



HERAPDF2.0 NNLOJets

Parton Distributions

A M Cooper-Sarkar
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HERAPDF2.0 was a Parton Distribution Function analysis based on HERA data alone using the H1 and ZEUS combined Neutral and Charged Current cross-sections from e^+ and e^- proton scattering. arXIV:1506.06042

A QCDfit for PDFs was done at LO, NLO and NNLO.

HERAPDF2.0 Jets extended this to use HERA charm and beauty data and HERA jets data, but this could only be done at NNLO because there were no NNLO DIS jet predictions.

Now updating HERAPDF2.0JetsNLO with NNLO predictions for jets from NNLOJET as implemented in the ApplFast grid system arXIV:2112.01120

New PDFs at NNLO are presented at $\alpha_s(M_Z) = 0.118$ (PDG value) and 0.1155

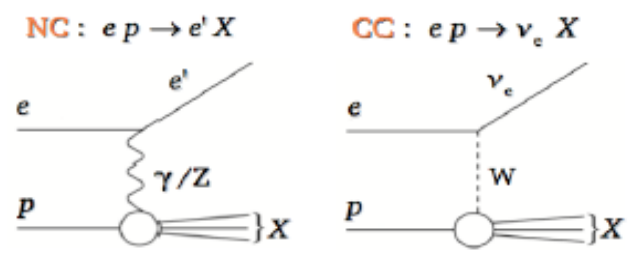
The lower value is used because $\alpha_s(M_Z)$ at NNLO is significantly lower than at NLO

A simultaneous PDF and $\alpha_s(M_Z)$ fit including the Jet data allow us to constrain $\alpha_s(M_Z)$

Free $\alpha_s(M_Z)$ fit at NNLO

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011_{(\text{exp})} {}^{+0.0001}_{-0.0002(\text{model/param})} \pm 0.0029_{(\text{scale})}$$

Deep Inelastic Scattering (DIS) is the best tool to probe proton structure



o Kinematic variables:

$Q^2 = -q^2 = -(k - k')^2$
Virtuality of the exchanged boson

$x = \frac{Q^2}{2p \cdot q}$ Bjorken scaling parameter

$y = \frac{p \cdot q}{p \cdot k}$ Inelasticity parameter

$s = (k + p)^2 = \frac{Q^2}{xy}$ Invariant c.o.m.

Neutral current:

$$\frac{d^2 \sigma_{NC}^{\pm}}{dx dQ^2} = \frac{2 \alpha \pi^2}{x Q^4} (Y_+ F_2 \mp Y_- x F_3 - y^2 F_L)$$

$F_2 \propto \sum_i e_i^2 (x q_i + x \bar{q}_i)$
quark distributions

$x F_3 \propto \sum_i (x q_i - x \bar{q}_i)$
valence quarks

$F_L \propto \alpha_s \times g$
gluon at NLO

LO expressions

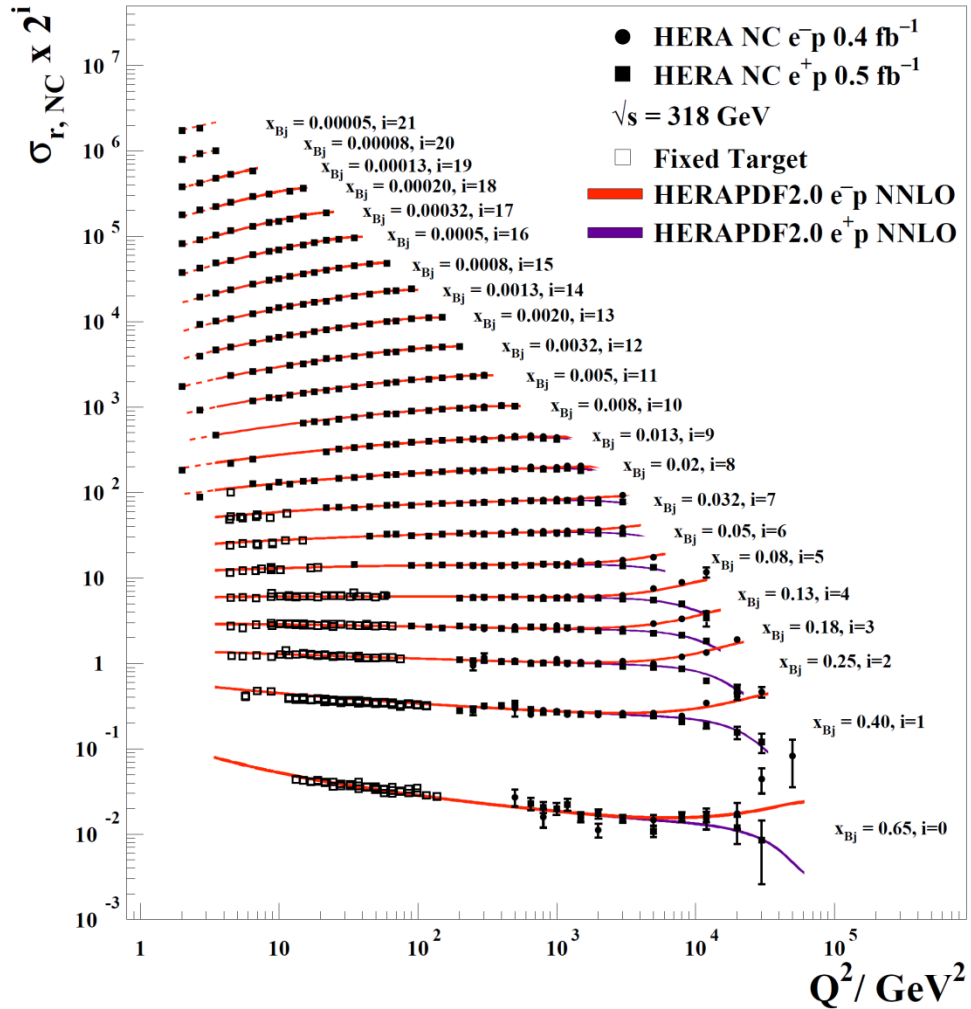
Charged current:

$$\frac{d^2 \sigma_{CC}^-}{dx dQ^2} = \frac{G_F^2}{2\pi} \frac{M_W^2}{M_W^2 + Q^2} (u + c + (1 - y^2)(\bar{d} + \bar{s}))$$

flavour decomposition

$$\frac{d^2 \sigma_{CC}^+}{dx dQ^2} = \frac{G_F^2}{2\pi} \frac{M_W^2}{M_W^2 + Q^2} (\bar{u} + \bar{c} + (1 - y^2)(d + s))$$

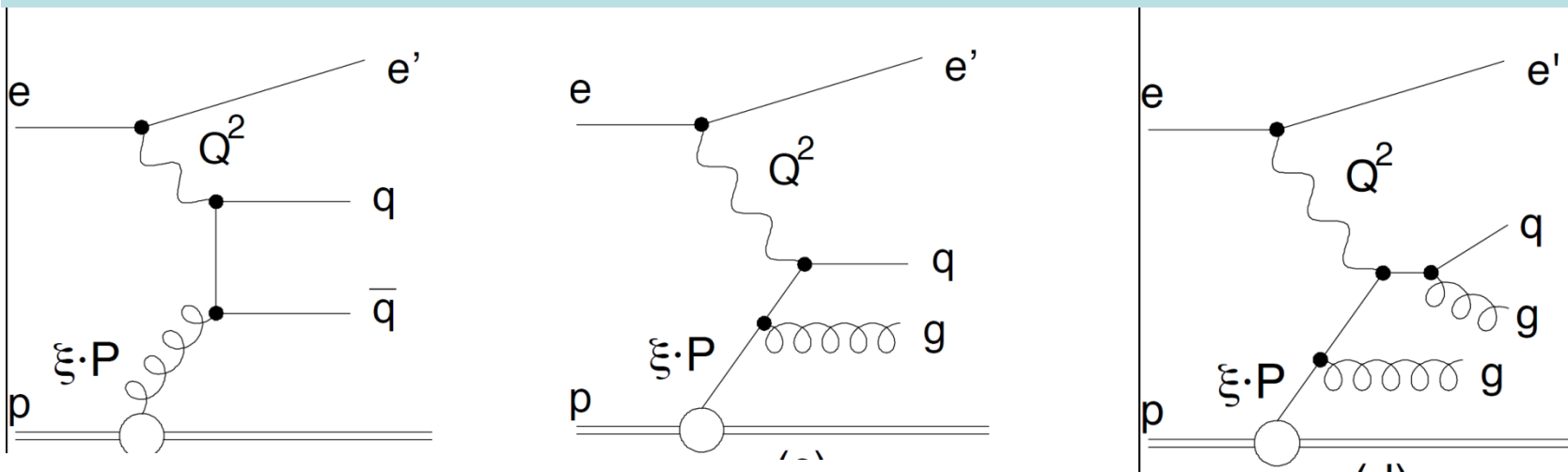
H1 and ZEUS



Gluon from the scaling violations: DGLAP equations tell us how the partons evolve

