa = \frac{1}{(2\pi)^3} \frac{1}{4\pi} \frac{1}{2\pi} \frac{\lambda^{1/2}\left(m_{\pi\pi}^2, m_\pi^2, m_\pi^2\right)^\star}{16 m_{\pi\pi}\left(s - m_p^2\right)^2} \frac{1}{2}. \tag{24}$$

The Moments of angular distributions can be defined as:

$$H^0(L,M) = \frac{1}{2\pi} \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi I(\Omega,\Phi) d^L_{M0}(\theta) \cos M\phi \tag{25}$$

$$H^1(L,M) = \frac{1}{\pi P_\gamma} \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi I(\Omega,\Phi) d^L_{M0}(\theta) \cos M\phi \cos 2\Phi \tag{26}$$

$^\star \lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$

# Results-Moments

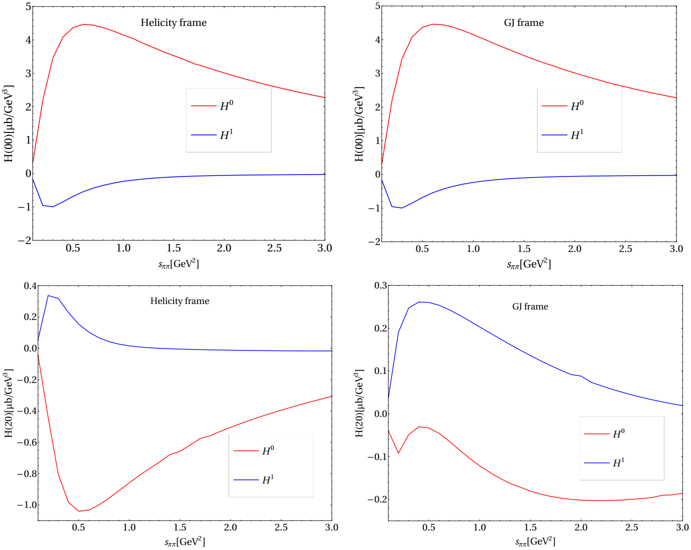



Figure: $\pi^+\pi^-$ Moments with $E_\gamma = 8.5GeV^2$ in both frames.

# Summary

- Search for hybrid mesons gives a greater insight into how quarks and gluons bind to form such states and hence increase our understanding of the fundamental strong force. gluon and origin of confinement which is crucial to complete our picture of strong interactions.
- We desribed the methodology used in studying double pion photoproduction process and computing the differential cross section as well as the Moments of $\pi^+\pi^-$ angular distributions for different $LM$ in two frames.
- It is still an open debate if we really need to include a correction factor to our partial waves for virtual pion-proton scattering, and what is the correct momentum dependence that we have to use if this is the case?

# Current Work

In the meantime we are working on adding the $P$ wave(Pomeron and $f_2$ exchange) amplitude to the photoproduction amplitude. †
The $P$ wave amplitude can be defined as:

$$A_{\lambda,\lambda_1,\lambda_2} = \frac{1}{s} g\beta^{\gamma\rho} e^{\beta} BW(m_{\pi\pi}) R(s,t) \bar{u}(p_2,\lambda_2)\gamma^{\alpha} u(p_1,\lambda_1) v^{\lambda}_{\alpha}. \tag{27}$$

Where $v^{\lambda}_{\alpha} = k_{\alpha}\epsilon^{\lambda}.(k_1 - k_2) - q.(k_1 - k_2)\epsilon^{\lambda}_{\alpha}$, and $k = k_1 + k_2$. The Breit-Wigner and Regge propagator are given by:

$$BW(m_{\pi\pi}) = \frac{1}{m_{\rho}^2 - m_{\pi\pi}^2 - i m_{\rho}\Gamma_{\rho}} \tag{28}$$

$$R(s,t) = \frac{1 + e^{i\pi\alpha(t)}}{\sin(\pi\alpha(t))}\left(\frac{s}{s_0}\right)^{\alpha(t)} \quad \alpha_P(t) = 1.08 + 0.2t \quad \alpha_{f_2}(t) = 0.5 + 0.9t \tag{29}$$

We used the following constants:‡

$$g = 5.96 \quad \beta^{\gamma\rho}_{P} = 2.506 \quad \beta_P = 3.6 \quad \beta^{\gamma\rho}_{f_2} = 2.476 \quad \beta_{f_2} = 0.55 \quad s_0 = 1 GeV^2$$

† arXiv:hep-ph/0304007


Thank you

# BACKUP SLIDES

# Threshold Correction

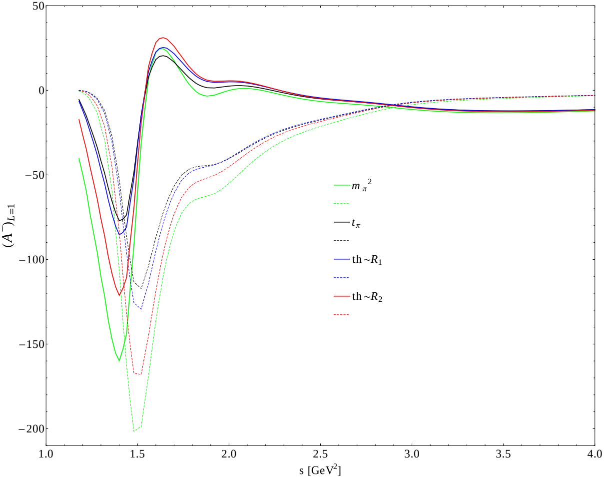



Figure: $A^-$ amplitude for $(L = 1)$ with onshell pion, offshell pion of virtuality $t_\pi = -0.1GeV^2$ in all cases i.e. with and without the different factor options; solid lines: Real, dashed lines: Imaginary.