QCD fit starts at minimum $Q^2 = 3.5 \text{ GeV}^2$

Adding more data to HERAPDF2.0: jet data



It is well known that jet data give a direct handle on the gluon PDF and can be used to measure $\alpha_s(M_Z)$

Data Set	taken from to	$Q^2[\text{GeV}^2]$ range from to	\mathcal{L} pb^{-1}	e^+/e^-	\sqrt{s} GeV	normalised	all points	used points	Ref.
H1 HERA I normalised jets	1999 – 2000	150 15000	65.4	e^+p	319	yes	24	24	[9]
H1 HERA I jets at low Q^2	1999 – 2000	5 100	43.5	e^+p	319	no	28	20	[10]
H1 normalised inclusive jets at high Q^2	2003 – 2007	150 15000	351	e^+p/e^-p	319	yes	30	30	[13,14]
H1 normalised dijets at high Q^2	2003 – 2007	150 15000	351	e^+p/e^-p	319	yes	24	24	[14]
H1 normalised inclusive jets at low Q^2	2005 – 2007	5.5 80	290	e^+p/e^-p	319	yes	48	37	[13]
H1 normalised dijets at low Q^2	2005 – 2007	5.5 80	290	e^+p/e^-p	319	yes	48	37	[13]
ZEUS inclusive jets	1996 – 1997	125 10000	38.6	e^+p	301	no	30	30	[11]
ZEUS dijets	1998 – 2000 & 2004 – 2007	125 20000	374	e^+p/e^-p	318	no	22	16	[12]

These data sets are new and were not used in the 2015 NLO analysis. Low Q^2 jet data are particularly sensitive to $\alpha_s(M_Z)$

However as well as adding new data sets we have subtracted some data

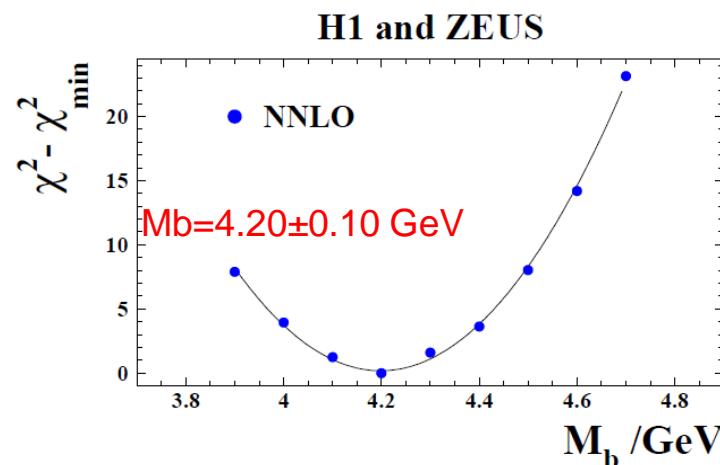
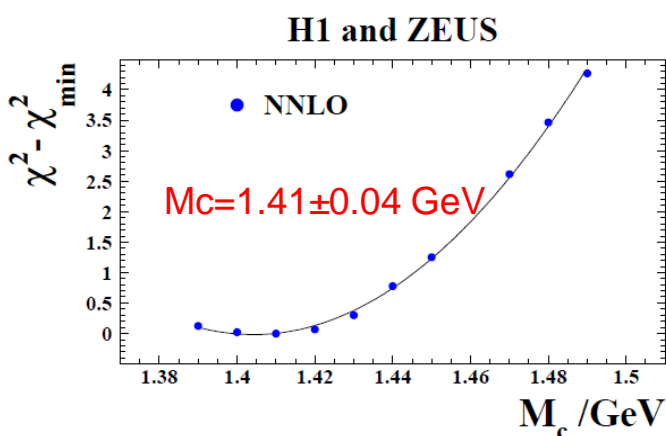
- Trijets- there are no NNLO predictions
- Data at low scale $\mu = (\text{pt}^2 + Q^2) < 10 \text{ GeV}$ for which scale variations are large (~25% NLO and ~10% NNLO)
- 6 ZEUS Dijet data points at low pt for which predictions are not truly NNLO

Adding more data to HERAPDF2.0: heavy flavour data

Since the publication of HERAPDF2.0 we also have NEW HERA combined charm and beauty data arXiv:2018.01019

This affects the evaluation of the optimal charm and beauty masses

Their 1σ variations are considered as model uncertainties



HERAPDF specifications: parameterisation at starting scale $\mu_{f0}^2 = 1.9\text{GeV}^2$

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

$$xg(x) = A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g},$$

$$xu_v(x) = A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}} \left(1 + E_{u_v} x^2\right),$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}},$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x),$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}. \quad \bar{D} = d + s$$

$$\bar{s} = fs. \quad \bar{D}$$

As usual we start with a minimal number of parameters and add more one at a time until the χ^2 no longer improves. 14 parameters

Parametrisation variations, adding extra parameters which can change PDF shape but do not improve χ^2 are part of the uncertainty.

A_{u_v}, A_{d_v}, A_g , from the number and momentum sum-rules
 $B_{u_{\text{bar}}}, A_{u_{\text{bar}}}$ constrained to produce $d_{\text{bar}} = u_{\text{bar}}$ at low- x .
Strangeness fraction $f_s = 0.4$ fixed, but varied as a model uncertainty

HERAPDF specifications: scale choice, hadronisation corrections, theoretical uncertainties

Factorisation scale

At NLO we used factorisation scale = Q^2 but this is not a good choice for low Q^2 jets, we have many more low Q^2 jet data points now – from the H1 2016 data- so we move to a choice factorisation scale = $(Q^2 + p_{t^2})$ for all jets- this makes almost no difference to high Q^2 jets

Renormalisation scale

For HERAPDF2.0Jets NLO we chose renormalisation = $(Q^2 + p_{t^2})/2$

For HERAPDF2.0Jets NNLO jets a choice of renormalisation = $(Q^2 + p_{t^2})$

Results in a lower χ^2 , $\Delta\chi^2 \sim -15$

In fact the ‘optimal’ scale choice for NLO and NNLO is different – if optimal is defined by lower χ^2 . At NLO $\Delta\chi^2 \sim -15$ for the old scale choice.

We will also explore the consequences of scale variation.

When jets are included the data are subject to hadronisation corrections. The uncertainties on these corrections are included along with the experimental systematic uncertainties. They are treated as 50% correlated and 50% uncorrelated between bins and data sets.

There are also (small) uncertainties on the theoretical predictions these are also applied 50% correlated and 50% uncorrelated as systematic uncertainties

Summary: model and parameterisation uncertainties

Model: Variation of input assumptions

Parameter	Central value	Downwards variation	Upwards variation
Q_{\min}^2 [GeV ²]	3.5	2.5	5.0
f_s	0.4	0.3	0.5
M_c [GeV]	1.41	1.37*	1.45
M_b [GeV]	4.20	4.10	4.30
μ_{f0}^2 [GeV ²]	1.9	1.6	2.2*

We require $\mu_{f0}^2 < M_c^2$ to generate charm perturbatively, hence the * (down/up) variations are not possible, thus the corresponding (up/down) variation is taken and symmetrised

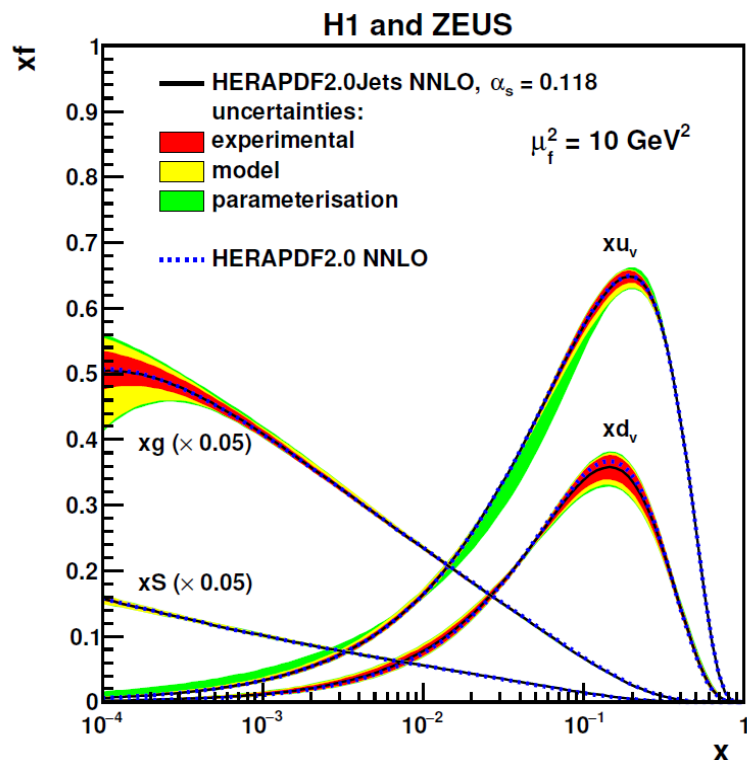
Parametrisation

Variation of $Q_0^2 = 1.9 \pm 0.3$ GeV² as well as addition of 15th D, E parameter(s)

Investigation of the effect of the negative gluon term

We determine new PDFs: HERAPDF2.0Jets NNLO

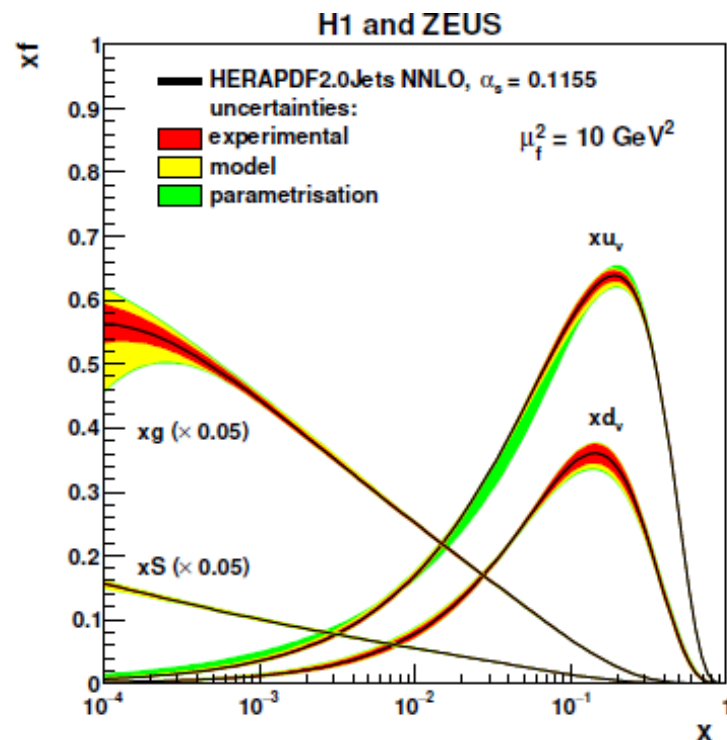
Compare older HERAPDF2.0 NNLO to
HERAPDF2.0Jets NNLO both with
 $\alpha_s(M_Z) = 0.118$



$\chi^2 = 1617$ for fixed $\alpha_s(M_Z) = 0.118$
1363 data points, 1349 degrees of
freedom,
 $\chi^2/\text{d.o.f} = 1.199$

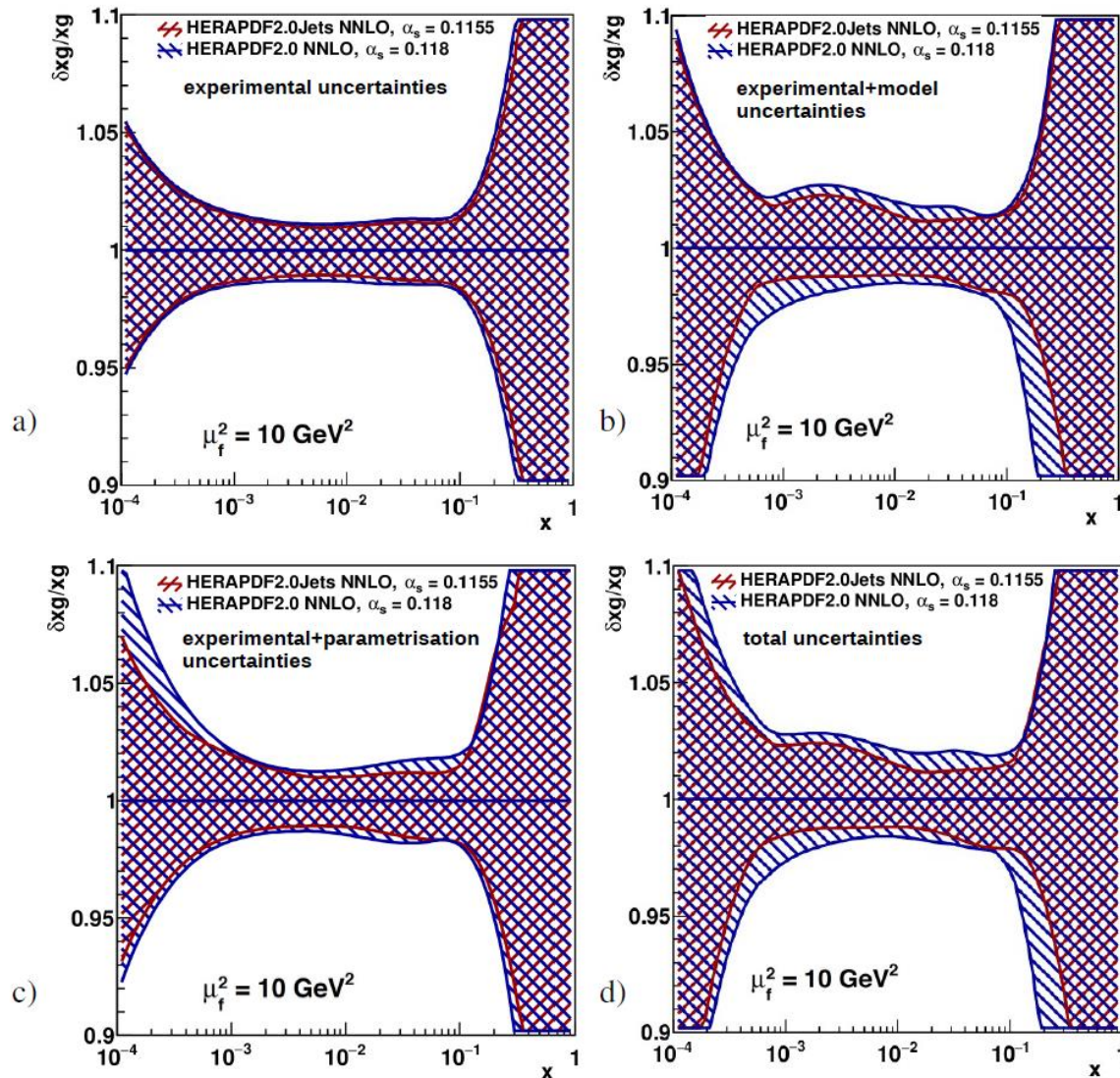
Compare $\chi^2/\text{d.o.f} = 1363/1131 = 1.205$ for
HERAPDF2.0NNLO