# Results-Moments

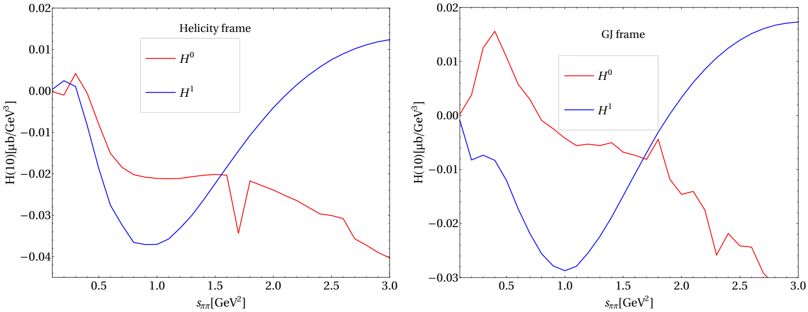



Figure: Moment of the $\pi^+ \pi^-$ angular distribution with $E_\gamma = 8.5GeV^2$ for $LM = (10)$ in both frames where $H^0$ is in red while $H^1$ is in blue .

# Results-Moments

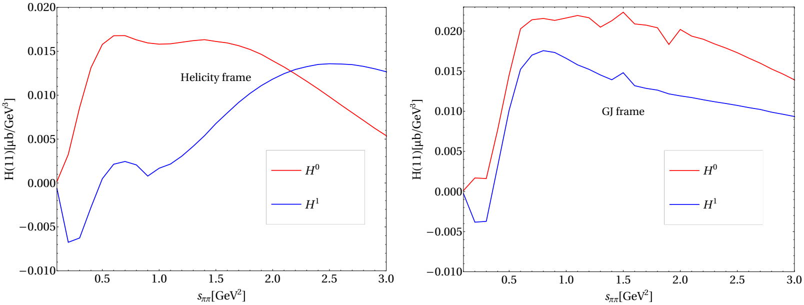



Figure: Moment of the $\pi^+\ \pi^-$ angular distribution with $E_\gamma = 8.5GeV^2$ for $LM = (11)$ in both frames where $H^0$ is in red while $H^1$ is in blue .

# Results-Moments

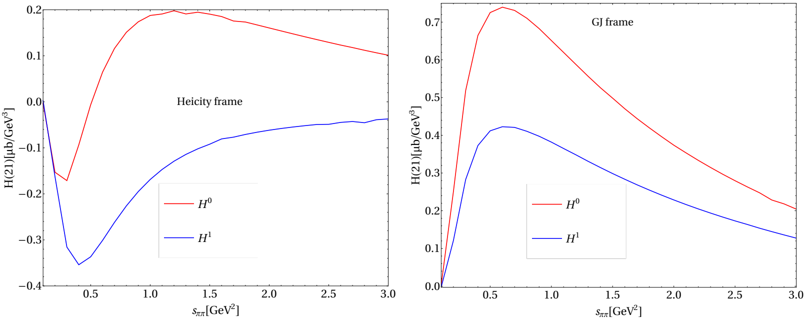



Figure: Moment of the $\pi^+ \ \pi^-$ angular distribution with $E_\gamma = 8.5GeV^2$ for $LM = (21)$ in both frames where $H^0$ is in red while $H^1$ is in blue .

# Results-Moments

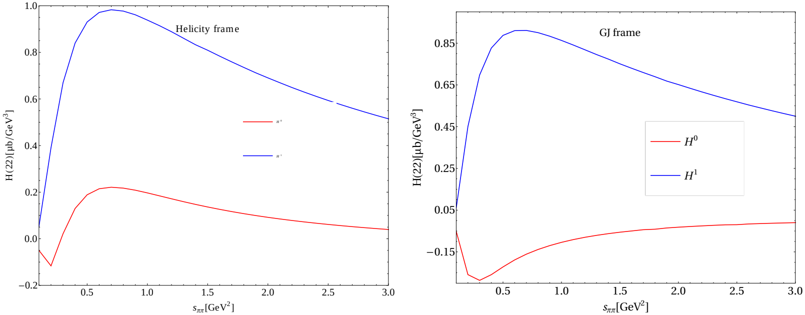



Figure: Moment of the $\pi^+ \pi^-$ angular distribution with $E_\gamma = 8.5GeV^2$ for$LM = (22)$ in both frames where $H^0$ is in red while $H^1$ is in blue .