Also look at the lower value
 $\alpha_s(M_Z) = 0.1155$ — the
change affects gluon PDF



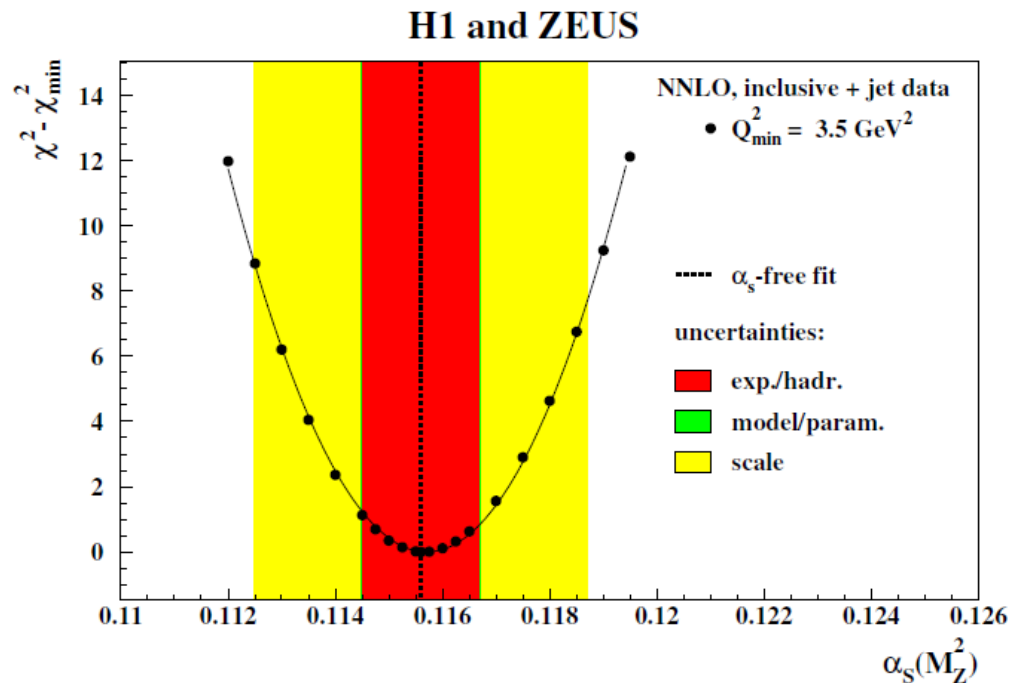
$\chi^2 = 1614$ for fixed $\alpha_s(M_Z) = 0.115$
1363 data points, 1349 degrees
of freedom,
 $\chi^2/\text{d.o.f} = 1.197$

We also compare the uncertainties of the **new Jets fit** and the **inclusive NNLO fit**



Reduction of high- x gluon uncertainties is due to the jet data

Reduction of low- x gluon uncertainties is due to reduced model uncertainties in variations of M_c and μ_{f0}^2



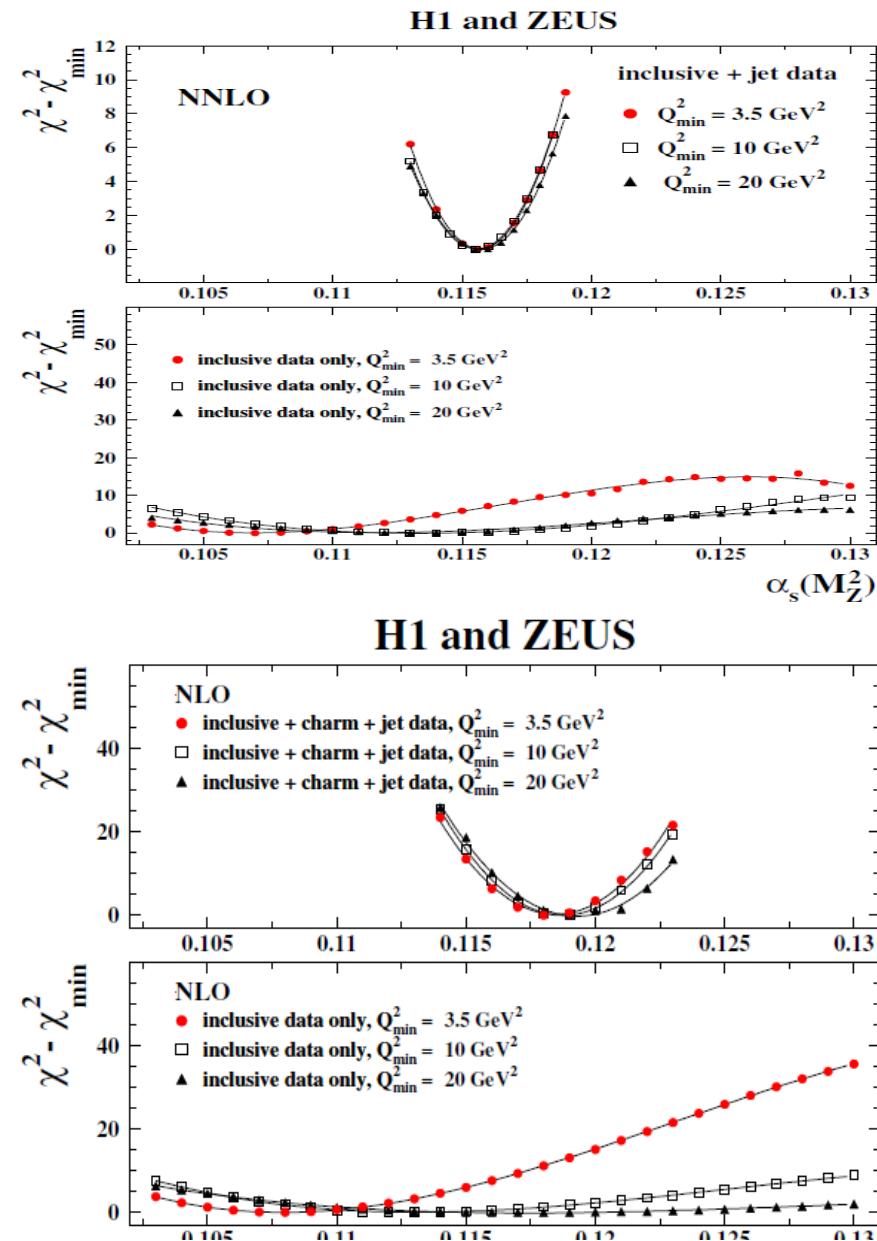
The black points show the result of a scan of the χ^2 of the PDF fit for fixed values of $\alpha_s(M_Z)$. This is in perfect agreement with a **simultaneous fit of $\alpha_s(M_Z)$ and PDF params.** The fits are repeated with changes in model parameter choices and parametrisation choices and with changes in the choice of scale as discussed on the next slide

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011(\text{exp}) + {}^{+0.0001}_{-0.0002}(\text{model+parametrisation}) \pm 0.0029(\text{scale})$$

NOTE that (exp) now includes hadronisation uncertainties and Scale uncertainties contain the full 7-point variation of factorisation and renormalisation scales by a factor of 2

Sensitivity to Q2 cut and comparison to NLO

We perform scans of the χ^2 vs $\alpha_s(M_Z)$ for harder cuts on the minimum Q^2 entering the fit and compare it with a similar plot in which inclusive only data are used—illustrating the power of jets.



These scans over the NNLO inclusive +jet data are compared to the published scans done at NLO.

But note we are using a different scale choice and slightly different jet data sets. In fact harmonising these choices only serves to increase the NLO to NNLO difference.

With common choices we obtain

$0.1186 \pm 0.0014(\text{exp})$ NLO and

$0.1144 \pm 0.0013(\text{exp})$ NNLO.

The change of the NNLO value from the preferred value of 0.1156 is mostly due to the exclusion of the H1 low Q^2 data and the low- p_T points at high Q^2

Comparison to HERAPDF2.0 NLO

In our previous NLO analysis we had applied **the scale uncertainties as ½ correlated and ½ uncorrelated** between bins and data sets, and if we follow this procedure the scale uncertainty on $\alpha_s(M_Z)$ is **NOW ± 0.0022**

Our present NNLO result using ½ correlated and ½ uncorrelated scale uncertainty

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011(\text{exp})^{+0.0001}_{-0.0002}(\text{model+parametrisation}) \pm 0.0022(\text{scale})$$

Maybe compared with the NLO result

$$\alpha_s(M_Z) = 0.1183 \pm 0.0008(\text{exp}) \pm 0.0012(\text{had})^{+0.0003}_{-0.0005}(\text{mod/param})^{+0.0037}_{-0.0003}(\text{scale})$$

Here we see a considerable reduction in scale uncertainty from NLO to NNLO

Comparison to other HERA DIS jet results

We can compare to the H1 result making a simultaneous PDF and $\alpha_s(M_Z)$ fit to just H1 inclusive and jet data,

This was done for $Q^2 > 10 \text{ GeV}^2$ on both inclusive and jets hence we have re-evaluated our result using this cut (rather than our default 3.5 GeV^2 cut)

The HERA comparable result is

$$\alpha_s(M_Z) = \mathbf{0.1156} \pm 0.0011(\text{exp, had, PDF}) \pm 0.0002(\text{mod/par}) \pm \mathbf{0.0021}(\text{scale})$$

And the earlier H1 result is

$$\alpha_s(M_Z) = \mathbf{0.1147} \pm 0.0011(\text{exp, had, PDF}) \pm 0.0002(3)\text{mod(par)} \pm \mathbf{0.0023}(\text{scale})$$

Conclusions

We have completed the HERAPDF2.0 family by performing an NNLO fit including jet data.

This results in two new PDF sets:

HERAPDF2.0JetsNNLO $\alpha_s(M_Z) = 0.118$ – the PDG value

HERAPDF2.0JetsNNLO $\alpha_s(M_Z) = 0.1155$ – The value favoured by our own fit

The Jet data allow us to constrain $\alpha_s(M_Z)$. Our NNLO value is

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011_{(\text{exp})} {}^{+0.0001}_{-0.0002(\text{model/param})} \pm 0.0029_{(\text{scale})}$$

If we want to compare the NLO result we have to use the same scale uncertainty evaluation,

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011_{(\text{exp})} {}^{+0.0001}_{-0.0002(\text{model/param})} \pm 0.0022_{(\text{scale})}$$

to be compared to the NLO result

$$\alpha_s(M_Z) = 0.1183 \pm 0.0009_{(\text{exp})} \pm 0.0005_{(\text{model/param})} \pm 0.0012_{(\text{had})} {}^{+0.0037}_{-0.0030(\text{scale})}$$

There is a systematic shift of $\alpha_s(M_Z)$ downwards at NNLO and a reduction in scale uncertainties

The HERAPDF approach uses only HERA data

The combination of the HERA data yields a very accurate and consistent data set for four different processes: e^+p and e^-p Neutral and Charged Current reactions; and for e^+p Neutral Current at four different beam energies

The use of the single consistent data set allows the usage of the conventional χ^2 tolerance $\Delta\chi^2 = 1$ when setting 68%CL experimental errors

NOTE the use of a pure proton target means no need for heavy target/deuterium corrections.

d-valence is extracted from CC e^+p without assuming d in proton = u in neutron

All data are at high W (> 15 GeV), so high- x , higher twist effects are negligible.

HERAPDF evaluates model uncertainties and parametrisation uncertainties in addition to experimental uncertainties

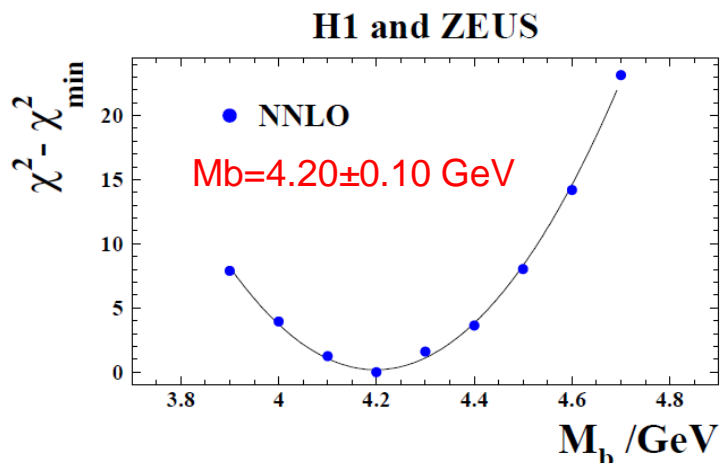
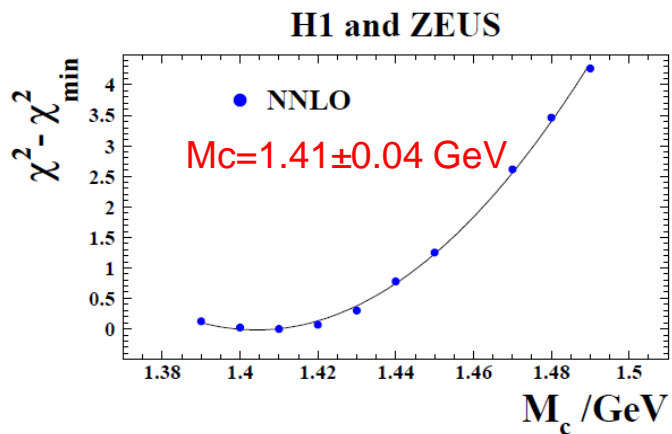
HERAPDF2.0 is based on the new final combination of HERA-I and HERA-II data which supersedes the HERA-I combination and supersedes all previous HERAPDFs

HERAPDF2.0Jets fits add HERA inclusive jet and dijet data to this at both low and high- Q^2

Since the publication of HERAPDF2.0 we also have **NEW HERA combined charm and beauty data Eur.Phys.J C78(2018)473**

This affects the evaluation of the optimal charm and beauty masses

Heavy quark coefficient functions are evaluated by the Thorne Roberts Optimized Variable Flavour Number scheme



We perform χ^2 scans against M_c and M_b using inclusive and heavy flavour data:

- We start with $\alpha_s(M_Z) = 0.118$ as usual and the standard HERAPDF 2.0 parametrisation.
- perform the scan, adopt the resulting values
- And then fit for $\alpha_s(M_Z)$ including jet data
- Since a new value $\alpha_s(M_Z) = 0.1156$ is obtained (See slide 9) we then revisit these scans obtaining very slightly different M_c , M_b values shown here and then
- refit for $\alpha_s(M_Z)$ using these new M_b , M_c value – $\alpha_s(M_Z) = 0.1156$ unchanged
- Then re-check the parametrisation with the new M_c, M_b , $\alpha_s(M_Z) = 0.1156$ AND jet data added—(after all there are 218 new jet data points)
- Previous parametrisation confirmed
- Hence no further iterations needed

HERAPDF specifications: parameterisation

$$xf(x) = Ax^B(1-x)^C(1+Dx+Ex^2)$$

$$xg(x) = A_g x^{B_g} (1-x)^{C_g} - A'_g x^{B'_g} (1-x)^{C'_g},$$

The effect of this negative term is investigated

$$xu_v(x) = A_{u_v} x^{B_{u_v}} (1-x)^{C_{u_v}} (1 + E_{u_v} x^2),$$

$$xd_v(x) = A_{d_v} x^{B_{d_v}} (1-x)^{C_{d_v}},$$

$$x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1-x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x), \quad \text{Ubar=ubar}$$

$$x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1-x)^{C_{\bar{D}}}. \quad \text{Dbar=dbar+sbar}$$

- Additional constraints

- A_{u_v}, A_{d_v}, A_g : constrained by the quark-number sum rules and momentum sum rule

- $B_{\bar{U}} = B_{\bar{D}}, \quad A_{\bar{U}} = A_{\bar{D}}(1-f_s) \quad \text{dbar=ubar at low-x}$

- $x\bar{s} = f_s x\bar{D} \quad \text{at starting scale, } f_s = 0.4$

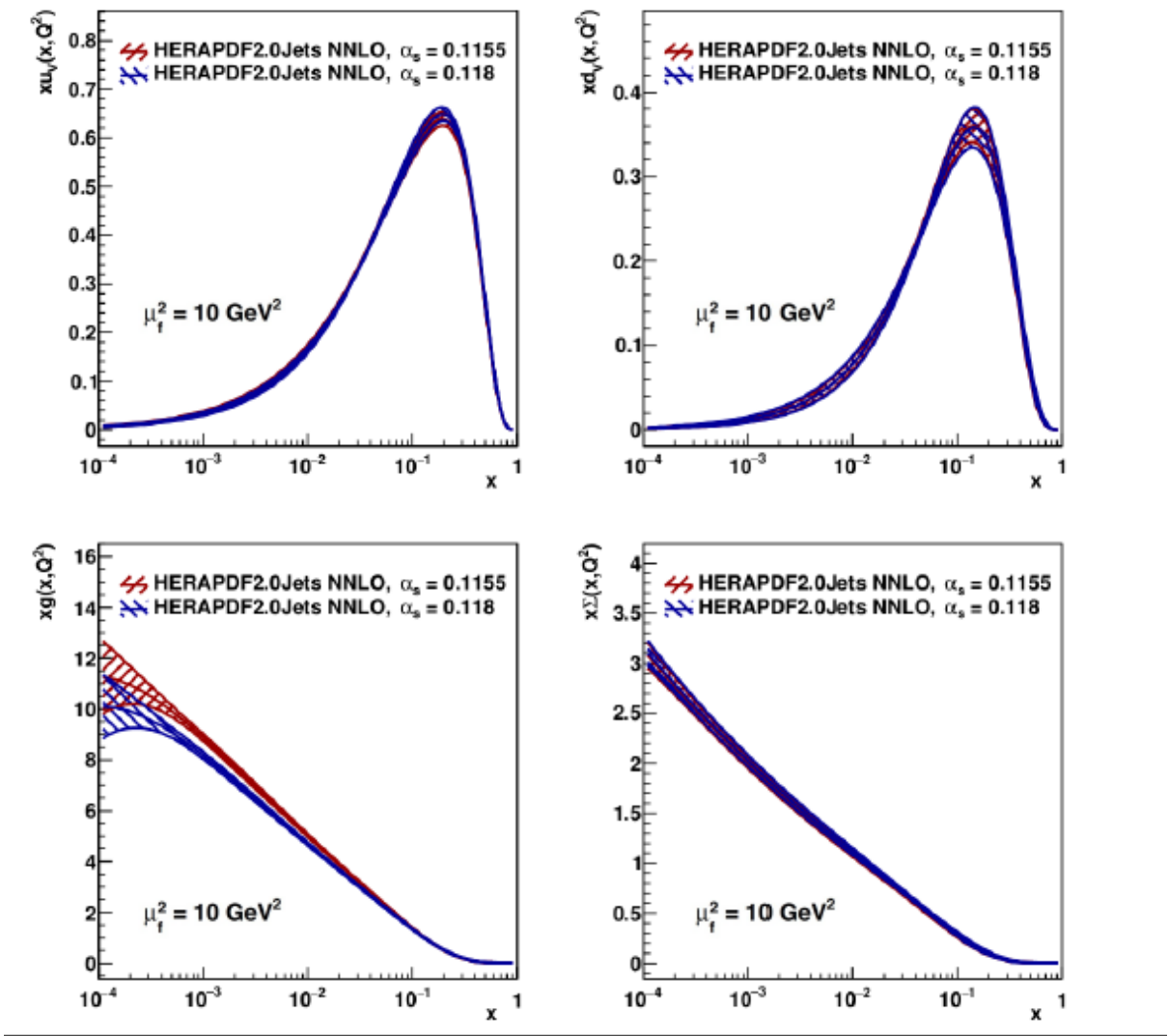


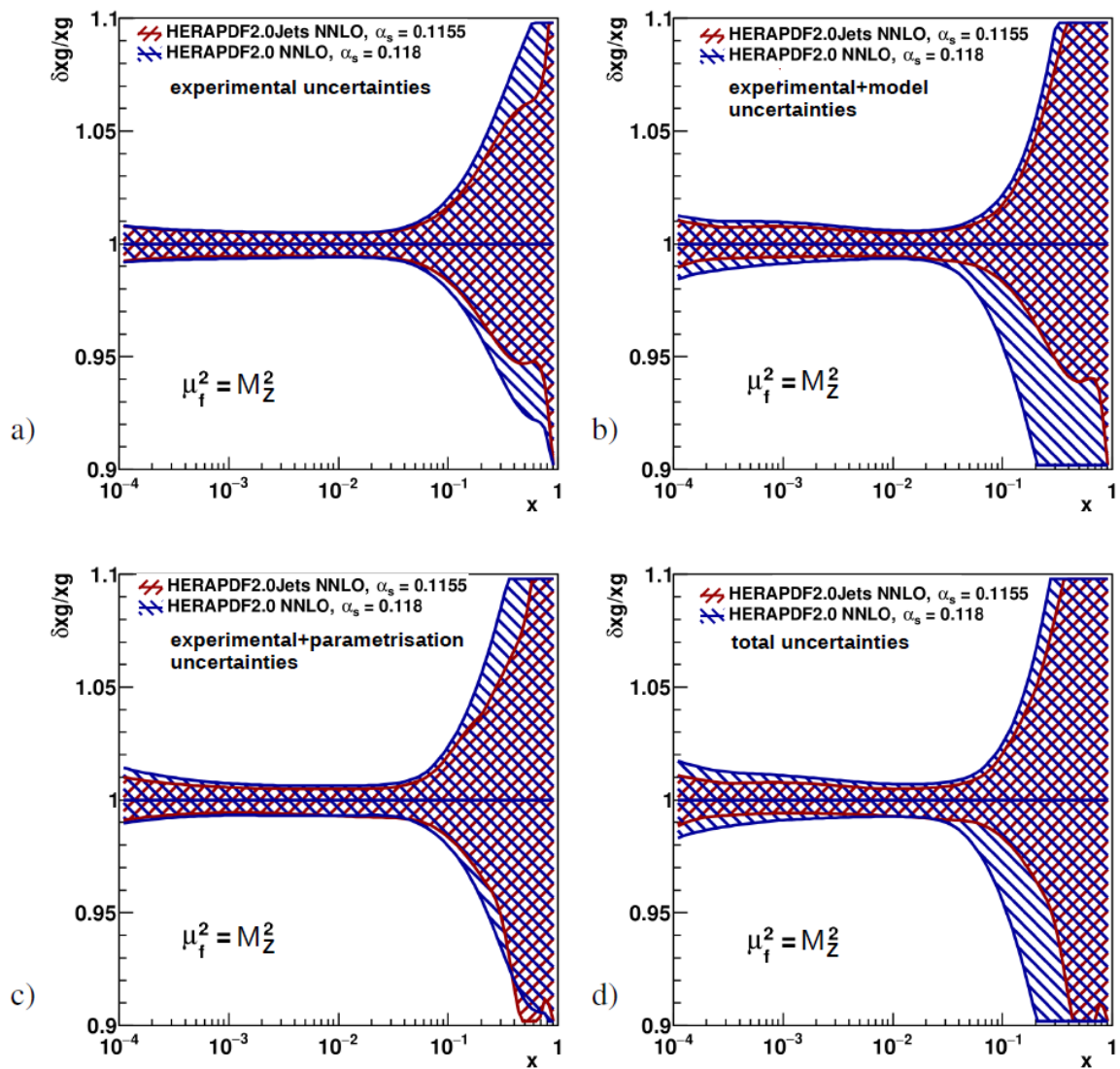
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As usual we start with a minimal number of parameters and add more one at a time until the χ^2 no longer improves. **Parametrisation variations adding extra parameters which can change PDF shape but do not improve χ^2 are part of the uncertainty**

Compare PDFs for

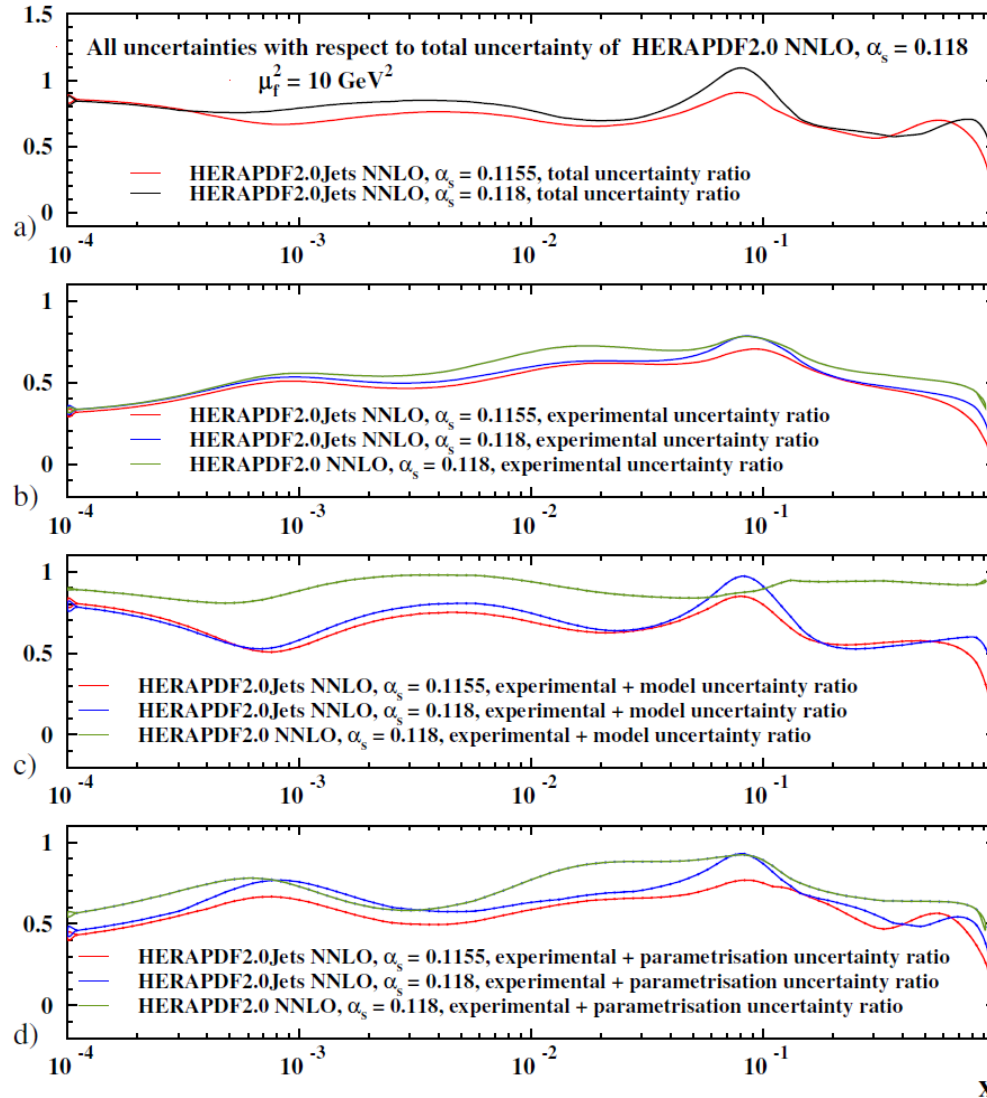
$\alpha_s(M_Z) = 0.1155$ and
 $\alpha_s(M_Z) = 0.118$





Here are some new ways of showing this, where ratios of uncertainties for the new fits to the published HERAPDF2.0 NNLO at $\alpha_s(M_Z) = 0.118$ are shown

H1 and ZEUS



For total uncertainties

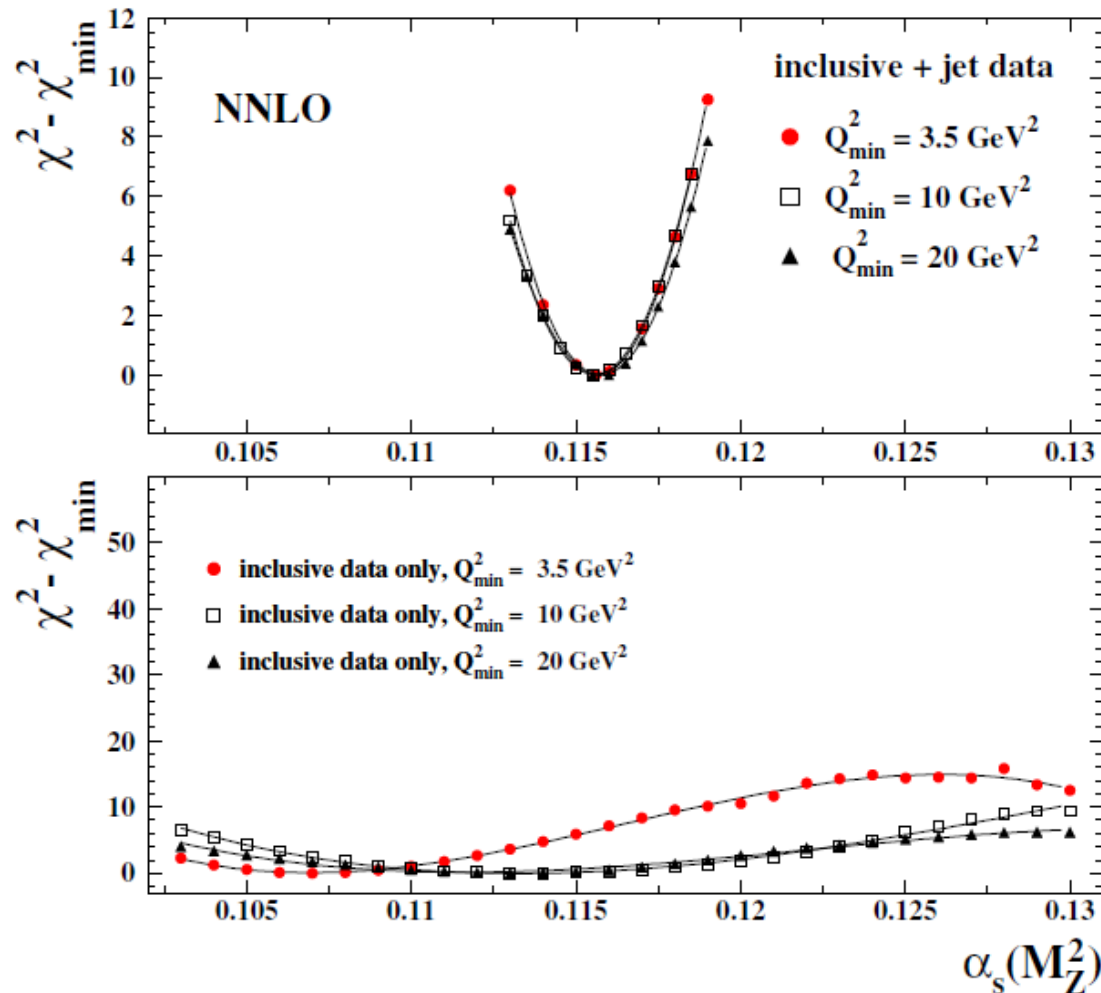
For the experimental uncertainties, which have barely changed

For the exp + model uncertainties, which have improved

For the exp+parametrisation uncertainties, which have improved a little

There is little difference between the uncertainties of the new fit for the two values of $\alpha_s(M_Z)$, but the best fit value gives marginally smaller uncertainties

H1 and ZEUS



We perform scans of the χ^2 vs $\alpha_s(M_Z)$ for harder cuts on the minimum Q^2 entering the fit and compare it with a similar plot in which inclusive only data are used— illustrating the power of jets.

A further check on the dependence of the value of $\alpha_s(M_Z)$ on the parametrisation was made such that the negative term in the gluon parametrisation was removed. The value $\alpha_s(M_Z) = 0.1151 \pm 0.0010(\text{exp})$ was obtained. The addition of a further $(1+Dx)$ term multiplied into the main gluon term was also tried resulting in $\alpha_s(M_Z) = 0.1151 \pm 0.0010(\text{exp})$, both compatible with our central result.

Some remarks on NLO to NNLO comparison- (not in the paper)

Our present NNLO result using $\frac{1}{2}$ correlated and $\frac{1}{2}$ uncorrelated scale uncertainty

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011(\text{exp})^{+0.0001}_{-0.0002}(\text{model+parametrisation} \pm 0.0022(\text{scale}))$$

where “exp” denotes the experimental uncertainty which is taken as the fit uncertainty, including the contribution from hadronisation uncertainties.

Maybe compared with the NLO result

$$\alpha_s(M_Z) = 0.1183 \pm 0.0008(\text{exp}) \pm 0.0012(\text{had})^{+0.0003}_{-0.0005}(\text{mod/param})^{+0.0037}_{-0.003}(\text{scale})$$

BUT

- the choice of scale was different;
- the NLO result did not include the recently published H1 low- Q^2 inclusive and dijet data [28];
- the NLO result did not include the newly published low p_T points from the H1 high- Q^2 inclusive data;
- the NNLO result does not include trijet data;
- the NNLO result does not include the low p_T points from the ZEUS dijet data;
- the NNLO analysis imposes a stronger kinematic cut $\mu > 10 \text{ GeV}$
- the treatment of hadronisation uncertainty differs.

All these changes with respect to the NLO analysis had to be made to create a consistent environment for a fit at NNLO. at the same time, an NLO fit cannot be done under exactly the same conditions as the NNLO fit since the H1 low Q^2 data cannot be well fitted at NLO. However, an NLO and an NNLO fit can be done under the common conditions:

An NLO and an NNLO fit can be done under the common conditions:

- choice of scale, $\mu_f^2 = \mu_r^2 = Q^2 + p_T^2$;
- exclusion of the H1 low- Q^2 inclusive and dijet data;
- exclusion of the low- p_T points from the H1 high- Q^2 inclusive jet data;
- exclusion of trijet data;
- exclusion of low- p_T points from the ZEUS dijet data;
- exclusion of data with $\mu < 10$ GeV
- hadronisation uncertainties treated as correlated systematic uncertainties as done in the NNLO analysis.

The values of $\alpha_s(M_Z)$ obtained for these conditions are:

$0.1186 \pm 0.0014(\text{exp})$ NLO and $0.1144 \pm 0.0013(\text{exp})$ NNLO.

The change of the NNLO value from the preferred value of 0.1156 is mostly due to the exclusion of the H1 low Q^2 data and the low- p_T points at high Q^2

What do we mean when we say the H1 low Q^2 jets cannot be well fitted at NLO?

Simply this, that at NNLO the increase in overall χ^2 of the fit when the 74 data pts of these data are added is ~ 80 (exact value depends on $\alpha_s(M_Z)$ and on scale choice)

Whereas at NLO the increase in overall χ^2 of the fit when the 74 data pts of these data are added is ~ 180 .

Comparison to other HERA DIS jet results

The H1 NNLO jet study using fixed PDFs

H1 jets	$\mu > 2m_b$	0.1170	(9) _{exp}	(7) _{had}	(5) _{PDF}	(4) _{PDFα_s}	(2) _{PDFset}	(38) _{scale}
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Using a similar break up of uncertainties and similar μ cut **our result is**

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011(\text{exp+had+PDF})^{+0.0001}_{-0.0002}(\text{model+parametrisation}) \pm 0.0029(\text{scale})$$

But these are results for fixed PDFs so we also compare to the H1 result making a simultaneous PDF and $\alpha_s(M_Z)$ fit to just H1 inclusive and jet data,

0.1147	(11) _{exp,NP,PDF}	(2) _{mod}	(3) _{par}	(23) _{scale}
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This was done for $Q^2 > 10 \text{ GeV}^2$ on both inclusive and jets hence we have re-evaluated our result using this cut (rather than the default 3.5 GeV^2 cut)

Our comparable result is

$$\alpha_s(M_Z) = \mathbf{0.1156} \pm 0.0011(\text{exp, had, PDF}) \pm 0.0002(\text{mod/par}) \pm \mathbf{0.0021}(\text{scale})$$

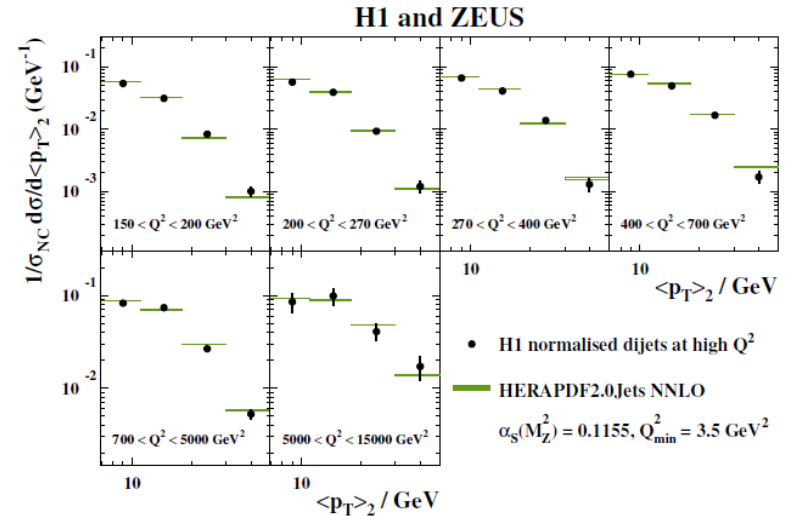
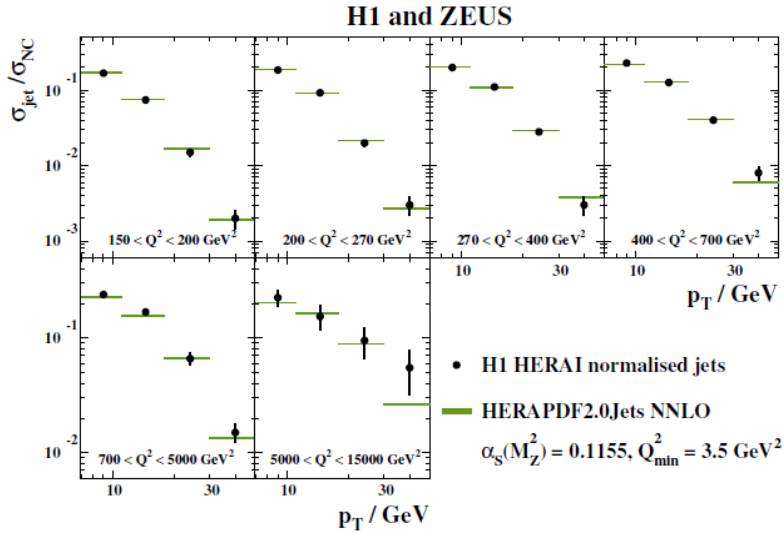
The NNLOjet $\alpha_s(M_Z)$ extraction using fixed PDFs

HERA inclusive jets	$\mu > 2m_b$	0.1171	(9) _{exp}	(5) _{had}	(4) _{PDF}	(3) _{PDFα_s}	(2) _{PDFset}	(33) _{scale}
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Our result (again) can be compare to the NNLOjet result for $\mu > 2m_b$

$$\alpha_s(M_Z) = 0.1156 \pm 0.0011(\text{exp+had+PDF})^{+0.0001}_{-0.0002}(\text{model+parametrisation}) \pm 0.0029(\text{scale})$$

Examples of data and theory prediction and ratios for a couple of data sets—



Or in ratio

